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APPLYING ENGINEERING MATHEMATICS

UNIT CODE: CON/CU/BUT/CC/01/6

Relationship to Occupational Standards

This unit addresses the unit of competency: **Apply engineering mathematics**

1.1 Introduction of the Unit of Learning / Unit of Competency

This unit describes the competencies required by a technician in order to apply algebra apply trigonometry and hyperbolic functions, apply complex numbers, apply coordinate geometry, carry out binomial expansion, apply calculus, solve ordinary differential equations, carry out mensuration, apply power series, apply statistics, apply numerical methods, apply vector theory and apply matrix.

1.2 Summary of Learning Outcomes

1. Apply Algebra
2. Apply Trigonometry and hyperbolic functions
3. Apply complex numbers
4. Apply Coordinate Geometry
5. Carry out Binomial Expansion
6. Apply Calculus
7. Solve Ordinary differential equations
8. Carry out Mensuration
9. Apply Power Series
10. Apply Statistics
11. Apply Numerical methods
12. Apply Vector theory
13. Apply Matrix

1.2.1 Learning Outcome No.1 Apply Algebra

1.2.1.1 Introduction to the learning outcome

This learning outcome specifies the content of competencies required to Apply Algebra.It includes; Base and Index, Law of Indices, Indicial equations, Law of logarithms, Logarithmic equations, Conversion of bases, Use of calculator, Reduction of equations, Solution of equations reduced to quadratic form, Solutions of simultaneous linear equations in three unknowns, Solutions of problems involving AP and GP.

1.2.1.2 Performance Standard

1.1 Calculations involving Indices are performed as per the concept.
1.2 Calculations involving Logarithms are performed as per the concept.
1.3 Scientific calculator is used in solving mathematical problems in line with manufacturer's manual.
1.4 Simultaneous equations are performed as per the rules.
1.5 Quadratic equations are calculated as per the concept

1.2.1.3 Information Sheet

a) Definitions

Algebra is the study of mathematical symbols and the rules for manipulating these symbols; it is a unifying thread of almost all of mathematics. It includes everything from elementary equation solving to the study of abstractions such as groups, rings, and fields.

Well, in Algebra we don't use blank boxes, we use a **letter** (usually an x or y, but any letter is fine). So we write:

$$x - 2 = 4$$

It is really that simple. The letter (in this case an x) just means "we don't know this yet", and is often called the **unknown** or the **variable**.

And when we solve it, we write:

$$x = 6$$

b) Content

❖ **Indices**

An index number is a number which is raised to a power. The power, also known as the index, tells you how many times you have to multiply the number by itself. For example, 2^5 means that you have to multiply 2 by itself five times = 2×2×2×2×2 = 32

- **Laws of indices-**

There are six laws of indices-:

Law no. 1

This is the First law of Indices, which demonstrates that **when multiplying two or more numbers having the same base, the indices are added**.

$$x^n . x^m = x^{n+m}$$

Law no. 2

This is the second law of indices, which demonstrates that **when dividing two numbers having the same base, the index in the denominator is subtracted from the index in the numerator**.

$$\frac{x^n}{x^m} = x^n - x^m = x^{n-m}$$

Law no. 3

This is the third law of indices, which demonstrates that **when a number which is raised to a power is raised to a further power, the indices are multiplied**.

$$(x^n)^m = x^{m.n}$$

Law no. 4

This is the fourth law of indices, which states that **when a number has an index of 0, its value is 1**.

$$x^0 = 1$$

Law no. 5

This is the fifth law of indices, which demonstrates that **a number raised to a negative power is the reciprocal of that number raised to a positive.**

$$x^{-n} = \frac{1}{x^n}$$

Law no. 6

This is the sixth law of indices, which demonstrates that **when a number is raised to a fractional power the denominator of the fraction is the root of the number and the numerator is the power**.

$$x^{\frac{n}{m}} = \sqrt[m]{x^n}$$

Application of rules of indices in solving problems.

$y^a \times y^b = y^{a+b}$

Examples -1 $y^a \div y^b = y^{a-b}$

$2^4 \times 2^8 = 2^{12}$

$5^4 \times 5^{-2} = 5^2$

Example - 2 $\frac{ym}{n} = (n\sqrt{y})m$

$2-3 = \frac{1}{23} = \frac{1}{8}$

$3-1 = \frac{1}{3}$

Example- 3 $(y^n)^m = y^{nm}$

$16^{1/2} = \sqrt{16} = 4$

$8^{2/3} = (\sqrt[3]{8})^2 = 4$

Example- 4 $(x^n)^m = x^{m.n}$

$2^5 + 8^4$

$= 25 + (2^3)^4$

$= 2^5 + 2^{12}$

Example- 5 $y^0 = 1$

$5^0 = 1$

- **Logarithms**

If a is a positive real number other than 1, then the logarithm of **x** with base **a** is defined by:

$y = \log_a x$ or $x = a^y$

- **Laws of logarithms**

There are Three Laws of Logarithms -:

(i) ***To multiply two numbers:***

$\mathbf{\log (A \times B) = \log A + \log B}$

or $\boldsymbol{log_a(xy) = log_a x + log_a y}$

The following may be checked by using a calculator:

$\lg 10 = 1$

Also, $\log 5 + \log 2 = 0.69897...$

$+0.301029... = 1$

Hence, $\log(5 \times 2) = \log 10 = \log 5 + \log 2$

(ii) ***To divide two numbers:***

$\boldsymbol{Log(\frac{A}{B}) = \log A - \log B}$

Or $\boldsymbol{log_a\left(\frac{x}{y}\right) = log_a x - log_a y}$

The following may be checked using a calculator:

$In(\frac{5}{2}) = \ln 2.5 = 0.91629...$

Also, $\ln 5 - \ln 2 = 1.60943... - 0.69314...$

$= 0.91629...$

Hence,

$In(\frac{5}{2}) = \ln 5 - \ln 2$

(iii) To raise a number to a power:

log *An* = *n*log *A*

or $\boldsymbol{log_a(x^n) = nlog_a x}$ for every real number

The following may be checked using a calculator:

lg 52 = lg 25 = 1.39794. . .

Also, 2 lg 5 = 2×0.69897. . .= 1.39794. . .

Hence, lg 52 = 2lg 5

- **Simultaneous equations with three unknowns**
  Simultaneous equations are equations which have to be solved together to find the unique values of the unknown quantities which are time for each of the equations. Two methods of solving simultaneous equations analytically are:
  (i) By substitution
  (ii) By elimination

Example-1

Solve the following simultaneous equation by substitution methods

$3x - 2y + \ 2y + z = 1 \ldots \ldots \ldots .. (\text{i})$

$x - 3y + 2z = 13 \ldots \ldots \ldots \ldots \ldots .. (ii)$

$4x - 2y + 3z = 17 \ldots \ldots \ldots \ldots . \ldots (iii)$

From equation (ii) $x = 13 + 3y - 2z$

Substituting these expression $(13 + 3y - 2z \quad for\ x \ \ gives)$

$3(13 + 3y - 2z) + 2y + z = 1$

$39 + 9y - 6z + 2y + x = 1$

$11y - 5z = \ -38 \ldots \ldots \ldots \ldots \ldots .. (iv)$

$4(13 + 3y - 2z) + 3z - 2y = 17$

$52 + 12y - 8z + 3z - 2y = 17$

$10y - 5z = -35 \ldots .. (v)$

Solve equation (iv) and (v) in the usual way,

From equations (iv) $5z = 11y + 38; z = \frac{11y+38}{5}$

Substituting this in equation (v) gives

$$10y - 5\left(\frac{11y + 38}{5}\right) = -35$$

$$10y - 11y - 38 = -35$$

$$-y = -35 + 38 = 3$$

$$y = -3$$

$$z = \frac{11y + 38}{5} = \frac{-33 + 38}{5} = \frac{5}{5} = 1$$

But $x = 13 + 3y - 2z$

$$x = 13 + 3(-3) - 2(1)$$

$$=13 - 9 - 2$$

$$=2$$

Therefore, $x = 2,\ y = -3\ and\ z = 1$ is the required solution?

For more worked examples on substitution and elimination method refer to Engineering Mathematics by A.K Stroud.

- **Quadratic Equations**

Quadratic equation is one in which the highest power of the unknown quantity is 2. For example, $2x^2 - 3x - 5 = 0$ is a quadratic equation? The general form of a quadratic equation is $ax^2 + bx + c = 0, where\ a, b\ and\ c$ are constants and $a \neq 0\ of\ solving\ quadratic\ equations$.

- By factorization (where possible)
- By completing the square
- By using quadratic formula
- Graphically

- ***Solution of Quadratic equations by factorization method***

Multiplying out $(x+1)(x-3)$ gives $x^2-3x+x-3$ i.e. x^2-2x-3. The reverse process of moving from x^2-2x-3 to $(x+1)(x-3)$ is called **factorising**.

Example -1

Solve the quadratic equation $x^2 - 4x + 4 = 0$ by factorization method

Solution

$$x^2 - 4x + 4 = 0$$

$$x^2 - 2x - 2x + 4 = 0$$

$$x(x - 2) - 2(x - 2) = 0$$

$$(x - 2)(x - 2) = 0$$

i.e., $x - 2 = 0\ \ or\ x - 2 = 0$

$$x = 2\ \ or\ x = 2$$

i.e., the solution is $x = 2$(twice)

- ***Solution of Quadratic equations by Completing the Square method***

An expression such as x^2 or $(x+2)^2$ or $(x-3)^2$ is called a **perfect square**.

If $x^2 = 3$ then $x = \pm\sqrt{3}$

If $(x+2)^2 = 5$ then $x+2 = \pm\sqrt{5}$ and $x = -2\pm\sqrt{5}$

If $(x-3)^2 = 8$ then $x-3 = \pm\sqrt{8}$ and $x = 3\pm\sqrt{8}$

Hence, if a quadratic equation can be rearranged so that one side of the equation is a perfect square and the other side of the equation is a number, then the solution of the equation is readily obtained by taking the square roots of each side as in the above examples. The process of rearranging one side of a quadratic equation into a perfect square before solving is called '**completing the square**'.

$(x+a)^2 = x^2 + 2ax + a^2$

Thus, in order to make the quadratic expression x^2+2ax into a perfect square, it is necessary to add (half the coefficient of x)^2, i.e. $\left(\frac{2a}{2}\right)^2$ or a^2

For example, $x^2 + 3x$ becomes a perfect square by Adding $\left(\frac{3}{2}\right)^2$,i.e.

$$x^2+3x+\left(\frac{3}{2}\right)^2 = \left(x+\frac{3}{2}\right)^2$$

Example-1

Solve $2x^2 + 5x = 3$ by completing the square

The procedure is as follows.

(i) Rearrange the equation so that all terms are on the same side of the equals sign (and the coefficient of the $x2$ term is positive).

Hence,

$$2x^2 + 5x - 3 = 0$$

(ii) Make the coefficient of the $x2$ term unity. In this case this is achieved by dividing throughout by 2.

Hence,

$$\frac{2x^2}{2} + \frac{5x}{2} + \frac{3}{2} = 0$$

i.e. $$x^2 + \frac{5x}{2} + \frac{3}{2} = 0$$

(iii) Rearrange the equations so that the x^2 and x terms are on one side of the equals sign and the constant is on the other side.

Hence,

$$x^2 + \frac{5}{2}x = \frac{3}{2}$$

(iv) Add to both sides of the equation (half the coefficient of x)^2. In this case the coefficient of x is $\left(\frac{5}{2}\right)$ Half the coefficient squared is therefore $\left(\frac{5}{4}\right)^2$

Thus,

$$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = \frac{3}{2} + \left(\frac{5}{4}\right)^2$$

The LHS is now a perfect square, i.e.

$$\left(x + \frac{5}{4}\right)^2 = \frac{3}{2} + \left(\frac{5}{4}\right)^2$$

(v) Evaluate the RHS. Thus,

$$\left(x + \frac{5}{4}\right)^2 = \frac{3}{2} + \frac{25}{16} = \frac{24+25}{2} = \frac{49}{16}$$

(vi) Take the square root of both sides of the equation (remembering that the square root of a number gives a ± answer). Thus,

$$\sqrt{\left(x + \frac{5}{4}\right)^2} = \sqrt{\left(\frac{49}{16}\right)}$$

i.e. $x + \frac{5}{2} = \pm\frac{7}{4}$

(vii) Solve the simple equation. Thus,

$$.\ x = -\frac{5}{2} \pm \frac{7}{4}$$

$$.\ x = -\frac{5}{2} + \frac{7}{4} = \frac{2}{4} = \frac{1}{2} \text{ or } 0.5$$

$$x = -\frac{5}{2} - \frac{7}{4} = -\frac{12}{4} = -3$$

Hence, $\boldsymbol{x = 0.5}$ or $\boldsymbol{x = -3}$; i.e. **the roots of the equation** $\boldsymbol{2x^2 + 5x = 3}$ are **0.5** and **−3**

For more worked examples on how to solve quadratic equations using, factorization, completing the square, quadratic formula refers to basic engineering mathematics by J.O Bird, Engineering mathematical by K.A Stroud etc.

- **Arithmetic Projection(A.P.) and Geometric Projection (G.P.)**
  - ***Arithmetic projection (A.P.)***

    When a sequence has a constant difference between successive terms it is called an **arithmetic progression** (often abbreviated to AP).

Examples-1

Find the general term of the A.P given by x + b, x+3b, x+5b….

Solution: here a= x + b,

d=x+3b - (x + b)

d=x+3b-x-b

Therefore, d=2b

The general term is given by the formula

$a_n = a + (n-1) d$

$a_n = x + b + n (n - 1)2b$

therefore, $a_n = x+(2n-1) b$

Example-2

show that a^2, b^2, c^2 are in A.P if $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP

Solution: since $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP

The common difference between the terms will be the same

Therefore $\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

$$\frac{b-a}{(c+a)(b+c)} = \frac{c-b}{(c+a)(a+b)}$$

$$\frac{b-a}{b+c} = \frac{c-b}{a+b}$$

Cross multiply

(b + a) (a -b) = (b + c) (c- b)

i.e., $b^2 - a^2 = c^2 - b^2$

Thus a^2, b^2, c^2 are in AP

Example-3

The sum of 4^{th} and 8^{th} term of an AP is 24 and the sum of the 6^{th} and 10^{th} terms is 34. Find the first term and the common difference of the AP

Solution: Here $a_4 + a_8 = 24$ and $a_6 + a_{10} = 34$

A +3d +a + 7d = 24……$[used\ the\ formula\ an = a + (n-1)d]$

2a + 10d =24

Divide by 2

a +5d = 12………… (1)

now we have, a6 + a10 = 34

a + 5d + a + 7d = 34

2a + 14d = 34

a + 7d = 17 ………… (2)

solving equation 1 and 2 simultaneously we get

d = $\frac{5}{2}$ and a = $\frac{-1}{2}$

Example-4

Write GP if a = 128 and common ratio r = - $\frac{1}{2}$

Solution: general form of GP = a, ar, ar^2, ar^3………

128, -64, 32, -16, ……………

Formula for n^{th} term GP is $a_n = ar^{n-1}$

Example-5

Find the 10^{th} and the n^{th} term of a geometric sequence $\frac{7}{2}, \frac{7}{4}, \frac{7}{8}, \frac{7}{16}$,………

Formula for the nth term GP is $a_n = ar^{n-1}$

10^{th} term of the GP = $\left(\frac{7}{2}\right) \times \left(\frac{1}{9}\right)^9 = \left(\frac{7}{2}\right) \times \left(\frac{1}{514}\right) = \left(\frac{7}{1024}\right)$

1.2.1.4 Learning Activities.

Learning Outcome, No 1: Apply Algebra	
Learning Activities	**Special Instructions**
• Perform calculations involving indices as per the concept • Perform calculations involving logarithms as per the concept • Solving simultaneous as per the rules • Solving quadratic equations as per the concept	Carryout various exercises in order to gain confidence in algebra

1.2.1.5 Self-Assessment

This section must be related with the Performance Criteria, Required Knowledge and Skills and the Range as stated in the Occupational Standards.

Q1. (a) Solve the following by factorization

$(i)\ \ x^2 + 2x - 8 = 0$

(b) Solve by completing the square the following quadratic equations

(i) $2x^2 + 3x - 6$

Q.2 Simplify as far as possible

(i) $log(x^2 - 4x + 3) - log(x - 1)$

(ii) $2log(x - 1) - log(x^2 - 1)$

Q.3 Solve the following simultaneous equations by the method of substitution

$$x + 3y - z = 2$$

$$2x - 2y + 2z = 2$$

$$4x - 3y + 5z = 5$$

Q.4 Simplify the following

$$F = (2_x^{\frac{1}{2}} y^{\frac{1}{4}})^4 \div \sqrt{\frac{1}{9} x^2 y^6}\ x\ (4\sqrt{x^2 y^4})^{-\frac{1}{2}}$$

1.2.1.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

HELM - Helping Engineers Learn Mathematics Workbooks

SIGMA - Summary sheets covering key areas in Mathematics

Lecture Notes - For the engineering analysis modules

Links to (online) textbooks available in the LIS

Links to external sites

1.2.1.7 References (APA)

Greenberg, M.D. (1998), Advanced Engineering Mathematics, 2nd ed., Prentice Hall (Upper Saddle River, N.J).

Hildebrand, F.B. (1974), Introduction to Numerical Analysis, 2nd ed., McGraw-Hill (New York).

Hildebrand, F.B. (1976), Advanced Calculus for Applications, 2nd ed., Prentice-Hall (Englewood Cliffs, NJ).

Hoyland, A., Rausand, M. (1994), System Reliability Theory: Models and Statistical Methods, John Wiley (New York).

Kaplan, W. (1984), Advanced Calculus, 3rd ed., Addison-Wesley (Cambridge, MA).

Kreyszig, E. (1999), Advanced Engineering Mathematics, 8th ed., John Wiley (New York).

O'Neil, P.V. (1995), Advanced Engineering Mathematics, 4th ed., PWS-Kent Pub. (Boston).

1.2.1.8 Suggested answers to self-assessment

Q1. (a) Solve the following by factorization $x^2 + 2x - 8 = 0$

Solution

$$x^2 - 2x + 4x - 8 = 0$$

$$x(x-2) + 4(x-2) = 0$$

$$(x+4)(x-2) = 0$$

$$(x+4) = 0$$

$$x = -4 \quad or$$

$$(x-2) = 0$$

$$x = 2$$

(b) Solve by completing the square the following quadratic equations $2x^2 + 3x - 6 = 0$

$$\frac{2x^2}{2} + \frac{3x}{2} - \frac{6}{2} = 0$$

$$x^2 + \frac{3}{2}x - \frac{6}{2} = 0$$

$$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \frac{6}{2} + \left(\frac{3}{4}\right)^2$$

$$\left(x + \frac{3}{4}\right)^2 = \frac{6}{2} + \left(\frac{3}{4}\right)^2$$

$$\left(x + \frac{3}{4}\right)^2 = \frac{6}{2} + \frac{9}{16} = \frac{48+9}{16} = \frac{57}{16}$$

$$\sqrt{\left(x + \frac{3}{4}\right)^2} = \sqrt{\left(\frac{57}{16}\right)}$$

$$x + \frac{3}{4} = \pm 3.5625 \text{ ans}$$

$x = -\frac{3}{4} \pm 3.5625$

$x = -\frac{3}{4} + 3.5625$ = 2.8125 ans

or $x = -\frac{3}{4} - 3.5625 = -4.3125\ ans$ =

Q.2 Simplify as far as possible $log(x^2 - 4x + 3) - log(x - 1)$

Solution

$$log\left(\frac{x^2-4x+3}{(x-1)}\right)$$

$$log\left(\frac{x^2-x-3x+3}{(x-1)}\right)$$

$$log\left(\frac{x(x-1)-3(x-1)}{(x-1)}\right)$$

$$log\left(\frac{(x-3)(x-1)}{(x-1)}\right)$$

$$log\left(\frac{(x-3)1}{1}\right)$$

$log(x - 3)$ ans

Q.3 Solve the following simultaneous equations by the method of substitution

$$x - y + z = 10 \ldots\ldots(i)$$

$$3x + y + 2z = 34\ldots\ldots\ldots(ii)$$

$$-5x + 2y - z = -14 \ldots\ldots(iii)$$

<u>Solution;</u>

Step 1- make x the subject formula in eq. (i)

$x + y - z = 10$

$x = 10 + y - z$ ……….(i)

Step 2- substitute for x in eq. (ii) and form eq. (iv)

$3(\ 10 + y - z\ ) + y + 2z = 34$

$30 + 3y - 3z + y + 2z = 34$

$4y - z - 4 = 0$ ……(iv)

Step 3- substitute for x in eq. (iii) and form eq. (v)

$-5(\ 10 + y - z\ ) + 2y - z = -14$

$-50 - 5y + 5z + 2y - z = -14$

$-3y + 4z - 36 = 0$……*(v)*

Step 4- Solve for y and z in eq. (iv) and eq.(v)

$$4y - z - 4 = 0 \text{ ......(iv)}$$

$$-3y + 4z - 36 = 0 \text{.......}(v)$$

Step 5- make z the subject in eq. (iv)

$$z = 4y - 4$$

Step 6- substitute for z in eq. (v)

$$-3y + 4(4y - 4) - 36 = 0$$

$$-3y + 16y - 16 - 36 = 0$$

$$13y = 52$$

$$y = \left(\frac{52}{13}\right) = 4$$

$$y = 4 \text{ ans}$$

step 7- substitute for y in eq. (iv) , hence solve for z

$$4y - z - 4 = 0$$

$$4(4) - z - 4 = 0$$

$$16 - z - 4 = 0$$

$$z = 16 - 4$$

$$z = 12 \text{ ans}$$

step 8- substitute for y an z in eq. (i) , hence solve for x

$$x = 10 + y - z$$

$$x = 10 + 4 - 12$$

$$x = 2 \text{ ans}$$

1.2.2 Learning Outcome No.2 - Apply trigonometry and hyperbolic functions

1.2.2.1 Introduction to the learning outcome

This learning outcome specifies the content of competencies required to Apply Trigonometry and hyperbolic functions. It includes; Half -angle formula, Factor formula, Trigonometric functions, Parametric equations, Relative and absolute measures, Measures calculation, Definition of hyperbolic equations, Properties of hyperbolic functions, Evaluations of hyperbolic functions Hyperbolic identities, Osborne's Rule, Ashx + bshx = C equation, One-to-one relationship in functions, Inverse functions for one-to-one relationship, Inverse functions for trigonometric functions, Graph of inverse functions and Inverse hyperbolic functions.

1.2.2.2 Performance Standard

2.1 Calculations are performed using trigonometric rules.
2.2 Calculations are performed using hyperbolic functions.

1.2.2.3 Information Sheet

a) Trigonometric ratios

The three trigonometric ratios derived from a right - angled triangle are the same, cosine and tangent refer to basic engineering mathematics by J.0 Bird to read move about trigonometry ratios.

Solution of right-angled triangles:

To solve a triangle means to find the unknown sides and angles, this is achieved by using the theorem of Pythagoras and or using trigonometric ratios.

<u>Example-1</u>

In a triangle PQR shown below find the length of PQ and PR

$$t\ on\ 38^0 = \frac{PQ}{QR} = \frac{PQ}{7.5}$$

Hence

$$PQ = 7.5\ X\ \tan 38^0 \qquad = 7.5\ x\ 0.7813\ = 5.86\ cm$$

$$\cos 38^0 = \frac{QR}{PR} = \frac{7.5}{PR}$$

$$PR = \frac{7.5}{cos38^0} = \frac{7.5}{0.7880} = 9.518\text{cm}$$

For more worked examples refer to basic engineering mathematics by J.) Bird. Also use it to learn about angles of elevation and depression.

Compound angle formulae

- $\mathbf{Sin}\ (\boldsymbol{A} \pm \boldsymbol{B}) = \boldsymbol{Sin\ A}\ \mathbf{cos}\ \boldsymbol{B} \pm \boldsymbol{Cos\ A}\ \mathbf{sin}\ \boldsymbol{B}$

- $\boldsymbol{Cos}(\boldsymbol{A} \pm \boldsymbol{B}) = \boldsymbol{Cos\ Acos\ B} \mp \boldsymbol{Sin\ A\ Sin\ B}$

- $\mathbf{tan}(\boldsymbol{A} \pm \boldsymbol{B}) = \frac{\mathbf{tan}\ \boldsymbol{A} \pm \mathbf{tan}\ \boldsymbol{B}}{\mathbf{1} \mp \mathbf{tan}\ \boldsymbol{A\ tanB}}$

Refer to technician mathematics book 3 by J.O Bird and learn more about compound angle formulae

Conversion of a cos wt + b sin wt into the general form $R\ Sin(wt + 2)$

Let a Cos wt + b Sin wt = $R\ Sin(wt{+}\propto)$

Expanding the right-hand side using the compound angle formulae gives

A cos wt + b Sin wt = $R(Sinwt\ Cos \propto\ + coswt\ sin \propto)$ =R cos $\propto$ $|Sinwt + R\ Sin\ \propto Coswt$

Equating the coefficients of Cos wt on both sides gives.

$a = R\ Sin \propto i.e\ Sin\ \propto = \frac{a}{R}$

Equating the coefficients of Sin wt on both sides gives

$b = R\ cos \propto\ \ i.e\ Cos\ \propto = \frac{b}{R}$

If the values of $a\ and\ b$ are known, the values of $R\ and\ \alpha$ can be calculated.

From Pythagoras theorem.

Diagram

$R = \sqrt{a^2} + b^2$

And from trigonometric ratios,

$\propto = tan^{-1}\left(\frac{a}{b}\right)$

Example -2

Express 3 $Sin\ \theta + 4\ Cos\theta$ in the general form $R\ Sin(\theta + \alpha)$

Let 3 $Sin\ \theta + 4\ Cos\ \theta = R\ Sin\ (\theta + \alpha)$

Expanding the right-hand side using the compound angle formulae gives

$3\ Sin\ \theta + 4\ Cos\ \theta = R\ [Sin\theta\ Cos\alpha + Cos\ \theta\ Sin\alpha\ ]$

= $R\ cos\alpha\ Sin\ \theta + R\ Sin\ \alpha\ Cos\ \theta$

Equating the coefficient of:

$Cos\theta; == R\ Sin\ \alpha\ i.e\ Sin\ \alpha = \frac{4}{R}$

Sin θ : $3 = R\ Cos\ \alpha\ \ i.e\ \ Cos\ \alpha = \frac{3}{R}$

This the values of $R\ and\ \alpha$ can be evaluated.

Diagram

$$R = \sqrt{4^2 + 3^2 = 5}$$

$$\alpha = tan^{-1}\frac{4}{3} = 53.13^0 or\ \ 233.13^0$$

Since both $Sin\ \alpha\ and\ Cos\ \alpha$ are positive, r lies in the first quadrant where all are positive, hence 233.13^0 is neglected.

Hence

$3\ Sin\theta + 4\ Cos\ \theta = 5\ Sin\ (\theta + 53.13^0)$

Example-3

Solve the equation $3\ Sin\ \theta + 4\ Cos\ \theta = 2$ for values of θ between 0^0 and 360^0 inclusive

Solution

From the example above

$3\ Sin\theta + 4\ Cos\ \theta = 5\ Sin\ (\theta + 53.13^0)$

Thus

$5\ Sin(\alpha + 53.13^0) = 2$

$Sin(\theta + 53.13^0) = \frac{2}{5}$

$\theta + 53.13^0 = Sin^{-1}\frac{2}{5}$

$\theta + 53.13^0 = 23.58^0$ or 156.42^0

$\theta = 23.58^0 - 53.13^0 = -29.55^0$

$=330.45^0$

OR $\theta = 156.42^0 - 53.13^0$

$=103.29^0$

Therefore, the roots of the above equation are 103.29^0 *or* 330.45^0

For more worked examples refer to Technician mathematics book 3 by J.) Bird.

b) Double /multiple angles

For double and multiple angles refer to Technician mathematics by J.O Bird

c) Factor Formulae

For worked exampled refer to Technic mathematics book 3 by J. O Bird, Pure mathematics by backhouse and Engineering mathematics by KA Stroud.

d) Half –angle formulae

Refer to pure mathematics by backhouse and Engineering mathematics by K.A STROUD

e) Hyperbolic functions

Definition of hyperbolic functions, $Sinhx$ $\cosh x$ and $\tanh x$

Evaluation of hyperbolic functions

Hyperbolic identities

Osborne's Rule

Solve hyperbolic equations of the form a $coshx + bSinhx = C$

For all the above refer, to Engineering mathematics by KA Stroud.

1.2.2.4 Learning Activities

Learning Outcome No. 2: Apply trigonometry and hyperbolic functions	
Learning Activities	**Special Instructions**
Perform calculations using trigonometric rules Perform calculations using hyperbolic functions	Carry out various exercises in order to gain confidence in trigonometry and hyperbolic functions

1.2.2.5 Self-Assessment

Q1. A surveyor measures the angle of elevation of the top of a perpendicular building as19^0. He moves 120m nearer the building and measures the angle of elevation as47^0. Calculate the height of the building to the nearest meter.

Q2. Solve the equation $5\cos\theta + 4\,Sin\,\theta = 5$ for values of θ between 0^0 and 360^0 Inclusive.

Q3. Prove the following identifies;

$$(i)\ Cosh\,2x = cosh^2x + Sinh^2x$$

$$(ii) Sinh(x+y) Sinh\,Coshy + coshy\,Sinhx$$

Q4. Solve the equation $3\,Sin\,hx + 4\,Coshx = 5$

1.2.2.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

HELM - Helping Engineers Learn Mathematics Workbooks

SIGMA - Summary sheets covering key areas in Mathematics

Lecture Notes - For the engineering analysis modules

Links to (online) textbooks available in the LIS

Links to external sites

1.2.2.7 References (APA)

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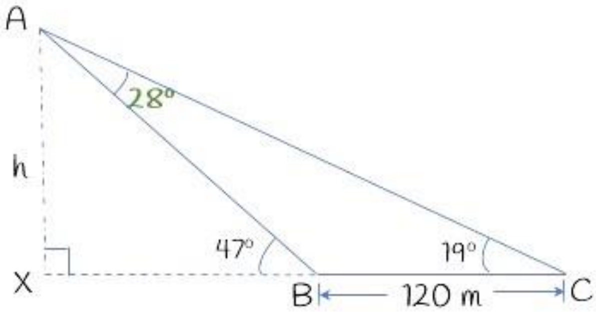
O'Neil, P.V. (1995), Advanced Engineering Mathematics, 4th ed., PWS-Kent Pub. (Boston).

1.2.2.8 Suggested responses to self-assessment

Q1. *A surveyor measures the angle of elevation of the top of a perpendicular building as* 19^0. *He moves 120m nearer the building and measures the angle of elevation as* 47^0. *Calculate the height of the building to the nearest meter.*

Solution 1 ;



$\angle BAC = 47° - 19° = 28°$ (exterior angle = sum of opposite interior angles)

Applying the Sine Rule to $\triangle ABC$,

we have $\frac{BC}{\sin 28°} = \frac{AB}{\sin 19°}$

=> $AB = \frac{BC \sin 19°}{\sin 28°}$

$= 120\,(0.6935) = 83.22$

In right angled $\triangle AXB$,

$h = AB \sin 47°$
$= 83.22 \sin 47°$
$= 60.86$ m

https://ask.manytutors.com/questions/70856

Q2. *Solve the equation* $4\,cos\,\theta + 3\,Sin\,\theta = 5$ *for values of* θ *between* 0^0 *and* 360^0 *Inclusive*

Solution 2

$4\cos\theta + 3\,Sin\,\theta = 5$

$4cos\theta + 3\sqrt{1 - cos^2}\theta = 5$

$3\sqrt{1 - cos^2}\theta = 5 - 4cos\theta$

Squaring both sides

$9(1 - cos^2\theta) = (5 - 4cos^2\theta)^2$

$9(9 - 9cos^2\theta) = (5 - 4cos^2\theta)^2$

$9 - 9cos^2\theta = 25 + 16cos^2\theta - 40cos\theta$

$25 - 16cos^2\theta - 40cos\theta - 9 + 9cos^2\theta = 0$

$25cos^2\theta - 40cos\theta + 16 = 0$

$$(5cos\theta)^2 - 2 \times 5cos\theta \times 4 + 4^2 = 0$$

$$(5cos\theta - 4)^2 = 0$$

$$5cos\theta - 4 = 0$$

$$cos\theta = \frac{4}{5}$$

Substitute for $cos\theta$ in the equation

$$4\cos\theta + 3\,Sin\,\theta = 5$$

$$4 \times \frac{4}{5} + 3\,Sin\,\theta = 5$$

$$\frac{16}{5} - 5 = 3\,Sin\,\theta$$

$$3\,Sin\,\theta = \frac{16-25}{5}$$

$$Sin\,\theta = -\frac{9}{15}$$

$$Sin\,\theta = \frac{-3}{15}$$

Then, $$tan\,\theta = \frac{sin\,\theta}{cos\,\theta}$$

$$tan\,\theta = \frac{\frac{-3}{15}}{\frac{4}{5}}$$

$$tan\,\theta = \frac{-3}{4} = -0.75$$

(i) $\theta = -36.89\,^{o}$ ans

(ii) $\theta = 143.11\,^{o}$ ans

Q3. Prove the following identifies;

$(i)\ Cosh\,2x = cosh^2x + Sinh^2x$

solution

using formula $(1+x)^n = 1 + \frac{e^x+e^{-x}}{2} + \frac{+e^x}{2!} + \cdots$

$$Cosh\,x = \frac{e^x+e^{-x}}{2} \text{ and } sinh\,x = \frac{e^x-e^{-x}}{2}$$

Right hand $= cosh^2x + Sinh^2x$

$$\left(\frac{e^x+e^{-x}}{2}\right)^2 + \left(\frac{e^x-e^{-x}}{2}\right)^2$$

$$\frac{e^{2x}+2+e^{-2x}}{4} + \frac{e^{2x}-2+e^{-2x}}{4}$$

$$= \frac{e^{2x}+2+e^{-2x}+e^{2x}-2+e^{-2x}}{4}$$

$$= \frac{2e^{2x}+2e^{-2x}}{4}$$

$$= \frac{2(e^{2x}+e^{-2x})}{4}$$

$$= \frac{e^{2x}+e^{-2x}}{2} = Cosh\,2x\,(left\ hand\ side)$$

Q4. Solve the equation $3\,Sin\,hx + 4\,Coshx = 5$

Solutions

1.2.3 Learning Outcome No.3- Apply Complex Number

1.2.3.1 Introduction to the learning outcome

This learning outcome specifies the content of competencies required to apply complex numbers. It includes; Definition of complex numbers, Stating complex numbers in numbers in terms of conjugate argument and, Modulus, Representation of complex numbers on the Argand diagram, Arithmetic operation of complex numbers Application of De Moivre's theorem and Application of complex numbers to engineering

1.2.3.2 Performance standard

3.1 Calculations are performed using trigonometric rules
3.2 Calculations are performed using hyperbolic functions
3.3 Calculations involving complex numbers are performed using De Moivre's theorem

1.2.3.3 Information Sheet

a) APPLY COMPLEX NUMBER

A number of the form $a + ib$ is called complex number where $a\ and\ b$ are real numbers and $i = \sqrt{-1}$ we call $'a'$ the real part and 'b' the imaginary part of the complex $a + ib$ if $a = o$ then ib is said to be purely imaginary, if $b = 0$ the number is real.

Pair of complex number $a + ib$ are said to be conjugate of each other.

- **Addition and subtraction of complex numbers**

Addition and subtraction of complex numbers is achieved by adding or subtracting the real parts and the imaginary parts.

Example 1

$(4 + j5) + (3 - j2)$

$(4 + j5) + (3 - j2)$=$4 + j5 + 3 - j2$

=$(4 + 3) + j(5 - 2)$

=$7 + j3$

Example 2

$(4 + j7) - (2 - js) = 4 + j7 - 2 + js = (4 - 2) + j(7 + 5) = 2 \ + j12$

- **Multiplication of complex numbers**

Example 1

$(3 + j4)(2 + j5)$

$6 + j8 + j15 + j^2 20$

$6 + j23 - 20 \ (since j^2 = -1)$

=$-14 + j^{23}$

Examples 2

$(5 + j8)(5 - j8)$

$(5 + j8)(5 - j8) = 25 + j40 - j40 - j^2 64$

$= 25 + 64$ =89

A pair of complex numbers are called conjugate complex numbers and the product of two conjugate. A Complex number is always entirely real.

$$cos\theta + jsin\theta$$

- **Argand diagram**

Although we cannot evaluate a complex number as a real number, we can represent diagrammatically in an argand diagram. Refer to Engineering Mathematics by K.A Stroud to learn more on how to represent complex numbers on an argand diagram. Use the same back learn three forms of expressing a complex number.

- **Demoivre's Theorem**

Demoivre's theorem states that $[r(cos\theta + jsin\theta)]^n = \ r^n(cosn\theta + jsinn\theta)$

It is used in finding powers and roots of complex numbers in polar

Example -1

Find the three cube roots of $z = 5(cos225^0 + jsin225^0)$

$$Z_1 = Z^{\frac{1}{3}}\left(Cos\frac{225^0}{3} + jsin\frac{225^0}{3}\right)$$

$1.71\ (cos75^0 + jsin75^0$

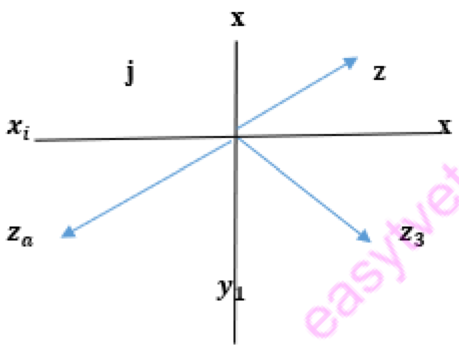
$z_1 = 1.71\ (cos75^0 + jSin75^0)$be roots are the same size (modules) i.e., 1.71 and separated at intervals of $\frac{360^0}{3}$, i.e 120^0

$z_{1=1.71—}\ 75^0$

$z_{2=1.71\ \cos(195^0+jSin195^0}$

z_s = 1.71 $(315^+ + jSin315^0)$

Sketched Argand diagram.



Refer to engineering mathematics by K.A Stroud and learn more on how to find the expansion of :

$cos^n\theta$ and $cos^n\theta$

- **Loci problems**

We sometimes required to find the locus of a point which moves in the Argand diagram according to some stated condition.

Examples-1

If $Z = x + jy$, find the equation of the locus $\left[\frac{z+1}{z-1}\right] = 2$

$sin\theta Z = x + jy.$

$\therefore \left(\frac{Z+1}{z-1}\right) = \frac{r_1}{r_2} = \left(\frac{z_1}{z_2}\right) = \frac{[(x+1)^2 + y^2]}{[(x-1)^2) + y^2]}$

$$\frac{[(x+1)^2 + y^2]}{(x-1)^2 + y^2}$$

$$\therefore \frac{(x+1)^2 + y^2}{(x-1)^2 + y^2} + 4$$

$\therefore (x+1)^2 + y^2 = 4\left((x-1)^2 + y^2\right)$

$x^2 + 2x + 1 + y^2 = 4(x^2 - 2x + 1 + y^2)$

$=4x^2 - 8x + 4 + 4y^2$

$\therefore 3x^2$-10x+3+$3y^2 = 0$

1.2.3.4 Learning Activities

Learning Outcome No. 3 APPLY COMPLEX NUMBER	
Learning Activities	**Special Instructions**
Preparing complex numbers using Argand diagrams Performing operations involving complex number Performing calculations involving complex number using De Moirés theorem	Carryout various exercises in order to gain confidence in complex numbers

1.2.3.5 Self-Assessment

1. Find the fifth roots of $-3 + j3$ in polar form and in exponential form
2. Determine the three cube roots of $\frac{2-j}{2+j}$ giving the results in a modulus/ argument form.
3. Express the principal root in the form $a + jb$
4. If $z = x + jy,\ where\ x\ and\ y$ are real, show that the locus $\left(\frac{Z-2}{z+2}\right) = 2$ is a circle and
5. Determine its center and radius.

1.2.3.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

HELM - Helping Engineers Learn Mathematics Workbooks

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Lecture Notes - For the engineering analysis modules

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Links to external sites

1.2.3.7 References (APA)

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Hildebrand, F.B. (1974), Introduction to Numerical Analysis, 2nd ed., McGraw-Hill (New York).

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Kreyszig, E. (1999), Advanced Engineering Mathematics, 8th ed., John Wiley (New York).

1.2.4 Learning Outcome No.4 - Apply Co-ordinate Geometry

1.2.4.1 Introduction to the learning outcome

This learning outcome specifies the content of competencies required to Apply coordinate geometry. It includes; Polar equations, Cartesian equation, Graphs of polar equations, Normal and tangents, Definition of a point, Locus of a point in relation to a circle and Loci of points for given mechanism.

1.2.4.2 Performance standards

4.1 Polar equations are calculated using coordinate geometry.
4.2 Graphs of given polar equations are drawn using the Cartesian plane

4.3 Normal and tangents are determined using coordinate geometry

1.2.4.3 Information Sheet

a) Introduction

The position of a point in a plane can be represented in two forms

Cartesian co-ordinate (x, y)

Polar co-ordinate (r, θ)

The position of a point in the corresponding axis can therefore generate Cartesian and polar equations which can easily change into required form to fit the required result.

Example-1

Convert $r^2 = sin\theta$ into Cartesian form.

$$\boldsymbol{cos\theta} = \frac{x}{y} \quad ; \quad \boldsymbol{sin\theta} = \frac{y}{x}$$

Form Pythagoras theorem $r^2 = x^2 + y^2$

$$r^2 = sin\theta$$

$$(x^2 + y^2) = \frac{y}{x}$$

$$(x^2 + y^2)r = y$$

$$(x^2 + y^2)(x^2 + y^2)^{\frac{1}{2}} = y$$

$$(x^2 + y^2)^{\frac{3}{2}} = y$$

Example- 2

Find the Cartesian equation of

$r = a(1 + 2cos)$i) $\qquad rcos(\theta - \alpha) = p$

[The $Cos\theta$ suggest the relation$X = COS\theta$, so multiplying through by r}

$$\therefore r^2 = a(r + 2rcos\theta)$$

$$\therefore x^2 + y^2 = a\left(\sqrt{(x^2 + y^2)} + 2x\right)$$

$$\therefore x^2 + y^2 + 2x = a\sqrt{(x^2 + y^2)}$$

Therefore, the Cartesian equation of $r = a(1 + 2cos)$ $(x^2 + y^2 - 2ax)^2 = a^2(x^2 + y^2)$

$$rcos(\theta - \alpha) = p$$

$cos(\theta - \alpha)$ May be expanded

$\therefore rcos\theta cos\alpha + rsin\theta sin\alpha = p$

Therefore, the Cartesian equation of $rcos(\theta - \alpha) = p$ is $xcos\alpha + ysin\alpha = p$

Example- 3

Find the polar equation of the circle whose Cartesian equation is $x^2 + y^2 = 4x$

$x^2 + y^2 = 4x$

Put $x = rcos\theta, y = rsin\theta,$ then

$r^2cos^2\theta + r^2sin^2\theta = 4rcos\theta$

$\therefore r^2 = 4rcos\theta$

Therefore, the polar equation of the circle is $r^2 = 4rcos\theta.$

For more information on the conversion of Cartesian equation to polar equation and vice versa refer to pure mathematics by J.K Backhouse

$$r = 4acot\theta cosec\theta$$

b) Equation of a Tangent Line in Cartesian Coordinates

Suppose that a function y=f(x) is defined on the interval (a, b) and is continuous at $x_0 \in (a, b)$. At this point (the point M in Figure 1), the function has the value $y_0=f(x_0)$.

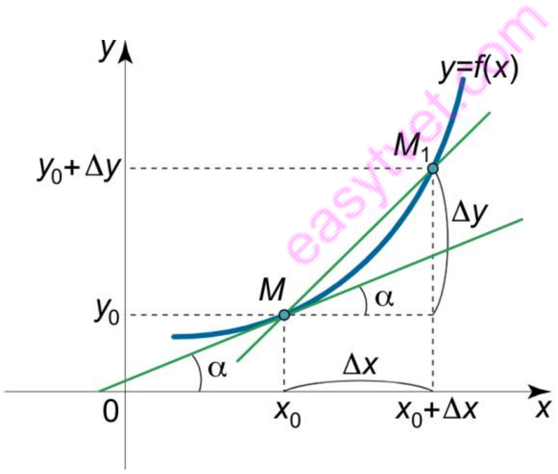

Figure 1

https://www.math24.net/tangent-normal-lines

Let the independent variable at x0 has the increment Δx. The corresponding increment of the function Δy is expressed as

Δy=f(x0+Δx)−f(x0).

In Figure 1, the point M1 has the coordinates (x0+Δx,y0+Δy). We draw the secant MM1. Its equation has the form

$y-y_0=k(x-x_0)$,

where k is the slope coefficient depending on the increment Δx and equal

$k=k(\Delta x)=\frac{\Delta y}{\Delta x}$.

When Δx decreases, the point M_1 moves to the point M: $M_1\to M$. In the limit $\Delta x\to 0$ the distance between the points M and M_1 approaches zero. This follows from the continuity of the function f(x) at x_0:

$\lim_{\Delta x\to 0}\Delta y=0, \Rightarrow \lim_{\Delta x\to 0}|MM_1|=\lim_{\Delta x\to 0}\sqrt{(\Delta x)^2+(\Delta y)^2}=0$.

The limiting position of the secant MM_1 is just the tangent line to the graph of the function y=f(x) at point M.

There are two kinds of tangent lines – oblique (slant) tangents and vertical tangents.

***Definition* 1.**

If there is a finite limit $\lim_{\Delta x\to 0}k(\Delta x)=k_0$, then the straight line given by the equation

$y-y_0=k(x-x_0)$,

is called the oblique (slant) tangent to the graph of the function y=f(x) at the point (X_0, Y_0).

***Definition* 2.**

If the limit value of k as $\Delta x\to 0$ is infinite: $\lim_{\Delta x\to 0}k(\Delta x)=\pm\infty$, then the straight line given by the equation

$x=x_0$,

is called the vertical tangent to the graph of the function y=f(x) at the point (x_0,y_0).

It is important that

$k_0=\lim_{\Delta x\to 0}k(\Delta x)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=f'(x_0)$,

that is the slope of the tangent line is equal to the derivative of the function $f(x_0)$ at the tangency point x_0. Therefore, the equation of the oblique tangent can be written in the form

$y-y_0=f'(x_0)(x-x_0)$ or $y=f'(x_0)(x-x_0)+f(x_0)$.

Since the slope of a straight line is equal to the tangent of the slope angle α, which the line forms with the positive direction of the x-axis, then the following triple identity is valid:

k=tanα=f′(x0).

Equation of a Normal Line in Cartesian Coordinates

A straight line perpendicular to the tangent and passing through the point of tangency (x0, y0) is called the normal to the graph of the function y=f(x) at this point (Figure 2).

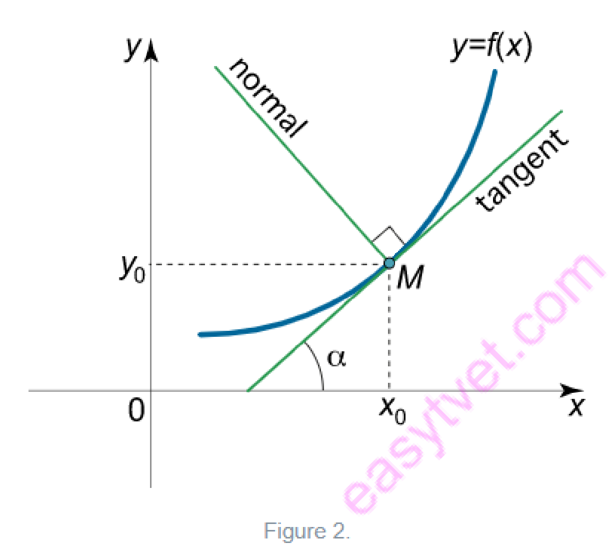

Figure 2.

https://www.math24.net/tangent-normal-lines

From geometry it is known that the product of the slopes of perpendicular lines is equal to −1. Therefore, knowing the equation of a tangent at the point (x0, y0):

y−y0=f′(x0) (x−x0),

we can immediately write the equation of the normal in the form

y−y0=−1f′(x0) (x−x0).

Suppose that a curve is defined by a polar equation r=f(θ), which expresses the dependence of the length of the radius vector r on the polar angle θ. In Cartesian coordinates, this curve will be described by the system of equations

{x=r cos θ=f(θ)cos θ y=r sin θ=f(θ)sin θ.

Thus, we have written the parametric equation of the curve, where the angle θ plays the role of a parameter. Next, it is easy to obtain an expression for the slope of the tangent to the curve at the point (x0, y0):

k=tan θ= y′ θ x′ θ= (r sin θ) ′ (r cos θ) ′= r′ θ sin θ + r cos θ r′ θ cos θ – r sin θ.

As a result, the equations of the tangent and normal lines are written as follows:

$y-y_0$=y′θ x′θ(x−x0) (tangent),

y−y0=−x′θ y′θ(x−x0) (normal).

The study of curves can be performed directly in polar coordinates without transition to the Cartesian system. In this case, instead of the angle θ with the polar axis (i.e. with the positive direction of the x-axis), it is more convenient to use the angle β with the line containing the radius vector r (Figure 3).

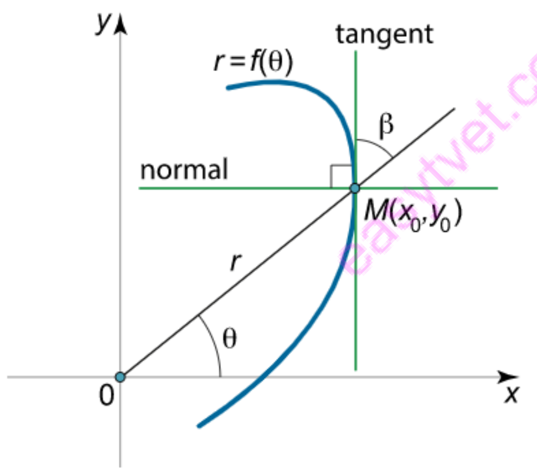

Figure 3.

https://www.math24.net/tangent-normal-lines

The tangent of the angle β is calculated by the formula

tanβ=r r′θ.

The angle formed by the normal and the extended radius vector is β+π2. Using the reduction identity, we get:

tan (β + π2) = -cot β = -tan β = r’ θr

Example 1

Find the equation of the tangent to the curve y = √x at the point (1,1)

Solution.

$y' = f'(x) = (\sqrt{x})' = 12\sqrt{x}$
$f'(x_0) = f'(1) = 12\sqrt{1} = 12$
$X_0 = 1$, $y_0 = 1$, $f'(X_0) = 12$
Substitute the 3 values into the equation of the tangent line:
$y - y_0 = f'(X_0)(X - X_0)$.
These yields:

$y - 1 = 12(X - 1)$
$y - 1 = X2 - 12$
$y = X2 - 12 + 1$
$y = X2 + 12$.

Answer: $y = X2 + 12$.

c) Point

A point is the most fundamental object in geometry. It is represented by a dot and named by a capital letter. A point represents position only; it has zero size (that is, zero length, zero width, and zero height).

d) Locus

A locus is the set of all points (usually forming a curve or surface) satisfying some condition. For example, the locus of points in the plane equidistant from a given point is a circle, and the set of points in three-space equidistant from a given point is a sphere.

1.2.4.4 Learning Activities

Learning Outcome No. 4 APPLY CO-ORDINATE GEOMETRY	
Learning Activities	**Special Instructions**
Calculating polar equations using coordinate geometry Drawing graphs of given polar equations using the Cartesian plane Determining normal and tangents using coordinate geometry	Carryout various exercises in order to gain confidence in Geometry

1.2.4.5 Self-assessment

1. Obtain the polar equation of the following loci

$$x^2 + y^2 = a^2$$

$$x^2 - y^2 = a^2$$

$$y = 0$$

$$y^2 = 4a(a - x)$$

$$x^2 + y^2 - 2y = 0$$

$$xy = c^2$$

2. Obtain the Cartesian equation of the following loci

$$r = 2$$

$$a(1+cos\theta)$$

$$r = acos\theta$$

$$r = atan\theta$$

$$r = 2a(1 + sin2\theta)$$

$$2r^2 sin2\theta = c^2$$

$$\frac{l}{r} = 1 + 8cos\theta$$

1.2.4.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

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1.2.4.7 References

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1.2.4.8 Suggested responses to self-assessment

1.2.5 Learning Outcome No.5 Carry out Binomial Expansion

1.2.5.1 Introduction to the learning outcome

This learning outcome specifies the content of competencies required to carry out binomial Expansion. It includes; Binomial theorem Power series using binomial theorem Roots of numbers using binomial theorem and Estimation of errors of small changes using binomial theorem.

1.2.5.2 Performance Standard

5.1 Roots of numbers are determined using binomial theorem.
5.2 Errors of small changes are determined using binomial theorem

1.2.5.3 Information Sheet

a) Carry out Binomial expansion

Binomial is a formula for raising a binomial expansion to any power without lengthy multiplication. It states that the general expansion of $(a+b)^n$ is given as

$$(a+b)^n = a^n b^0 + na^{n-1}n^1 + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \ldots$$

Where n can be a fraction, a decimal fraction, positive or negative integer.

Example- 1

Use binomial theorem to expand $(2+x)^3$

Solution

$$(a+b)^n = a^n b^0 + na^{n-1}n^1 + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \ldots$$

$$A = 2, b = x \; and \; n = 3$$

$$(2+x)^3 = 2^3x^0 + 32^2x^1 + \frac{3(3-1)2^1x^2}{2!} + \frac{3(3-1)(3-2)2^0x^3}{3!} + \ldots$$

$$= 18 + 12x + 6x^2 + x^3$$

For more examples on positive power refer to Technician Mathematic Book by J.O Bird.

b) Binomial theorem for any index

It has been shown that

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots.$$

The series may be continued indefinitely for any value of n provide $-1 < x < 1$

<u>Example -1</u>

Use the binomial theorem to expand $\frac{1}{1-x}$ in ascending power of x as far as the term in x^3.

Solution :

Since $\frac{1}{1-x}$ may be written$(a - x + x)^{-1}$, the binomial theorem may be used. Thus

$(1-x)^{-1} = 1 \pm 1(-x) + \frac{-1(-2)}{2!}x^2 + \frac{-1(-2)(-3)}{3!} + \ldots.$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

Provided $-1 < x < 1$

c) Practical application of binomial theorem

<u>Example- 1</u>

The radius of a cylinder is reduced by 4% and its height increased by 2%. Determine the appropriate percentage change in its volume neglecting products of small quantities.

Solution

Volume $V = \pi r^2 h$

Let original values be, $radius = r$

$Height = h$

New values $radius = (1 - 0.04)r$

$Height = (1 + 0.02)h$

New volume $= \pi(1 - 0.004)^2 r^2 (1 + 0.04)h)$

Using binomial theorem, $(1 - 0.04)^2 = 1 - 2(0.04) + (0.04)^2 = 1 - 0.08$

$=\pi r^2 h(1 - 0.08)(1.02) = \pi r^2 h(0.94)$

Percentage change $= \frac{(0.94-1)100\%}{1} = -6\%$

The new volume decreased by 6%

d) Estimation of errors of small changes using binomial theorem.

<u>Example-1</u>

An error of +1.5% was made when measuring the radius of a sphere. Ignoring the products of small quantities, determine the approximate error in calculating ;

(i) the volume and

(ii) the surface area.

(i) ***Volume of sphere,*** $V=\frac{4}{3}\pi r^3$

New volume = $\frac{4}{3}\pi(1.015r)^3 = \frac{4}{3}\pi r^3(1+0.015)^3 = \frac{4}{3}\pi r^3[(1+3(0.015)] = \frac{4}{3}\pi r^3(1+0.015)^3$

= 1.045V

i.e., the new volume has increased by 4.5%

(ii) ***Surface area of sphere***, $A = 4\pi r^2$

New surface area = $4\pi(1+0.015)^2r^2 = 4\pi r^2[1+2(0.015)] = 4\pi r^2(1+0.03)$

= 1.03 A

i.e., the surface area has increased by 3%

Example -2

The radius of a cone is increased by 2.7% and its height reduced by 0.9%. Determine the approximate percentage change in its volume, neglecting the products of small terms.

Solution;

Volume of cone = $\frac{1}{3}\pi r^2 h$

New volume = $\frac{1}{3}\pi(1+0.027)^2 r^2(1-0.009)h = \frac{1}{3}\pi r^2 h(1+(2)0.027-0.009)$

= $\frac{1}{3}\pi r^2 h(1+0.045)$

i.e., 104.5% of the original volume

Thus, the approximate percentage change in volume is an increase of 4.5%

1.2.5.4 Learning Activities

Learning Outcome No.5: CARRY OUT BINOMIAL EXPANSION	
Learning Activities	**Special Instructions**
Determining roots of numbers using binomial theorem Determining errors and small changes using bionomical theorem	Carryout various exercises in order to gain confidence in Binomial numbers

1.2.5.5 Self-Assessment

1. Expand $\frac{1}{(4-x)^2}$ in the scending powers of x as far as x^3, using the binomial theorem.

2. Show that if higher power of x are neglected

$$\sqrt{\frac{1+x}{1-x}} = 1 + x + \frac{x^2}{2}$$

3. The second moment of area of a rectangular section through its centroid is given by $\frac{bl3}{12}$. Determine the appropriate change in the second moment of area if b is increased by 3.5% and l is reduced by 2.5%

1.2.5.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

- HELM - Helping Engineers Learn Mathematics Workbooks
- SIGMA - Summary sheets covering key areas in Mathematics
- Lecture Notes - For the engineering analysis modules
- Links to (online) textbooks available in the LIS
- Links to external sites

1.2.5.7 References

Greenberg, M.D. (1998), Advanced Engineering Mathematics, 2nd ed., Prentice Hall (Upper Saddle River, N.J).

Hildebrand, F.B. (1974), Introduction to Numerical Analysis, 2nd ed., McGraw-Hill (New York).

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O'Neil, P.V. (1995), Advanced Engineering Mathematics, 4th ed., PWS-Kent Pub. (Boston).

1.2.5.8 Suggested responses to self-assessment

Q1. Expand $\frac{1}{(4-x)^2}$ in the scending powers of x as far as x^3, using the binomial theorem.

Solution -

$$\frac{1}{(4-x)^2} = \frac{1}{[4\left(1-\frac{x}{4}\right)]^2} = \frac{1}{4^2\left(1-\frac{x}{4}\right)^2}$$

$$= \frac{1}{16}\left(1 - \frac{x}{4}\right)^{-2}$$

Using the expansion of $(1 + x)^n$

$$\frac{1}{(4-x)^2} = \frac{1}{16}\left(1 - \frac{x}{4}\right)^{-2}$$

$$= \frac{1}{16}\left[1 + (-2)\left(-\frac{x}{4}\right) + \frac{(-2)(-3)}{2!}\left(-\frac{x}{4}\right)^2 + \frac{(-2)(-3)(-4)}{3!}\left(-\frac{x}{4}\right)^3 + \cdots \right]$$

$$= \frac{1}{16}\left(1 + \frac{x}{2} + \frac{3x^2}{16} + \frac{x^3}{16} \ldots\ldots\right) \text{ ans}$$

Q 2. Show that if higher power of x are neglected

$$\sqrt{\frac{1+x}{1-x}} = 1 + x + \frac{x^2}{2}$$

Solution:

The binomial theorem says that, if r is any real number and $|x| < 1$,then

$$(1 + x)^r = 1 + rx + \frac{r(r-1)}{2!}x^2 + \frac{r(r-1)(r-2)}{3!}x^3 + \cdots$$

Since is close enough to zero so that its cube and higher powers can be neglected, this can be shortened to say that;

$$(1 + x)^r \approx 1 + rx + \frac{r(r-1)}{2!}x^2$$

Now, as can write

$$\sqrt{\frac{1+x}{1-x}} = (1 + x)^{½} \times (1 + x)^{-½}$$

We can therefore use the binomial theorem to say that ;

$$\sqrt{\frac{1+x}{1-x}} \approx \left(1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}x^2\right) \times \left(1 + \left(-\frac{1}{2}\right)(-x) + \frac{-\frac{1}{2}\left(-\frac{3}{2}\right)}{2!}(-x)^2\right)$$

$$= \left(1 + \frac{1}{2}x - \frac{1}{8}x^2\right) \times \left(1 + \frac{1}{2}x + \frac{3}{8}x^2\right)$$

$$\approx 1 + \left(\frac{1}{2} + \frac{1}{2}\right)x + \left(\frac{3}{8} + \frac{1}{4} - \frac{1}{8}\right)x^2$$

$$= 1 + x + \frac{x^2}{2} \text{ ans}$$

3. The second moment of area of a rectangular section through its centroid is given by $\frac{bl3}{12}$. Determine the appropriate change in the second moment of area if b is increased by 3.5% and l is reduced by 2.5%

Solution -

New values of b and l are (1+ 0.035)b and (1- 0.025)l respectively

New second moment of area ;

$\frac{1}{12}$ $\frac{1}{12}[(1+0.035)b][(1-0.025)l]^3$

$= \frac{bl^3}{12}(1+0.035)\,(1-0.025)^3$

$\approx \frac{bl^3}{12}(1+0.035)\,(1-0.075)$,neglecting power of small terms

$\approx \frac{bl^3}{12}(1+0.035-0.075)$, neglecting products of small terms

$\approx \frac{bl^3}{12}(1-0.040)$ or $(0.96)\frac{bl^3}{12}$, i.e. 96% of the original second moment of are.

Hence the second moment of area is reduced by approximately 4%

1.2.6 Learning Outcome No.6 Apply Calculus

1.2.6.1 Introduction to the learning Outcome

This learning outcome specifies the content of competencies required to Apply calculus. It includes; Definition of derivatives of a function, Differentiation from first principle, Tables of some common derivatives, Rules of differentiation, Rate of change and small change, Stationery points of functions of two variables, Definition of integration, Indefinite and definite integral, Methods of integration application of integration and Integrals of hyperbolic and inverse functions.

1.2.6.2 Performance Standard

6.1 Derivatives of functions are determined using Differentiation.
6.2 Derivatives of hyperbolic functions are determined using Differentiation
6.3 Derivatives of inverse trigonometric functions are determined using Differentiation
6.4 Rate of change and small change are determined using Differentiation.
6.5 Calculation involving stationery points of functions of two variables are performed using differentiation.
6.6 Integrals of algebraic functions are determined using integration
6.7 Integrals of trigonometric functions are determined using integration
6.8 Integrals of logarithmic functions are determined using integration
6.9 Integrals of hyperbolic and inverse functions are determined using integration.

1.2.6.3 Information Sheet

Calculus is a branch of mathematics involving calculations dealing with continuously varying functions. The subject falls into two parts namely differential calculus (differentiation) and integral calculus (integration)

a) Differentiation

The central problem of the differential calculus is the investigation of the rate of change of a function with respect to charges in the variables on which it depends.

❖ **Differentiation from first principles.**

To differentiate from first principles means to find $f'(x)$ using the expression.

$$f'(r) = \lim_{\delta x \to 0} \left\{ \frac{f(x+\delta x)}{\delta x} \right\}$$

$$\delta x \to \; 0 \left\{ + \frac{(x+\delta x) - f(x)}{\delta x} \right\}$$

$$f(x) = x^2$$

$$f(x+\delta x) = (x+\delta x)^2 = x^2 + 2x\delta x + \delta x^2$$

$$f(x+\delta x) - f(x) = x^2 + 2\,x\delta x + \delta x^2 - x^2$$

$=2x\delta x + \delta x^2$

$$\frac{f(x+\delta x) - f(x)}{\delta x} = \frac{2x\delta x + \delta x^2}{\delta x}$$

$=2x + \delta x$

As $\delta x \to 0$, $\frac{f(x+\delta x)-f(x)}{\delta x} \to 2x + 0$

$\therefore f'(x) = \lim_{\delta x \to 0} \left\{ \frac{f(x+\delta x)-f(x)}{\delta x} \right\} = 2x$

At $x = 3$, the gradient of the curve i.e $f'(x) = 2(3) = \; 6$

Hence if $f(x) = x^2$, $\quad f'(x) = 2x$. The gradient at $x = 3$ is 6

- **Methods of differentiation**

  There are several methods used to differentiate different functions which include:

  (i) ***Product Rule***
  (ii) ***Quotient Rule***
  (iii) ***Chain Rule***
  (iv) ***Implicit Rule***

__**Example: 1**__

Determine ${dy}/{dx}$ given that $y = x^2 Sinx$

Solution;

From product rule: $uv(x) = u\frac{dv}{dx} + v\frac{du}{dx}$

$$u = x^2 \text{and } v = Sinx$$

$\frac{du}{dx} = 2x$ $\qquad\qquad$ $\frac{dv}{dx} = Cosx$

$\therefore \frac{dy}{dx} = x^2(Cosx) + Sin\,(2x)$

$= x^2 Cos\, x + 2x\, Sin\, x$

$$y = \frac{x^2+1}{x-3}$$

Soln. Using Quotient rule:

$$\frac{u(x)}{v(x)} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = x^2+1$ $\quad\quad v = x - 3$

$\frac{du}{dx} = 2x$ $\quad\quad \frac{dv}{dx}=1$

$\therefore$dy⁄dx$= \frac{(x-3)(2x)-(x^2+1)(1)}{(x-3)^2}$

$= \frac{2x^2-6x-x^2-1}{(x-3)^2}$

$= \frac{x^2-6x-1}{(x-3)^2}$

$$\frac{x^2-6x-1}{x^2-6x+9}$$

For more examples on the cases of application of the other, highlighted rates refer to Engineering Mathematics by K Stroud.

- **Picture of some common derivatives**

Derivative	Integral (Antiderivative)
$\frac{d}{dx}n = 0$	$\int 0\, dx = C$
$\frac{d}{dx}x = 1$	$\int 1\, dx = x + C$
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x\, dx = e^x + C$

$$\frac{d}{dx}\ln x = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \ln x + C$$

$$\frac{d}{dx}n^x = n^x \ln x \qquad \int n^x\,dx = \frac{n^x}{\ln n} + C$$

Derivative	**Integral (Antiderivative)**
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \tan x \sec x\,dx = \sec x + C$
$\frac{d}{dx}\csc x = -\csc x \cot x$	$\int \cot x \csc x\,dx = -\csc x + C$
$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$
$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$	$\int -\frac{1}{\sqrt{1-x^2}}\,dx = \arccos x + C$
$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = \arctan x + C$
$\frac{d}{dx}\mathrm{arc}\cot x = -\frac{1}{1+x^2}$	$\int -\frac{1}{1+x^2}\,dx = \mathrm{arc}\cot x + C$
$\frac{d}{dx}\mathrm{arc}\sec x = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \mathrm{arc}\sec x + C$
$\frac{d}{dx}\mathrm{arc}\csc x = -\frac{1}{x\sqrt{x^2-1}}$	$\int -\frac{1}{x\sqrt{x^2-1}}\,dx = \mathrm{arc}\csc x + C$

https://www.dummies.com/education/math/calculus/the-most-important-derivatives-and-antiderivatives-to-know/

- **Application of differentiation**

  Differentiation can be used to determine velocity and acceleration of a moving body. It can also be applied to determine maximum and minimum values.

  Example: A rectangular area in formed using a pieced of wire 30cm long. Find the length and breadth of the rectangle if it is to enclose the maximum possible area.

  Solution.

  Let the dimension a rectangle be $x \text{ and } y$

  Perimeter of rectangle = $2x + 2y = 36$

  i.e., $x + y = 18$..................(i)

  Since it's the maximum area that is required a formula for the area A must be obtained in terms of one variable only.

  Area = $A = xy$

  From equation (i) $y = 18 - x$

  Hence $A = x(18 - x) = 18x - x^2$

  Now that an expression for the area has been obtained in terms of one variable it can be differentiated with respect to that variable

$\frac{dA}{dx} = 18 - 2x$ $for\ maximum$ or minimum value i.e. $x = 9$

$$\frac{d^2A}{dx^2} = -2, which\ is\ negative\ \ giving\ a\ maximum\ value$$

$$y = 18 - x = 18 - 9 = 9$$

  Hence the length and breadth of the rectangle of maximum area both 9 cm i.e a square gives the maximum possible area for a given perimeter length When perimeter is 36cm maximum area, possible is $81cm^2$.

- **Integration**

  Process of integration reverses the process of differentiation. In differentiation if $f(x) = x^2$ then $f'(x) = 2x$.

  Since integration reverse the prices of moving from $f(x)$ to f'(x), it follows that the integral of $2x$ $is\ x^2\ i.e\ its\ the\ process\ of\ moving\ from\ f'$(x) to $f(x)$. Similarly, if $y = x^2$ then $\frac{dy}{dx} = 3x^2$. Reversing this process shows that the integral of $3x^2$ and x^3.

  Integration is also the process of summation or adding parts together and on elongate $'s'$ shown a $\int$ is used to replace the words 'integrated of "Thus $\int 2x$=x^2 and $\int 3x^2 = x^3$

Refers to engineering mathematics by K.A Stroud and learn those on definite and indefinite integrals.

- **Methods of integration and application of integration**
  The methods available are:

  (i) *By using an algebraic substitution*
  (ii) *Using trigonometric identifies and substitutions*
  (iii) *Using partial fraction*
  (iv) *Using the* $t = tan\frac{\theta}{2}$ *substitution*
  (v) *Using integration by parts*

Refer to Engineering mathematics by K.A Stroud and learn more about methods of integration,

- **Picture of integrals of hyperbolic and inverse functions**

constant $c \neq 0$

$$\int \sinh(cx)\,dx = \frac{1}{c}\cosh(cx)$$

$$\int \cosh(cx)\,dx = \frac{1}{c}\sinh(cx)$$

$$\int \sinh^2(cx)\,dx = \frac{1}{4c}\sinh(2cx) - \frac{x}{2}$$

$$\int \cosh^2(cx)\,dx = \frac{1}{4c}\sinh(2cx) + \frac{x}{2}$$

$$\int \frac{dx}{\cosh(cx)} = \frac{2}{c}\arctan e^{cx}$$

$$\int \frac{dx}{\sinh^n(cx)} = \frac{\cosh(cx)}{c(n-1)\sinh^{n-1}(cx)} - \frac{n-2}{n-1}\int \frac{dx}{\sinh^{n-2}(cx)} \quad (\text{for } n \neq 1)$$

$$\int \frac{dx}{\cosh^n(cx)} = \frac{\sinh cx}{c(n-1)\cosh^{n-1}(cx)} + \frac{n-2}{n-1}\int \frac{dx}{\cosh^{n-2}(cx)} \quad (\text{for } n \neq 1)$$

https://math.info/Calculus/Integrals_Hyperbolic/

$$\int x^2 \cosh(cx)\,dx = -\frac{2x\cosh(cx)}{c^2} + \left(\frac{x^2}{c} + \frac{2}{c^3}\right)\sinh(cx)$$

$$\int \tanh(cx)\,dx = \frac{1}{c}\ln|\cosh(cx)|$$

$$\int \coth(cx)\,dx = \frac{1}{c}\ln|\sinh(cx)|$$

$$\int \tanh^n(cx)\,dx = -\frac{1}{c(n-1)}\tanh^{n-1}(cx) + \int \tanh^{n-2}(cx)\,dx \text{ (for } n\neq 1)$$

$$\int \coth^n(cx)\,dx = -\frac{1}{c(n-1)}\coth^{n-1}(cx) + \int \coth^{n-2}(cx)\,dx \text{ (for } n\neq 1)$$

https://math.info/Calculus/Integrals_Hyperbolic/

Examples

(i) $\int x \cosh(x^2)\,dx$
(ii) $\int \tanh x\,dx$

Solution

We can use u- substitution in both cases

Example -1

Let $u = x^2$ Then $du = 2xdx$

$$\int x\cosh(x^2)\,dx = \int \frac{1}{2}\cosh u\,du$$

$$= \frac{1}{2}\sinh u + C$$

$$= \frac{1}{2}\sinh(x^2) + C$$

Example -2

Let u= cosh x, then du = sinh x dx

$$\int \tanh x\,dx = \int \frac{\sinh x}{\cosh x}\,dx$$

$$= \int \frac{1}{u}\,du$$

$$= \ln[\cosh x] + C$$

Note that cosh x > 0 for all x, so we can eliminate the absolute value signs and obtain

$$\int \tanh x\,dx = \ln[\cosh x] + C$$

Also, we the above stated book to learn more on application on integration to find areas, volumes of revolutions etc.

1.2.6.4 Learning Activities

Learning Outcome No.6 APPLY CALCULUS	
Learning Activities	**Special Instructions**
• Determining Derivatives from first principles • Determining derivatives of inverse trigonometric functions • Determining the rate of change and small changes using differentiation • Determining integrals of algebraic functions • Determining integrals of trigonometric functions • Determining integrals of logarithmic functions • Determining integrals of hyperbaric and inverse hyperbaric functions.	Carryout various exercises in order to gain confidence in calculus

1.2.6.5 Self-Assessment

1. Find the co-ordinate, of the points on the curve Where the gradient is zero.

$$y = \frac{\frac{1}{3(5-6x)}}{3x^2+2}$$

2. If $y = \frac{4}{3x^3} - \frac{2}{x^2} + \frac{1}{3x} - \sqrt{x}$. Find $\frac{d^2y}{dx}$ and $\frac{d^3y}{dx}$

3. Find $\int cos6x\ Sin\ 2x\ \ dx$

4 . Evaluate $\int_3^4 \frac{x^3-x^2-5x}{x^2-2x+2}dx$

1.2.6.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

- HELM - Helping Engineers Learn Mathematics Workbooks
- SIGMA - Summary sheets covering key areas in Mathematics
- Lecture Notes - For the engineering analysis modules
- Links to (online) textbooks available in the LIS
- Links to external sites

1.2.6.7 References (APA)

Greenberg, M.D. (1998), Advanced Engineering Mathematics, 2nd ed., Prentice Hall (Upper Saddle River, N.J).

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1.2.6.8 Suggested responses to self-assessment

1.2.7 Learning Outcome No.7 Solve Ordinary Differential Equations

1.2.7.1 Introduction to the learning outcomes

This learning outcome specifies the content of competencies required to Solve Ordinary Differential Equations. It includes; Types of first order differential equations, Formation of first order differential equation, Solution of first order differential equations, Application of first order differential equations, Formation of second order differential equations for various systems, Solution of second order differential equations and Application of second order differential equations.

1.2.7.2 Performance Standards

7.1 First order and second order differential equations are solved using the method of undetermined coefficients.

7.2 First order and second order differential equations are solved from given boundary conditions

1.2.7.3 Information Sheet

An equation involves differential co-efficient is called a differential equation example;

$$\frac{dy}{dx} = \frac{1 + x^2}{1 - y^2}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = 0$$

The order of a differential equation is the order of the highest differential is coefficient present in the equation.

Differential equations represent dynamic relationship i.e., quantities that change, and are thus An equation which involves differential co-efficient is called a differential equation.

Example: 1

$$\frac{dy}{dx} = \frac{1 + x^2}{1 - y^2}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = 0$$

The order of a differential equation is the order of the highest differential coefficient present in the equation.

A differential equation represents dynamic relationships, i.e., quantities that change, and are thus frequently occurring in scientific and engineering problems.

a) Formation of a differential equation

Differential equations may be formed in practice from a consideration of the physical problem to which they refer. Mathematically, they can occur when arbitrary constants are eliminated from a given function.

Example-1

Consider$y = Asinx + Bcosx$, where A and B are two arbitrary constants. If we differentiate, we get

$\frac{dy}{dx} = ACosx - BSinx$ and $\frac{d^2y}{dx^2} = -ASinx - BCosx = -(aSinx + BCosx)$

i.e. $\frac{d^2y}{dx^2} = -y$

$\therefore \quad \frac{d^2y}{dx^2} - y = 0$

This is a differential equation of the second order.

For the formation of first order differential equations, refer to Technician Mathematics 4 and 5 by J.O Bird.

b) Types of first order differential equations:

- *By separating the variables*
- *Homogeneous first order differential equations*
- *Linear differential equations*
- *Exact differential equations*

Refer to Engineering Mathematics by K.A Stroud, Technician 4 and 5 by J.O Bird for worked out examples and further exercises.

Application of first order differential equations.

Differential equations of the first order have many applications in engineering and science.

Example:2

The rate at which a body cools is given by the equations $\frac{d\theta}{dt} = -k\theta$ where θ the temperature of the body above the surroundings is and k is a constant. Solve the equation for θ given that $t = 0$,

$\theta = \theta_0$

Solution

$$\frac{d\theta}{dt} = -k\theta$$

Rearranging gives

$$dt = \frac{-1}{k\theta}$$

Integrating both sides give $\int dt = \frac{-1}{k}\int \frac{d\theta}{\theta}$

i.e $t = \frac{-1}{k} ln\theta + c$……………………..(i)

Substituting the boundary conditions $t = 0$, $\theta = \theta_0$ to find c gives

$$0 = \frac{-1}{k} ln\theta_0 + c$$

i.e $$c = \frac{1}{k} ln\theta_0$$

Substituting $c = \frac{-1}{k} ln\theta_0$ in equation (i) gives

$$t = \frac{-1}{k} ln\theta + \frac{1}{k} ln\theta_0$$

$$t = \frac{1}{k}(ln\theta_0 + ln\theta) = \frac{1}{k}\ln\left(\frac{\theta_0}{\theta}\right)$$

$$kt = ln\left(\frac{\theta_0}{\theta}\right)$$

$$e^{kt} = \frac{\theta_0}{\theta}$$

$$e^{-kt} = \frac{\theta}{\theta_0}$$

Hence; $\theta = \theta_0 e^{-kt}$

Further problems on application of differential equations may be found in Engineering Mathematics by K.A Stroud, Technician 4 and 5 by J.O Bird.

c) Formation of the second order differential equation

For, formation of second order differential equations refer to Engineering Mathematics by K.A Stroud, Technician 4 and 5 by J.O Bird.

d) Application of second order differential equations

Many applications in engineering give rise to the second order differential equations of the form

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

Where a, b, c are constants coefficient and $f(x)$ is a given function of x.

Example include:

- *Bending of beams*
- *Vertical oscillations and displacements*
- *Damped forced vibrations*

For more worked examples refer to Engineering Mathematics by K.A Stroud, Technician 4 and 5 by J.O Bird.

1.2.7.4 Learning Activities

Learning Outcome No. 7 Solve Ordinary differential equations	
Learning Activities	**Special Instructions**
Solve first order and second order differential equations using the method of undetermined coefficients. Apply boundary conditions to find the particular solution	Carry out various exercises in order to gain confidence in solving different ordinary differential equation

1.2.7.5 Self-Assessment

1. Solve the following equations:

(i) $x(y-3)\frac{dy}{dx}4y$

(ii) $(xy + y^2) + (x^2 - xy)\frac{dy}{dx} = 0$

$$\frac{dy}{dx} + ytanx = sinx$$

2. The change, q, on a capacitor in an LCR circuit satisfies the second order differential equation

$$L\frac{d^2q}{dt^2} + b\frac{dq}{dt} + \frac{1}{c}q = E$$

Show that if $2L = cR^2$ the general solution of this equation is

$$q = e^{\frac{-t}{cR}}\left(Acos\frac{1}{cR}t + Bsin\frac{1}{cR}t\right) + cE$$

3. If $i = \frac{dq}{dt} = 0$ and $q = 0$ when $t = 0$. Show that the current in the circuit is

$$i = \frac{2E}{R}e^{\frac{-t}{cR}}sin\frac{1}{cR}$$

1.2.7.6 Tool, Equipment, Supplies and Materials for the specific Learning Outcome

- Calculator
- Tables
- Worked Examples

1.2.7.7 References

Greenberg, M.D. (1998), Advanced Engineering Mathematics, 2nd ed., Prentice Hall (Upper Saddle River, N.J).

Hildebrand, F.B. (1974), Introduction to Numerical Analysis, 2nd ed., McGraw-Hill (New York).

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1.2.7.8 Suggested responses to self-assessment

1.2.8 Learning Outcome No.8 Carry out Mensuration

1.2.8.1 Introduction to the learning outcome

This learning outcome specifies the content of competencies required to Carry out Mensuration. It includes; Units of measurements, Perimeter and areas of regular figures, Volume of regular solids, Surface area of regular solids, Area of irregular figures and Areas and volumes using Pappus theorem.

1.2.8.2 Performance Standard

8.1 Perimeter and areas of figures are obtained.
8.2 Volume and of Surface area of solids are obtained.
8.3 Area of irregular figures are obtained.
8.4 Areas and volumes are obtained using Pappus theorem

1.2.8.3 Information Sheet

a) Introduction

This Unit is concerned with measuring, calculating and estimating lengths, areas and volumes, as well as the construction of three-dimensional (3D) objects.

- **What is Mensuration?**

In the broadest sense, mensuration is all about the process of measurement. Mensuration is based on the use of algebraic equations and geometric calculations to provide measurement data regarding the width, depth and volume of a given object or group of objects. While the measurement results obtained by the use of mensuration are estimates rather than actual physical measurements, the calculations are usually considered very accurate. Mensuration is often based on making use of a model or base object that serves as the standard for making the calculations. From that point, advanced mathematics is employed to project measurements of length, width, and weight associated with like items. The end result is data that can help to make the best use of resources available today while still planning responsibly for the future. Units and Measuring Different units can be used to measure the same quantities. It is important to use sensible units. Some important units are listed below.

- **Conversion of Units**

It is useful to be aware of both metric and imperial units and to be able to convert between them.

Original Units	Converted units
1 km	1000m
1m	100cm
1m	1000mm
1cm	10mm
1 tonne	1000kg
1 kg	1000g
1 litre	1000ml
1 m^3	1000litres
1 cm^3	1 ml

Examples

(a) How many mm are there in 3.72 m?

(b) How many cm are there in 4.23 m?

(c) How many m are there in 102.5 km?

(d) How many kg are there in 4.32 tonnes?

Solutions

(a) 1 m = 1000 mm So, 3.72 m = 3.72 × 1000 = 3720 mm

(b) 1 m = 100 cm So, 4.23m = 4.23 × 100 = 423cm

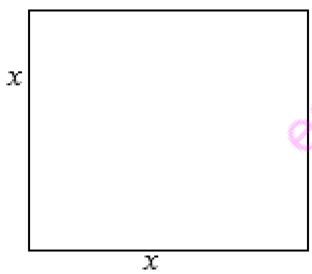
(c) 1 km = 1000 So, 102.5 km = 102 5 × 1000 = 102 500 m

(d) 1 ton = 1000 kg So 4.32 km = 4.32 × 1000 = 4320 km

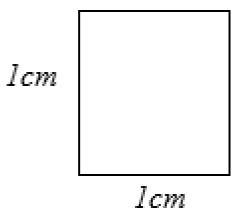
- **Estimating Areas and perimeters of Squares, Rectangles and Triangles**

For a square, the area is given by x multiplied by $x = \boldsymbol{x^2}$

The perimeter is given by $\boldsymbol{4x}$ where x is the length of a side

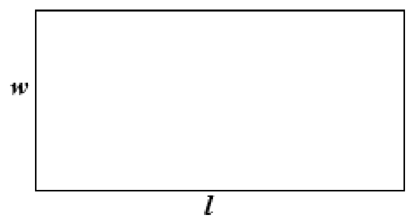


A square with sides of 1 cm has an area of $1cm^2$



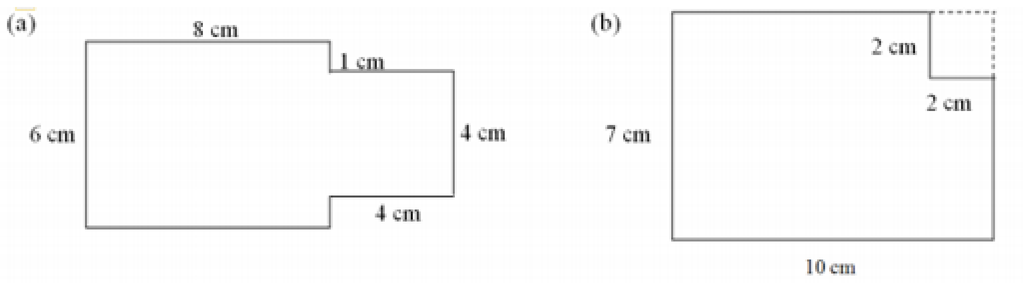
- ***Area and perimeter of a rectangle:***

For a rectangle, the area is given by $l\ w$ and the perimeter by $2(l + w)$, where l is the length and w the width



Example-1

Find the perimeter and area of each shape below.



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Solution

(a) The perimeter is found by adding the lengths of all sides.

P = 6+8+1+4+4+4+1+8

= 36 cm

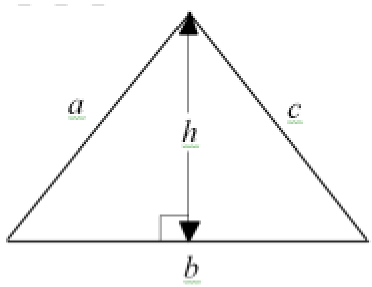
(b) Adding the lengths of the sides gives

P = 10+7+8+2+2+5

= 34 cm

- ***Area of a Triangle:***

For a triangle, the area is given by $^1/_2$bh and the perimeter by a + b + c, where b is the length of the base, h is the perpendicular height and a and c are the lengths of the other two sides.



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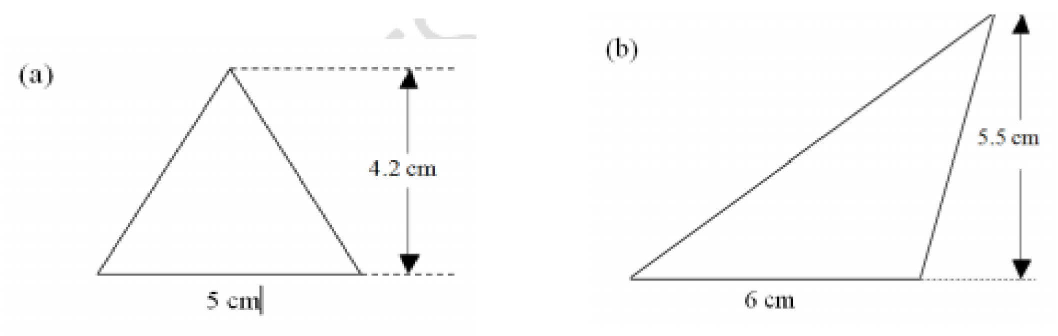
Example-1

Find the area of each triangle below



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Solution.

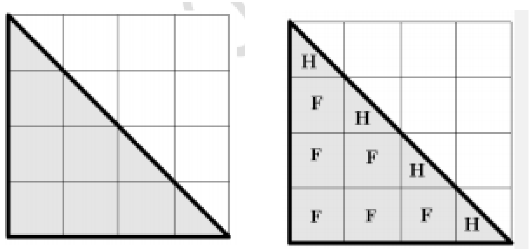
a) Use Area = ½ bh or ½ × base × height
A = ½ × 5 × 4.2

= 10.5 cm^2

b) Area = ½ × 6 × 5.5
=16.5cm^2

Example -2

Find the area of the shaded triangle



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Solution.

The triangle covers 6 full squares marked F, and 4 half squares marked H. Area 6 + 2 = 8 cm^2

- ***Area and Circumference of Circle***

The circumference of a circle can be calculated using

$C = 2\pi r$ or πd

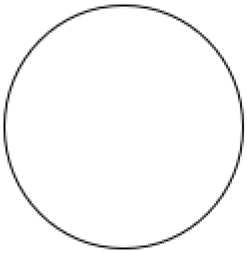
Where r is the radius and d the diameter of the circle

The area of the circle is found using

$A = \pi r^2$ or $A = \frac{\pi d^2}{4}$

Example-1

Find the circumference and area of a circle of radius 4 cm



Solution.

The circumference is found using $\mathbf{C=2\pi r}$

$C = 2\pi \times 4$

$C = 25.1cm$

The area is found using $A= \boldsymbol{\pi r^2}$

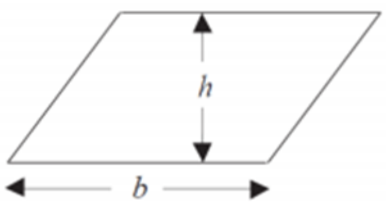
$A = \pi \times 4^2$

$= 50.3cm^2$

- ***Areas of Parallelograms, Trapeziums, Kites and Rhombuses***

The formulae for calculating the areas of these shapes are:

Parallelogram A= bh



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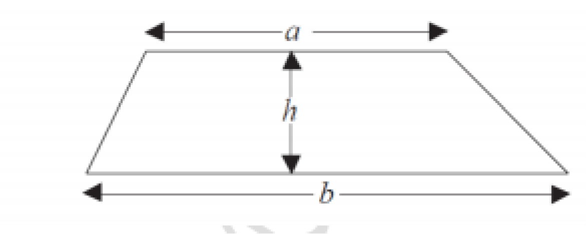
Trapezium A =1/2(a + b) h



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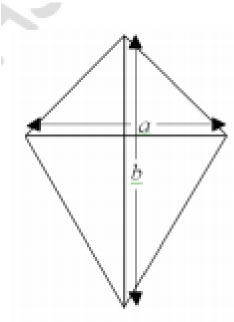
Kite A= ½ ab



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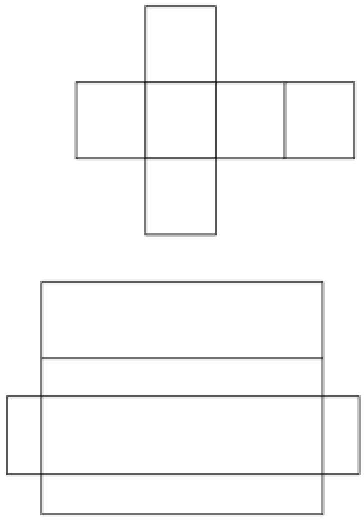
The area of a rhombus can be found using either the formula for a kite or the formula for a parallelogram.

- ***Surface Area of a cuboid***

The net of a cube can be used to find its surface area.

The net is made up of 6 squares, so the surface area will be 6 times the area of one square.

If x is the length of the sides of the cube its surface area will be 6 x 2.



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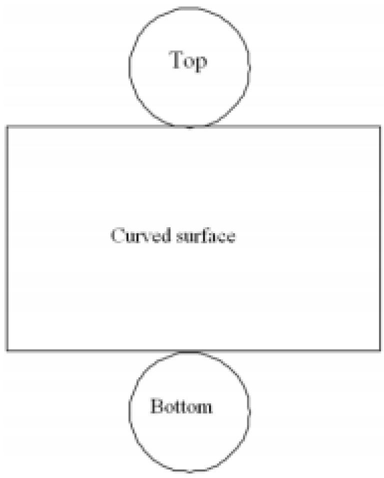
This diagram above shows the net for a cuboid.

To find the surface area the area of each of the 6 rectangles must be found and then added to give the total.

If x, y and z are the lengths of the sides of the cuboid, then the area of the rectangles in the net are as shown here

A= $2xy + 2xz + 2yz$

- ***Surface area of a cylinder***
  To find the surface area of a cylinder, consider how a cylinder can be broken up into three parts, the top, bottom and curved surface



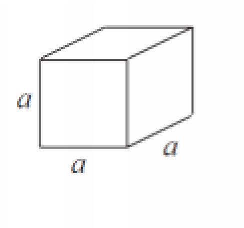
A= $2\pi r^2 + 2\pi rh$

- ***Volumes of Cubes, Cuboids, Cylinders and Prisms***
  - ***Volume of a Cube:***

    The volume of a cube is given by

    $V = a^3$

    A is the length of each side of the cube



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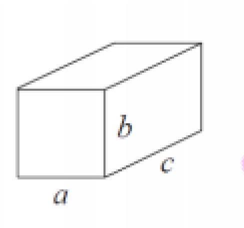
- ***Volume of Cuboid:***

  The volume of a cuboid is given by

  $V = abc$

  Where a, b and c are the lengths shown in the diagram



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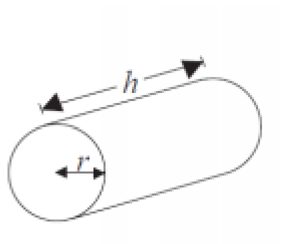
- ***Volume of a cylinder:***

  The volume of a cylinder is given by

  $V = \pi r^2 h$

  Where r is the radius of the cylinder and h its height



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➢ ***Volume of a Triangular prism***

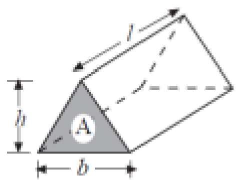
The volume of a triangular prism can be expressed in 2 ways

V = A *l*

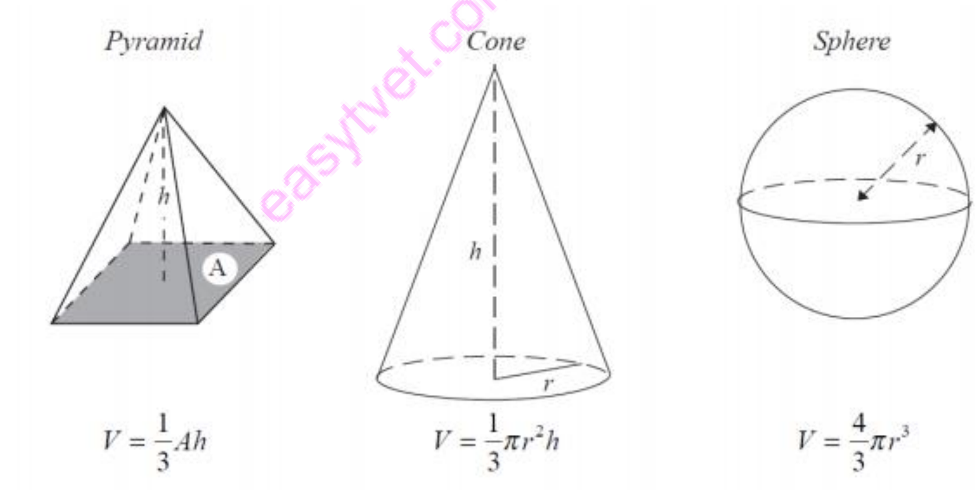
Where A is the area of the end and l is the length of the prism

Or V = ½*bh l*

Where b is the base of the triangle and h is the height of the triangle



Other forms of volumes that can be calculated are;



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- **Discrete and Continuous Measures**

  Quantities are said to be **discrete** if they can only take particular values.

  For example,

  (i) Number of trainees in a class 28, 29, 30, 31, 32…

  (ii) Shoe size 6 ½, 7, 7 ½ , 8, 8 1/2 , 9 …

(iii) Bottle of milk in a fridge 0, 1, 2, 3, …

Quantities that can take any value within a range are said to be **continuous**.

For example, height, weight, Time

1.2.8.4 Learning Activities

Learning outcomes, No .8 Apply Laplace transforms	
Learning Activities	**Special instruction**
Perform calculation on unit conversions Perform calculations on perimeters of different shapes Perform calculations on area of different shapes Perform calculations on surface area of different objects Perform calculations on volumes of different objects	Carryout various exercises in order to gain confidence in mensuration and calculations

1.2.8.5 Self-assessment

Q1 A barrel is a cylinder with a radius of 40 cm and height 80cm. it is full of water

a) Find the volume of the barrel
b) Find the mass of the water in the barrel

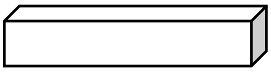
Q2 a metal bar has a cross section with an area of 3 cm^3 and a length of 40 cm. its mass is 300 grams

a) Find the volume of the bar
b) Find the density of the bar
c) Find the mass of another bar with the same cross section and a length of 50 cm
d) Find the mass of a bar made form the same material but with a cross section area of $5cm^3$ and length 80 cm

Q3 A bottle which holds $450cm^3$ of water has a mass of 530 grammes. What is the mass of the empty bottle?

Q4 A ream (500 sheets) of papers is shown

If the mass of the ream is 2.5 kg find the density of the paper.



Length= 30cm

Width= 21cm

Height = 5 cm

1.2.8.6 Tools, equipment's

- Calculator
- Laplace transform Tables
- HELM - Helping Engineers Learn Mathematics Workbooks
- SIGMA - Summary sheets covering key areas in Mathematics
- Lecture Notes - For the engineering analysis modules
- Links to (online) textbooks available in the LIS
- Links to external sites

1.2.8.7 References

Greenberg, M.D. (1998), Advanced Engineering Mathematics, 2nd ed., Prentice Hall (Upper Saddle River, N.J).

Hildebrand, F.B. (1974), Introduction to Numerical Analysis, 2nd ed., McGraw-Hill (New York).

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1.2.8.8 Suggested responses to self-assessment

1.2.9 Learning Outcome No.9 Apply Power Serries

1.2.9.1 Introduction to learning outcome

This learning outcome specifies the content of competencies required to Apply Power Series. It includes; Definition of the term power series, Taylor's theorem, Deduction of McLaurin's theorem to obtain power series and Application of Taylor's theorem and McLaurin's theorems in numerical work

1.2.9.2 Performance standard

9.1 Power series are obtained using Taylor's Theorem.

9.2 Power series are obtained using McLaurin's 's theorem

1.2.9.3 Information Sheet

The power series of Maclaurin's theorem different functions can be carried out using two theorems.

Taylor's Theorem

Maclaurin's theorem

Taylor's series states that;

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots$$

<u>Example-1</u>

Express$Sin(x+h)$ as a series of powers of h and hence evaluates $Sin\ 44^0$ correct to four decimal places.

Solution:

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots$$

$f(x) = Sinx$

$f'(x) = \cos x$

$f''^{(x)} = -sinx$

$f'''(x)$ =-cosx

$f^{iv}(x) = \sin x$

$\therefore Sin(x+h) = \sin x + h\cos x - \frac{h^2}{2}$ sin x - $\frac{h^3}{6}\cos x$ ....

$Sin44^0 = \sin(45^0 - 1^0)$

$= Sin(^{\pi}/_{4}) - 0.01745$

$= \sin {}^{\pi}/_{4} - 0.01745\ cos\frac{\pi}{4} - \frac{0.01745^2}{2}\ sin\,{}^{\pi}/_{4} + \frac{0.0174s^3}{6}\ cos\,{}^{\pi}/_{4}$

$But\ sin45 = cos\,45 = 0.707$

$= 0.707\ (1 - 0.01745 - 0.0001523 + 0.0000009)$

$= 0.707(0.982395)$

$= 0.69466$

$0.6947(4dp)$

For the use Maclaurin's theorem refer to Engineering Mathematics by Stroud.

1.2.9.4 Learning Activities

Learning Outcome No. 9 POWER SERIES	
Learning Activities	Special Instructions
Obtaining power series using Taylor's the Obtaining power series using Taylor's theorem Obtaining power series using Maclaurin's Theorem.	Carryout various exercises in order to gain confidence in power series

1.2.9.5 Self-Assessment

1. Use Maclaurin's theorem to expand $ln(3x+1)$. Hence use the expansion to evaluate $\int_0^1 \frac{\ln(3x+1)}{x^2}dx$ to four decimal places
2. Use Taylors series to expand $\cos\left(\frac{\pi}{3} + h\right)$ in terms of h as far as h^3. Hence evaluate $cos68^0$ correct to four decimal places

1.2.9.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

- HELM - Helping Engineers Learn Mathematics Workbooks
- SIGMA - Summary sheets covering key areas in Mathematics
- Lecture Notes - For the engineering analysis modules
- Links to (online) textbooks available in the LIS
- Links to external sites

1.2.9.7 References (APA)

Greenberg, M.D. (1998), Advanced Engineering Mathematics, 2nd ed., Prentice Hall (Upper Saddle River, N.J).

Hildebrand, F.B. (1974), Introduction to Numerical Analysis, 2nd ed., McGraw-Hill (New York).

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Kreyszig, E. (1999), Advanced Engineering Mathematics, 8th ed., John Wiley (New York).

1.2.9.8 Suggested responses to self-assessment

1.2.10 Learning Outcome No.10 Apply statistics

1.2.10.1Introduction to learning outcome

This learning outcome specifies the content of competencies required to Apply Statistics. It includes; Measures of central tendency mean, mode and median, Measures of dispersion, Variance and standard deviation, Definition of probability, Laws of probability, Expectation variance and S.D., Types of distributions, Mean, variance and SD of probability distributions and Application of probability distributions.

1.2.10.2Performance Standard

10.1 Mean, median, mode and Standard deviation are obtained from given data.
10.2 Calculations are performed based on Laws of probability.
10.3 Calculation involving ***probability distributions***, mathematical expectation sampling distributions are performed.
10.4 Sampling distribution methods are applied in data analysis.
10.5 Calculations involving use of standard normal table, sampling distribution, T-distribution and Estimation are done.
10.6 Confidence intervals are determined

1.2.10.3 Information sheet

a) Apply statistics

Statistics is discipline which deals with collection, organization, presentation and analysis of data.

Data consist of set of record observations that carry information on a particular setting with the availability of data.

A statistical exercise normally consists of four stages.

(i) Collection of data
(ii) Organization and presentation of data in convenient form
(iii) Analysis of data to make their meaning clear
Interpretation of results and
(iv) the conclusion.
In statistic we consider quantities that are varied.
These quantities are referred as variables.
Variables are denoted by letters

Examples

- Heights
- Ages
- Weighs
- Times

b) Types of data

- Quantitative data
- Qualitative data

❖ **Presentation of data**

The aim of presenting data is to communicate information.

The type of presentation chosen depend on the requirement and the interest of people receiving that particular information.

Frequently the first stage in presenting is preparing a table.

❖ **Tabulation of data**

➢ ***Frequency distribution***

Given a set of draw data we usually arrange into frequency distribution where we collect like quantities and display them by writing down their frequencies.

For those on data presentation refer to engineering mathematics by K.A stroud.

- ***Measures of central tendencies***

This is single value which used to represent entire set of data. It is typical value to which most observation fall closest than any other value.

There are mainly measures of central tendencies.

- *Arithmetic mean*
- *Median*
- *Mode*

Refer to Engineering mathematics for Engineers by H.K Dass

- ***Measurers of Dispersion***

They include:

- Range
- Standard deviation
- Quartiles

Read more about these measures in the above stated book

- **Normal Distribution**

This is a continuous distribution. It is derived as the limiting for of the Binomial distribution for large values of n and p and q are not very small.

The normal distribution is given by the equation

$$fx) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

Where μ is the mean and σ is the standard deviation, $\pi = 3.14159$ and $e = 2.71828$

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

On substitution $z = \frac{x-\mu}{\sigma}$ in (1) we get $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}z^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$

Here $mean = 0,\ standard\ deviation\ = 1$

Equation (2) is known as standard form of normal distribution.

- **Normal curve**

Shown graphically: the probabilities of heads in 1 losses are

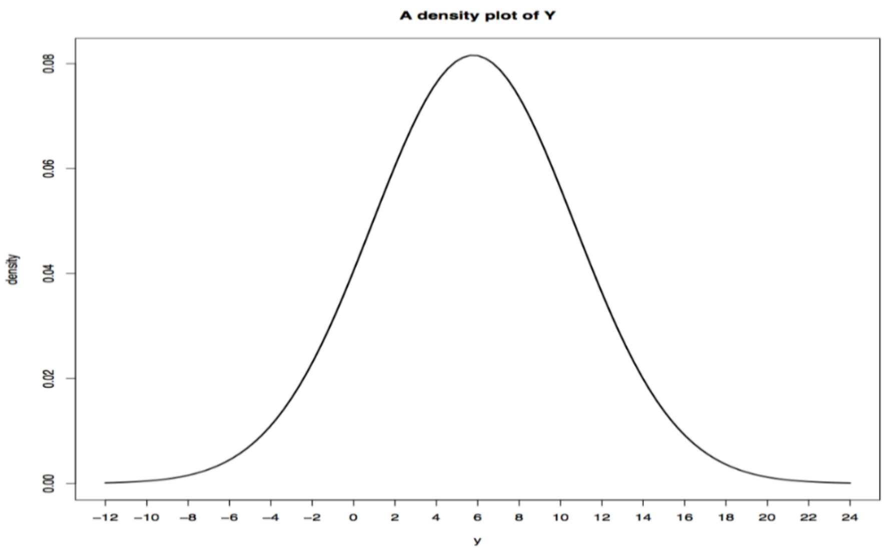

Figure 1A density Plot of Y

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- **Area under the normal curve**

  By taking $z = \frac{x-\mu}{\sigma}$, the standard normal curve is formed.

  The total area under this curve is 1. The area under the curve is divided into two equal parts by $z = 0$. Left had side area and right-hand side area to $z = 0$ is 0.5. The area between the ordinate $z = 0$ and any other ordinate can be calculated.

Example- 1

On the final examination in mathematics the mean was 72 and the standard deviation was15. Determine the standard scores of students receiving grades

60 b) 93 c) 72

Solution:

$$z = \frac{x-\mu}{\sigma} = \frac{60-72}{15} = -0.8$$

$$z = \frac{93-72}{15} = 1.4$$

$$z = \frac{72-72}{15} = 0$$

For more on variance and standard deviation refer to Mathematics for Engineers by H.K Dass.

Read on Poison and standard deviation of Binomial distribution.

1.2.10.4 Learning Activities

Learning outcomes No. 10 Apply Statistics	
Learning Activities	**Special instruction**
• Perform identification collection and organization of data • Perform interpretations analysis and presentation of data in appropriate format • Evaluate mean median mode and standard deviation obtained from the given data • Performing calculations based on laws of probability • Performing calculations involving probability distributions and mathematical expectation sampling.	Carryout various exercises in order to gain confidence in statistics

1.2.10.5 Self-Assessment

1. A machine produced components whose masses are normally distributed with mean μ and standard deviation σ if 89.8 % of the components have a mass of at least 88g and 3% have a mass less than 84.5g. Find the mean and the standard deviation of the distribution (6mks).
2. The diameters of bolts produced by a certain machine are distributed by a probability density function.

$$f(x) = \begin{cases} kx(3-x), & 0 \leq x \leq 3 \\ 0 & Otherwise \end{cases}$$

   Find the Constant k.

3. Probability that the diameter of a bolt-selected at a random will fall in the interval

$$1 < x < 2.5$$

   Mean and the variance of the distribution (14mks)

4. Tumaini Ltd, is supplied with petrol once a week. The weekly demand, x hundreds of liters, has the probability density function

$$f(x) = \begin{cases} k(1-x), & 0 \leq x \leq 1 \\ 0 & Otherwise \end{cases}$$

Where c is a constant, Determine the;

i. Value of c
ii. Mean of x

5. Minimum capacity of the petrol tank if the probability that it will be exhausted in a given week is not to exceed 0.02.
6. Metal bars produced in a factory have masses that are normally distributed with mean μ and standard deviationσ. Given that 9.4% have a mass less than 45kg and 7% have a mass above 75kg. Evaluate the values of μ and σ and
7. Find the probability that a mass of a metal bar selected at random will be less than 40kg.

1.2.10.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

- HELM - Helping Engineers Learn Mathematics Workbooks
- SIGMA - Summary sheets covering key areas in Mathematics
- Lecture Notes - For the engineering analysis modules
- Links to (online) textbooks available in the LIS
- Links to external sites

1.2.10.7 References

Greenberg, M.D. (1998), Advanced Engineering Mathematics, 2nd ed., Prentice Hall (Upper Saddle River, N.J).

Hildebrand, F.B. (1974), Introduction to Numerical Analysis, 2nd ed., McGraw-Hill (New York).

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1.2.10. **Suggested responses to self-assessment**

1.2.11 Learning Outcome No.11 Apply Numerical Methods

1.2.11.1Introduction to the unit of learning

This learning outcome specifies the content of competencies required to Apply Numerical Methods. It includes; Definition of interpolation and extrapolation, Application of interpolation, Application of interactive methods to solve equations and Application of interactive methods to areas and volumes

1.2.11.2 Performance Standards

11.1 Roots of polynomials are obtained using iterative ***numerical methods***.
11.2 Interpolation and extrapolation are performed using numerical methods.

1.2.11.3 Information Sheet

a) Apply numerical methods

The limitation of analytical methods, have led engineers and scientist to evolve graphical and numerical methods. The graphical methods though simple give results to a low degree of accuracy. Numerical methods can, however, be derived which are more accurate.

Numerical methods are often of a repetitive nature. These consist of repeated execution of the same process where at each step the results of the proceeding step are used. This is known as iteration process and is repeated till the result is obtained to a derived degree of accuracy.

The numerical methods for the solution of algebra and transcendental equations include:

- Method of false solution/ regular false method
- Newton-Raphson method.

<u>Example -1</u>

Find by Newton Raphson method the real roots of $xe^x - 2 = t$ correct to 3 decimal places.

Solution:

Let $f(x) = xe^x - 2$ and $f(1) = e - 2 = 0.7183$

So, a root lies between 0 and 1. Its near to 1 let's take $x_0 = 1$

Also $f'(x) = xe^x + e^x$ and $f'(1) = e + e = 5.4366$

Therefore, by Newton-Raphson rule, the first approximation x_1 is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{0.7183}{5.4366} = 0.8679$$

Therefore $f(x_1) = 0.0672, \quad f'(x_1) = 4.4491$

Thus, the first approximation x_2 is given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.8679 - \frac{0.0672}{4.4491} = 0.8528$$

Hence the required root is 0.853 correct to 3 dp

c) Finite differences and interpolation

Suppose we are given the following values for $y = f(x)$ for a set of values of x:

$$x: x_0 \; x_1 \; x_2 \; \ldots . x_n$$

$$y: y_0 \; y_1 \; y_2 \; \ldots . y_n$$

Then the process of finding any value of y corresponding to any value of $x = x_i$ between x_0 and x_n is called interpolation.

The interpolation is the technique of estimation the value of a function for any value of the intermediate value of the independent variable while the process of computing the value of the function outside the given range is called extrapolation.

Example -1

The table gives the distance in nautical miles of the visible horizon for the given height in feet above the earth's space.

Height (x)	100	150	200	250	300	350	400
Distance (y)	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find values of y when x =2.8 feet and 410 $feet$

The difference table is as given below

x	y	Δ	Δ^2	Δ^3	Δ^4
100	10.63				
		2.40			
150	13.03		-0.39		
		2.01		0.15	
200	15.04		-0.24		-0.07
		1.77		0.08	
250	16.81		-0.16		-0.05
		1.61		0.03	
300	18.42		-0.13		-0.01
		1.48		0.02	
350	19.90		-0.11		
		1.37			
400	21.27				

If we take $x_0 = 200$, then $y_0 = 15.04$, $\Delta_{y_0} = 1.77$ $\Delta^2{}_{y_0} = -0.16$ $\Delta^3{}_{y_0} = 0.03$ etc

Since $x = 218$ and $h = 50 \therefore p = \frac{x-x_0}{h} = \frac{18}{50} = 0.36$

Using Newton Raphson's forward interpolation formula, we get

$$y_{218} = y_0 + p\Delta_{y_0} + \frac{p(p-1)\Delta^2{}_{y_0}}{1.2} + \frac{p(p-1)(p-2)}{1.2.3} \quad \Delta^3{}_{y_0} + \cdots$$

$$f(218) = 15.04 + 036(1.77) + \frac{0.36(-0.64)}{2}(-0.16) + \frac{0.36(0.64)(-1.64)(0.03)}{6}(0.03) + \cdots$$

$$= 15.04 + 0.637 + 0.018 + 0.001 + \cdots. = 15.696\ nautical\ miles$$

Since $x = 410$ is near the end of the table, we use Newton backward interpolation formular.

Therefore taking $x_n = 400p = \frac{x-x_n}{h} = \frac{10}{50} = 0.2$

Using the backward differences

$y_n = 21.27$, $\nabla_{y_n} = 1.37$ $\nabla^2{}_{y_n} = 0.11$ $\nabla^3{}_{y_n} = 0.02$

Newton backward difference gives

$$y_{410} = y_{405} + p\Delta_{y_{400}} + \frac{p(p+1)\nabla^2{}_{y_{400}}}{1.2} + \frac{p(p+1)(p+2)}{1.2.3} \quad \nabla^3{}_{y_{400}} + \cdots$$

$$21.27 + 0.2(1.37) + \frac{0.2(1.2)}{2}(-0.11) + \cdots. = 21.53\ nautical\ miles$$

Refer to Higher Engineering Mathematics by Dr B.S and learn forward and backward interpolation formulae

1.2.11.4 Learning Activities

Learning outcomes No.14 Apply numerical methods	
Learning Activities.	**Special instruction**
Apply numerical methods in solving engineering works Roots of polynomials are obtained using iterative ***numerical methods*** Interpolation and extrapolation are performed using numerical methods	Carryout various exercises in order to gain confidence in numerical methods

1.2.11.5 Self-Assessment

1. Using Newton-Raphson forward formula find the values of $f(1.6)$, if

X	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.5

2. State Newton interpolation formula and use it to calculate the value of $\exp(1.85)$ given the following table

X	1.7	1.8	1.9	2.0	2.1	2.2	2.3
$f(x)$	5.474	6.000	6.686	7.389	8.166	9.025	9.914

1.2.11.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

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- SIGMA - Summary sheets covering key areas in Mathematics
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- Links to external sites

1.2.11.7 References

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1.2.11.8 Suggested responses to self-assessment

1.2.12 Learning Outcome No.12 Apply vector

1.2.12.1 Introduction to the learning outcome

This learning outcome specifies the content of competencies required to Apply Vectors. It includes; Vectors and scalar in two and three dimensions, Operations on vectors: Addition and Subtraction, Position vectors and Resolution of vectors.

1.2.12.2 Performance Standards

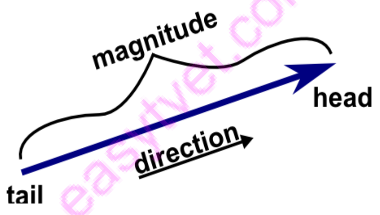
12.1 Vectors and scalar quantities are obtained in two and three dimensions,
12.2 ***Operations*** on vectors are performed,
12.3 Position of vectors is obtained.
12.4 Resolution of vectors is done

1.2.12.3 Information Sheet

a) Introduction

- ***Definition of a vector***

A vector is an object that has both a magnitude and a direction. Geometrically, we can picture a vector as a directed line segment, whose length is the magnitude of the vector and with an arrow indicating the direction. The direction of the vector is from its tail to its head.

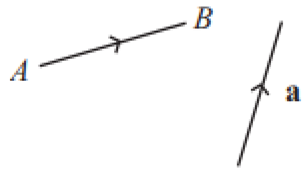


https://mathinsight.org/vector_introduction

Two vectors are the same if they have the same magnitude and direction. This means that if we take a vector and translate it to a new position (without rotating it), then the vector we obtain at the end of this process is the same vector we had in the beginning.
Two examples of vectors are those that represent force and velocity. Both force and velocity are in a particular direction. The magnitude of the vector would indicate the strength of the force or the speed associated with the velocity.

- ***Representing vector quantities:***

We can represent a vector by a line segment. This diagram shows two vectors

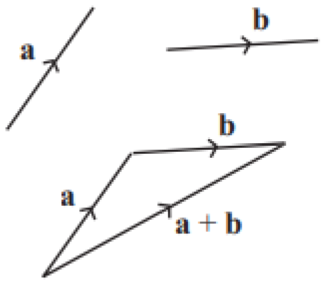


https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-introvector-2009-1.pdf

We have used a small arrow to indicate that the first vector is pointing from A to B. A vector pointing from B to A would be going in the opposite direction. Sometimes we represent a vector with a small letter such as a, in a bold typeface. This is common in textbooks, but it is inconvenient in handwriting. In writing, we normally put a bar underneath, or sometimes on top of, the letter: a or a. In speech, we call this the vector "a-bar"

https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-introvector-2009-1.pdf

- ***Adding two vectors :***
  One of the things we can do with vectors is to add them together. We shall start by adding two vectors together. Once we have done that, we can add any number of vectors together by adding the first two, then adding the result to the third, and so on. In order to add two vectors, we think of them as displacements. We carry out the first displacement, and then the second. So, the second displacement must start where the first one finishes.



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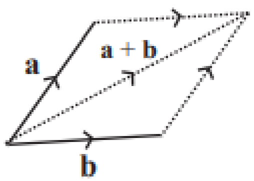
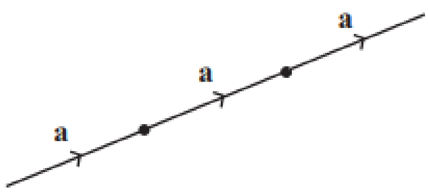
The sum of the vectors, a + b (or the resultant, as it is sometimes called) is what we get when we join up the triangle. This is called the triangle law for adding vectors. There is another way of adding two vectors. Instead of making the second vector start where the first one finishes, we make them both start at the same place, and complete a parallelogram. This is called the parallelogram law for adding vectors. It gives the same result as the triangle law, because one of the properties of a parallelogram is that opposite sides are equal and in the same direction, so that b is repeated at the top of the parallelogram.



https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-introvector-2009-1.pdf

- ***Adding a vector to itself***
  What happens when you add a vector to itself, perhaps several times? We write, for example, a + a + a = 3



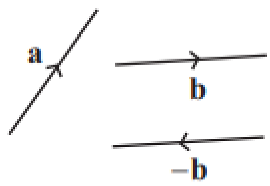
In the same way, we would write

$$n\mathbf{a} = \underbrace{\mathbf{a} + \ldots + \mathbf{a}}_{n \text{ copies}}.$$

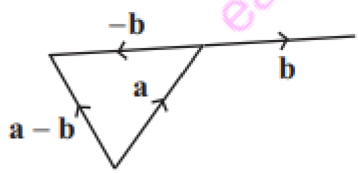
https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-introvector-2009-1.pdf

❖ ***Subtracting two vectors***

What is a − b? We think of this as a + (−b), and then we ask what −b might mean. This will be a vector equal in magnitude to b, but in the reverse direction.



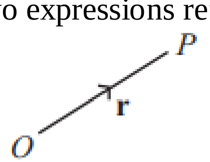
Now we can subtract two vectors. Subtracting b from a will be the same as adding −b to a.



https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-introvector-2009-1.pdf

❖ ***Position vectors***

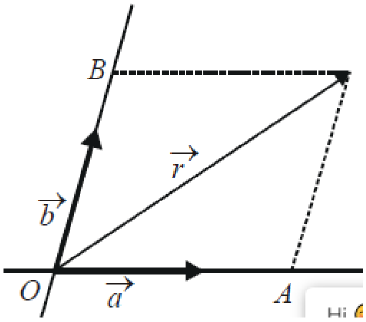
Sometimes vectors are referred to a fixed point, an origin. Such a vector is called a position vector. So, we might refer to the position vector of a point P with respect to an origin O. In writing, might put OP for this vector. Alternatively, we could write it as r. These two expressions refer to the same vector.



https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-introvector-2009-1.pdf

- ***Resolution of vectors.***

  Consider two non-collinear vectors →a and → b a→ and b→; as discussed earlier, these will form a basis of the plane in which they lie. Any vector → r r→ in the plane of →a and →b a→ and b→ can be expressed as a linear combination of →a and→ b a→ and b→:



$$\vec{r} = \overrightarrow{OA} + \overrightarrow{OB}$$

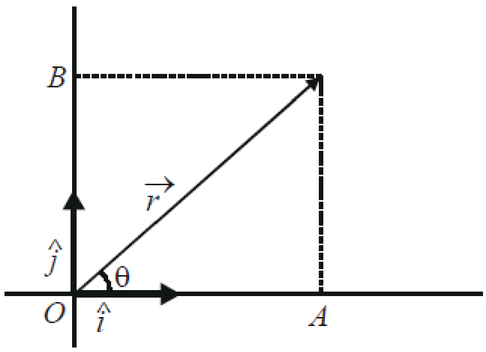
The vectors −→OA and −→OBOA→ and OB→ are called the components of the vector →rr→ along the basis formed by →a and→ ba→ and b→ . This is also stated by saying that the vector →when resolved along the basis formed by →a and→ ba→ and b→ , gives the components −→OA and−→OB OA→ and OB→ . Also, as discussed earlier, the resolution of any vector along a given basis will be unique.

https://www.cuemath.com/jee/resolution-vectors/

- ***Rectangular resolution***

  Let us select as the basis for a plane, a pair of unit vector ^i and ^ji^ and j^ perpendicular to each other.

  We can extend this to the three-dimensional case: an arbitrary vector can be resolved along the basis formed by any three non-coplanar vectors, giving us three corresponding components. Refer to Fig - 20 for a visual picture.



Let θ be the angle that $\vec{r}$ makes with $\hat{i}$

$\Rightarrow \quad OA = |\vec{r}| \cos\theta$

$\quad\quad OB = |\vec{r}| \sin\theta$

Fig - 22

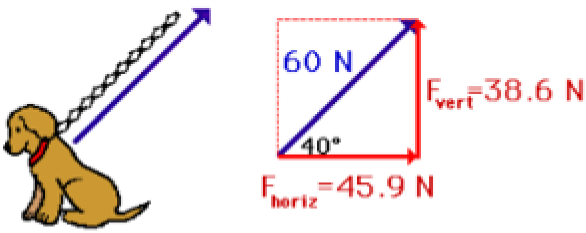
Any vector $\vec{r}$ in this basis can be written as

$$\begin{aligned} \vec{r} &= \overrightarrow{OA} + \overrightarrow{OB} \\ &= (|\vec{r}|\cos\theta)\,\hat{i} + (|\vec{r}|\sin\theta)\,\hat{j} \\ &= x\hat{i} + y\hat{j} \end{aligned}$$

where *x* and *y* are referred to as the *x* and *y* components of →rr→.

https://www.cuemath.com/jee/resolution-vectors

The above method is illustrated below for determining the components of the force acting upon Fido. As the 60-Newton tension force acts upward and rightward on Fido at an angle of 40 degrees, the components of this force can be determined using trigonometric functions.



$\text{Sin } 40^\circ = \dfrac{F_{vert}}{60N}$

$F_{vert} = 60N \times \sin 40^\circ$

$F_{vert} = 38.6\ N$

$\text{Cos } 40^\circ = \dfrac{F_{horiz}}{60N}$

$F_{horiz} = 60N \cos 40^\circ$

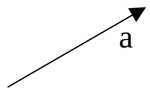
$F_{horiz} = 45.9\ N$

1.2.12.4 Learning Activities

Learning outcomes.No.12 APPLY VECTORS THEORY	
Learning Activities	**Special instruction**
➢ Vectors and scalar in two and three dimensions ➢ Operations on vectors: • Addition and Subtraction • Position vectors • Resolution of vectors	Carryout various exercises in order to gain confidence in vectors

1.2.12.5 Self-Assessment

1. The vector a is shown below



Sketch the vectors 2a, 3a, 1/2 a and −2a.

2. In ΔOAB, OA = a and OB = b. In terms of a and b,
   (a) What is AB?
   (b) What is BA?
   (c) What is OP, where P is the midpoint of AB?
   (d) What is AP?
   (e) What is BP?
   (f) What is OQ, where Q divides AB in the ratio 2:3?

3. What is meant by a unit vector?
4. If e is a unit vector, what is the length of 3e?
5. In ΔABC, AB = a, BC = b, CA = c. What is a + b + c?

1.2.12.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

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1.2.12.7 References

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1.2.12.8 Suggested responses to self-assessment

1.2.13 Learning Outcome No.13 Apply Matrix

1.2.13.1 Introduction to the learning outcome

This learning outcome specifies the content of competencies required to Apply Matrix. It includes; Matrix operation, Determinant of 3x3 matrix, Inverse of 3x3 matrix, Solution of linear simultaneous equations in 3 unknowns and Application of matrices.

1.2.13.2 Performance standards

13.1 Determinant and inverse of 3x3 matrix are obtained.
13.2 Solutions of simultaneous equations are obtained.
13.3 Calculation involving Eigen values and Eigen vectors are performed

1.2.13.3 Information Sheet

a) Introduction

A matrix is a set of real or complex numbers (or elements) arranged in rows and columns to form a rectangular array.

A matrix having M rows and N columns is called a $M \times N$ $(i.e\ M\ by\ N)$ matrix and is referred to as having order $M \times N$.

A matrix is indicated by writing the array with large square brackets e.g.

$\begin{bmatrix} 5 & 7 & 2 \\ 6 & 3 & 8 \end{bmatrix}$ is a 2×3 matrix i.e 2 by 3 matrix where $5,7,2,6,3$ and 8 are the elements of the matrix.

c) Operation on matrices

❖ Addition and subtraction

Matrices can be added or subtracted if they are of the same order.

Example-1

Given matrix $A = \begin{bmatrix} -1 & 2 & 5 \\ 3 & 0 & 4 \\ 1 & -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 1 & 6 \\ 1 & -4 & 5 \end{bmatrix}$

$$A + B = \begin{bmatrix} -1 & 2 & 5 \\ 3 & 0 & 4 \\ 1 & -3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & -1 \\ -2 & 1 & 6 \\ 1 & -4 & 5 \end{bmatrix} = \begin{bmatrix} (-1+3) & (2+2) & (3 \pm -1) \\ (3 \pm 2) & (0+1) & (4+6) \\ (1+1) & (-3 \pm 4) & (2+5) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 4 \\ 1 & 1 & 10 \\ 2 & -7 & 7 \end{bmatrix}$$

❖ Multiplication of Matrices

Example -1

Given matrix $A = \begin{bmatrix} -1 & 2 & 5 \\ 3 & 0 & 4 \\ 1 & -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 1 & 6 \\ 1 & -4 & 5 \end{bmatrix}$

$$A \times B = \begin{bmatrix} -1 & 2 & 5 \\ 3 & 0 & 4 \\ 1 & -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ -2 & 1 & 6 \\ 1 & -4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 3 + 2 \times -2 + 5 \div 1 & -1 \times 2 + 2 \times 1 + 6 \times -4 & -1 \times 1 + 2 \times 6 + 5 \times 5 \\ 3 \times 3 + 0 \times -2 + 4 \times 1 & 3 \times 2 - 0 \times 1 + 4 \times -4 & 3 \times -1 + 0 \times 6 + 4 \times 5 \\ 1 \times 3 \pm 3 \times -2 + 2 \times 1 & 1 \times 2 - 3 \times 1 + 2 \times -4 & 1 \times -1 + 6 \times -3 + 2 \times 5 \end{bmatrix}$$

$$= \begin{bmatrix} -3 - 4 + 5 & -2 + 2 - 20 & -1 + 12 + 25 \\ 9 + 0 + 4 & 6 + 0 - 16 & -3 + 0 + 20 \\ 3 + 6 + 2 & 1 - 3 - 8 & -1 - 1 + 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -20 & 36 \\ 13 & * 10 & 17 \\ 11 & -10 & -9 \end{bmatrix}$$

Note that in matrices $AB \neq BA$

For more examples on matrices operations refer to Pure Mathematics I.

❖ Determinant of a 3 × 3 matrix

Refer to engineering mathematics by K.A Stround and learn more on how to find the determinant of a 3 × 3 ("3 by 3") matrix.

❖ Inverse of a 3 × 3 matrix

Example- 1

To find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 6 & 0 & 2 \end{bmatrix}$

Evaluate the determinant of A i.e $|A|$

For $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 6 & 0 & 2 \end{bmatrix}$, $|A| = 1(2-0) - 2(8-30) + 3(0-6) = 28$

Now from the matrix of the cofactors

$$C = \begin{bmatrix} 2 & 22 & -6 \\ -4 & -16 & 12 \\ 7 & 7 & -7 \end{bmatrix}$$

Next, we have to write down the transpose of "C" to find the adjoint of "A"

$$Adj\ A = C^T = \begin{bmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{bmatrix}$$

Finally, we divide the elements of adj A by the value of $|A|$ i.e 28 to get A^{-1} the inverse of A.

$$A^{-1} = \begin{bmatrix} \frac{2}{28} & \frac{-4}{28} & \frac{7}{28} \\ \frac{22}{28} & \frac{-16}{28} & \frac{12}{28} \\ \frac{-6}{28} & \frac{7}{28} & \frac{-7}{28} \end{bmatrix} = \frac{1}{28}\begin{bmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{bmatrix}$$

❖ Solution of linear equation in three unknowns

Solve the set of equations

$x_1 + 2x_2 + x_3 = 4$

$3x_1 - 4x_2 - 2x_3 = 2$

$5x_1 + 3x_2 + 5x_2 = -1$

First write the set of equations in matrix form

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

Next step is to find the inverse of A where A is the square matrix on the left-hand side.

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 3 & 5 \end{vmatrix} = -14 - 50 + 29 = 29 - 64 = -35$$

Therefore $|A| = -35$

Matrix of co-factors $C = \begin{bmatrix} -14 & -25 & 29 \\ -7 & 0 & 7 \\ 0 & 5 & -10 \end{bmatrix}$

The matrix of the $Adj\ A = C^T = \begin{bmatrix} -14 & -7 & 0 \\ -25 & 0 & 5 \\ 29 & 7 & -10 \end{bmatrix}$

Now $|A| = -35,$ therefore $A^{-1} = \frac{adj\ A}{|A|} = \frac{1}{35}\begin{bmatrix} -14 & -7 & 0 \\ -25 & 0 & 5 \\ 29 & 7 & -10 \end{bmatrix}$

$$\therefore \quad x = A^{-1}.b = \frac{1}{35}\begin{bmatrix} -14 & -7 & 0 \\ -25 & 0 & 5 \\ 29 & 7 & -10 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$$

Finally, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$

$$\therefore x_1 = 2, x_2 = 3, x_3 = -4$$

- **Eigen values and Eigen vectors**

In many applications of matrices to technological problems involving coupled oscillations and vibrations equations are of the form

$$A.x = \lambda x$$

Where A is a scale matrix and λ is a number, the values of λ are called he eigen values. Characteristic values or latent roots of the matrix A and the corresponding solution of the given equation
$A.x = \lambda x$ are called the eigevectors or characteristics vector of$A$.

For more information refer to Engineering Mathematics by K.A Stroud.

1.2.13.4 Learning Activities

Learning outcomes.No.13 APPLY MATRIX	
Learning Activities	**Special instruction**
• Apply matrix • Apply addition and subtraction • Multiplication • Determinant and inverse of 3x3 matrix are obtained • Solutions of simultaneous equations are obtained • Calculation involving Eigen values and Eigen vectors are performed	Carryout various exercises in order to gain confidence in matrix

1.2.13.5 Self- Assessment

1. Solve the following set of linear equations by matrix method

$$x_1 + 3x_2 + 2x_3 = 3$$

$$2x_1 - x_2 - 3x_3 = -8$$

$$5x_1 + 2x_2 + x_2 = 9$$

2 Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 5 & 1 \\ 2 & 0 & 6 \end{bmatrix}$

If $A.x = \lambda x$, $A = \begin{bmatrix} 2 & 2 & -2 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. Determine the eigenvalues of the matrix A and an eigen vector corresponding to each eigenvalue.

1.2.13.6 Tools, Equipment, Supplies and Materials for the specific learning outcome

- HELM - Helping Engineers Learn Mathematics Workbooks
- SIGMA - Summary sheets covering key areas in Mathematics
- Lecture Notes - For the engineering analysis modules
- Links to (online) textbooks available in the LIS
- Links to external sites

1.2.13.7 References

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1.2.13.8 Suggested responses to self-assessment