1503/203
MATHEMATICS II
June/July 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

CRAFT CERTIFICATE IN AUTOMOTIVE ENGINEERING

MODULE II

MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have mathematical tables/scientific calculator for this examination.
*Answer any **FIVE** of the following **EIGHT** questions in the answer booklet.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the matrix $A=\begin{bmatrix}1 & 1 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$,

show that $A^3-3A^2-7A-3I=0$, where I is an identity matrix. (9 marks)

(b) The stability of a mechanical system requires the application of three forces which satisfy the simultaneous equations.

$$2F_1+2F_2+F_3=9$$
$$F_1+F_2+2F_3=9$$
$$2F_1+F_2+2F_3=10$$

$f_1 = 1$
$f_2 = 2$
$f_3 = 3$

where F_1, F_2 and F_3 are in newtons. Use the inverse matrix method to determine the values of the forces.
(11 marks)

2. (a) Prove the identities:

(i) $$\frac{\sin\theta+\cos\theta}{\sin\theta}-\frac{\cos\theta-\sin\theta}{\cos\theta}=\sec\theta\,\text{cosec}\,\theta$$

(ii) $$\frac{\cos\theta+\sin\theta-\sin^3\theta}{\sin\theta}=\cot\theta+\cos^2\theta$$

(6 marks)

(b) Given that A, B and C are angles of a triangle, prove that

$\sin A+\sin(B-C)=2\sin B\cos C$ (4 marks)

(c) Solve the equations:

(i) $2\sin\theta=\cos(\theta-60^\circ)$

(ii) $\cos\theta+\cos3\theta=0$, for values of θ between 0° and 360°.

(10 marks)

3. (a) (i) Determine the middle term in the binomial expansion of $(2y+3x)^{[illegible]}$.

(ii) Find the value of the term in (i) when $x=\frac{1}{2}$ and $y=\frac{1}{3}$.

(6 marks)

(b) Find the first four terms in the binomial expansion of $(1+\frac{1}{2}x)^{\frac{1}{3}}$. (3 marks)

(c) (i) Use the binomial theorem to expand $\sqrt{\left(\frac{1+3x}{1-3x}\right)}$ as far as the term in x^3.

(ii) By setting $x=\frac{1}{28}$ in the result in (i), determine the approximate value of $\sqrt{31}$, correct to four decimal places.

(11 marks)

4. (a) Find $\frac{dy}{dx}$ from first principles, given that $y = \frac{1}{4-x}$. (5 marks)

(b) Use implicit differentiation to determine the values of:

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

at the point (0, −1) on the curve $x^2 + 2y^2 - 4xy + 6x - 2y = 4$. (8 marks)

(c) Determine the stationary points of the curve $f(x) = x^3 + 9x^2 - 21x + 3$, and state their nature. (7 marks)

5. (a) Solve the equations:

(i) $\frac{3}{x+2} - \frac{2}{x+3} = \frac{1}{2}$

(ii) $3(3^{2x}) - 7(3^x) + 2 = 0$

(12 marks)

(b) Three currents I_1, I_2 and I_3 in amperes, in an electrical network satisfy the simultaneous equations:

$I_1 + I_2 - 2I_3 = 1$

$-I_1 + 3I_2 + I_3 = 6$

$I_1 - I_2 + 3I_3 = 2$

Use the method of elimination to determine the values of the currents. (8 marks)

6. (a) Determine the values of P and Q such that $5\sinh x - 3\cosh x = Pe^x + Qe^{-x}$. (4 marks)

(b) Prove the identities:

(i) $\cosh^2 x - \sinh^2 x = 1$

(ii) $\tanh 3x = \frac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$

(7 marks)

(c) Express $\cosh^{-1} x$ in logarithmic form, and hence determine the value of $\cosh^{-1}(2)$, correct to three decimal places. (9 marks)

7. (a) Given the vectors $\underline{A} = 2\underline{i} - 3\underline{j} + 2\underline{k}$ and $\underline{B} = 4\underline{i} + 2\underline{j} - 3\underline{k}$, determine:

(i) a unit vector perpendicular to $\underline{A}$ and $\underline{B}$.

(ii) the angle between $\underline{A}$ and $\underline{B}$.

(14 marks)

(b) Temperature distribution in a workshop is given by the scalar function $T(x, y, z) = x^2y + 2xz^2$. Determine the magnitude of ∇T at the point (1, 2, 3).

(6 marks)

8. (a) Evaluate the integrals:

(i) $\int_{-1}^{1} \frac{x^{-2} - 2x^{-1} + 3}{x^{-2}} dx$

(ii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3\sin^2 x - 4\cos x + 2x)dx$

(9 marks)

(b) Use integration to determine the area of the region below the curve $y = 4 - x^2$ between the points $x = -2$ and $x = 2$.

(4 marks)

(c) Locate the stationary points of the function $x^2 - 2y^2 + 4xy - 6x + 12y$, and determine their nature.

(7 marks)

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