1. (a) The expression $y = ax^2 + bx + c$ has a value 4 when $x = 1$, 13 when $x = 2$ and 26 when $x = 3$. Determine the values of a, b and c. (8 marks)

(b) Solve the equation:

$$25^x - 5^{x+2} + 100 = 0.$$ (7 marks)

(c) Obtain the first **four** terms in the expression of $\left(1+\frac{x}{2}\right)^{10}$ and hence find the value of $(1.005)^{10}$. Correct to four decimal places. (5 marks)

2. (a) Given that:

$A = \begin{pmatrix} 2 & x \\ x & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$ and that AB is a singular matrix, determine the possible values of x. (8 marks)

(b) When Kirchhoff's Laws are applied to an electrical network, the following simultaneous equations for currents flowing in amperes in various closed loops are obtained:

$$\begin{aligned} 2i_2 + 5i_3 &= -5 \\ 3i_1 + i_2 + 2i_3 &= -7 \\ i_1 + 3i_2 + 4i_3 &= 5 \end{aligned}$$

Determine the values of i_1, i_2 and i_3 using Cramer's rule. (12 marks)

3. (a) Given that $\underset{\sim}{A} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{B} = -4\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$,

Determine:

(i) $\underset{\sim}{A}.\underset{\sim}{B}$;

(ii) $\underset{\sim}{A} \times \underset{\sim}{B}$;

(iii) the angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(8 marks)

(b) If $A = x^2yz + xz^2$ and $B = xy^2z - z^3$, find ∇AB at the point (1, 2, -1). (5 marks)

(c) Given that $\underset{\sim}{V} = 2xy\underset{\sim}{i} - 3zy\underset{\sim}{j} + x^2z\underset{\sim}{k}$, determine:

(i) Curl $\underset{\sim}{V}$;
(ii) Div $\underset{\sim}{V}$.

(7 marks)

4. (a) In a certain type of ammeter, the current i amperes when the deflection of the needle is θ degrees is given by the equation $i^2 \text{Sin}(\theta + 30°) = 0.845 \text{Cos}(\theta + 30°)$ where i is the current. Determine the deflection θ when the current is 1.75 A.

(4 marks)

(b) If Sin A = 0.8, where A is acute, determine Cos 4 A. (4 marks)

(c) Given that $8 \text{Cos}\,\theta + 21 \text{Sin}\,\theta = R \text{Cos}(\theta - \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$:

(i) find the values of R and α and hence;

(ii) solve the equation:

$8 \text{Cos}\,\theta + 21 \text{Sin}\,\theta = 18$ for $0° \leq \theta \leq 360°$.

(12 marks)

5. (a) If $5e^x - 2e^{-x} = A \text{Sinh}\,x + B \text{Cosh}\,x$, determine the values of A and B. (6 marks)

(b) Solve the equation $3 \text{Cosh}\,x + 2 \text{Sinh}\,x = 14.31$ correct to **four** decimal places.

(7 marks)

(c) The radius of the base of a cone and the height are measured 1% too large and 1.5% too small respectively. Determine, using binomial expansion, the percentage error involved in calculating the volume. (7 marks)

6. (a) Differentiate the following functions with respect to x:

(i) $y = \text{Sin}(2x + 1)$;
(ii) $y = x^3 \text{Cos}\,x$;
(iii) $y^2 + 2y^3 = 3x^4 + 5$.

(9 marks)

(b) Determine the co-ordinates of turning points of $y = x^3 - 6x^2 + 9x$ and hence distinguish between them. (11 marks)

7. (a) If $z = 3x^2y^2 + 3x^2 + 2y^3$ find, at the point (-1, 1):

(i) $\frac{d^2z}{dx^2}$;

(ii) $\frac{d^2z}{dydx}$.

(5 marks)

(b) The radius of a right circular cylinder is increasing at a rate of 3 cm/s and the height is decreasing at a rate of 2 cm/s. Find the rate at which the volume is changing when the radius is 6 cm and the height is 4 cm, using partial differentiation.

(8 marks)

(c) If $\underset{\sim}{V}_1 = 20$ m/s at 70°, $\underset{\sim}{V}_2 = 30$ m/s at 160° and $\underset{\sim}{V}_3 = 25$ m/s at 210°, determine the resultant of $\underset{\sim}{V}_1 - \underset{\sim}{V}_2 + \underset{\sim}{V}_3$. (7 marks)

8. (a) Evaluate the integrals:

(i) $\int (2x - 4)^3 dx$;

(ii) $\int Sin^2 2x \, dx$;

(iii) $\int_0^1 (x^3 + 3x^2 + 2)dx$.

(9 marks)

(b) Find the area enclosed between the curve $y = 2x^2 + x - 3$ and the x-axis. (6 marks)

(c) The distance S in metres of a fixed point is given by $S = 2t + 6t^2$. Determine the:

(i) distance S, when t = 3 seconds;
(ii) time at which its velocity is 62 m/s.

(5 marks)