1521/203 1601/203
1522/203 1602/203
MATHEMATICS II
Oct./ Nov. 2016
Time: 3 Hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

CRAFT CERTIFICATE IN ELECTRICAL AND ELECTRONIC TECHNOLOGY (POWER OPTION) (TELECOMMUNICATION OPTION)

MODULE II

MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have mathematical tables/ non-programmable scientific calculator for this examination:
Answer ***FIVE*** *of the following* ***EIGHT*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no question is missing.

1. (a) Write down the middle term in the binomial expansion of $(4x + 3y)^8$, and determine its value when $x = \frac{1}{3}$ and $y = \frac{1}{4}$. (7 marks)

(b) Find the first four terms in the binomial expansion of $\left(1 + \frac{1}{2}x\right)^{\frac{1}{2}}$, and state the values of x for which the expansion is valid. (5 marks)

(c) (i) Expand $(1 + x)^{-\frac{1}{2}}$ as far as the term in x^3.

(ii) By setting $x = \frac{1}{8}$ in the result in (i), determine the value of $\frac{1}{2\sqrt{2}}$, correct to four decimal places.

(8 marks)

2. (a) Given $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$, where A and B are acute, determine the values of:

(i) $\sin(A - B)$;

$4x + 3y$

$4x^8, 4x^7 3y^1, 4x^6 3y^2, 4x^5 3y^3, 4x^4 3y^4$

(ii) $\tan(A + B)$.

(7 marks)

1 1
1 2 1
1 3 3 1
1 4 6 4 1

$= 1, 4, 6, 4, 1$

$(1) 4x^8 + 4(4x^7 3y^1) + 6$

(b) Prove the identities:

(i) $\dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta} = 2\sec\theta$

(ii) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

(6 marks)

(c) (i) Given that $t = \tan 22\frac{1}{2}^\circ$, use the formula for $\tan 2\theta$ to show that $t^2 + 2t - 1 = 0$.

(ii) Hence, by solving the equation in (i), determine the value of $\tan 22\frac{1}{2}^\circ$, giving the answer in surd form.

(7 marks)

3. (a) By completing the square, determine the values of r, s and t such that $3x^2 + 4x + 10 = r(x + s)^2 + t$. (5 marks)

(b) Solve the equation:

$3^{2x+1} - 11(3^x) + 6 = 0$, correct to two significant figures. (6 marks)

(c) Currents I_1, I_2 and I_3 in an electric circuit satisfy the simultaneous equations:

$$I_1 - 2I_2 + 2I_3 = 3$$
$$2I_1 + 3I_2 - I_3 = 7$$
$$-3I_1 + 4I_2 + 5I_3 = 29.$$

Use elimination method to determine the values of the currents. (9 marks)

4. (a) Given the vectors $\underset{\sim}{A} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$, $B = -i + 3j + 4k$, determine the:

(i) angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(ii) area of the parallelogram spanned by the vectors $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(10 marks)

(b) The electric field $\underset{\sim}{E} = x^2\underset{\sim}{i} + z^2\underset{\sim}{j} + y^2\underset{\sim}{k}$ exists in a region of space. Determine, at the point (-1, 2, 1):

(i) $\nabla \times \underset{\sim}{E}$;

(ii) $\nabla \cdot \underset{\sim}{E}$.

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(10 marks)

5. (a) Given the matrices $A = \begin{bmatrix} 1 & -3 & 1 \\ -1 & 1 & 3 \\ 1 & 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, determine $(AB)^{-1}$. (9 marks)

(b) Three forces F_1, F_2 and F_3 in a mechanical system satisfy the simultaneous equations

$$2F_1 - F_2 + 3F_3 = 7$$
$$F_1 + F_2 - F_3 = 0$$
$$-F_1 + F_2 + 2F_3 = 4$$

$\int_0^{\pi/2} (3\cos x - 2\sin x + 1)\,dx$

Use Cramer's rule to solve the equations. (11 marks)

6. (a) Prove the identities:

(i) $\dfrac{\sin B - \sin C}{\sin B + \sin C} = \cot\left(\dfrac{B+C}{2}\right)\tan\left(\dfrac{B-C}{2}\right)$;

(ii) $\cosh^2 x - \sinh^2 x = 1$.

(4 marks)

(b) Solve the equations:

(i) $3\cot 2x + \cot x = 1$, for $0° \leq x \leq 360°$;

(ii) $5\cosh\theta + \sinh\theta = 5$.

(16 marks)

7. (a) Given $y = \frac{1}{x+3}$, find $\frac{dy}{dx}$ from first principles. (5 marks)

(b) Use implicit differentiation to determine the equation of the tangent to the curve $x^2 - 3y^2 + 4xy = 6$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$. (7 marks)

(c) Determine the stationary points on the curve $y = x^3 + 3x^2 - 72x + 6$, and state their nature. (8 marks)

8. (a) Given $z = \frac{x+y}{x-y}$, show that $x\frac{\partial^2 z}{\partial x^2} - y\frac{\partial^2 z}{\partial y^2} = 0$. (8 marks)

(b) Locate the stationary points of the curve $z = 2x^2 + 3y^2 - 8xy + 6x - 6y$, and determine their nature. (7 marks)

(c) Evaluate the integrals:

(i) $\int_0^1 \frac{x^{-3} + 2x^{-2} + x^{-1}}{x^{-4}}\, dx$;

(ii) $\int_0^{\pi/2} (3\cos x - 2\sin x + 1)\, dx$.

(5 marks)

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