1521/203
1601/203
1602/203
MATHEMATICS II
Oct./Nov. 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

CRAFT CERTIFICATE IN ELECTRICAL AND ELECTRONIC TECHNOLOGY (POWER OPTION) (TELECOMMUNICATION OPTION)

MODULE II

MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
This paper consists of ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Solve the equation $3^{2x+3} - 7(3^{x+1}) + 2 = 0$. (8 marks)

(b) Use the binomial theorem to expand $(2+3x)^{\frac{1}{2}}$ as far as the term in x^3, and state the range of values of x for which the expansion is valid. (4 marks)

(c) (i) Determine the first three terms in the binomial expansion of $(1+8x)^{\frac{1}{3}}$.

(ii) By putting $x = \frac{1}{27}$ in the result in (i), determine the value of $\sqrt[3]{35}$, correct to four decimal places. (8 marks)

2. (a) Given the matrices $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 3 \\ 1 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix}$, determine $(AB)^{-1}$.

(10 marks)

(b) Three currents I_1, I_2 and I_3 in an electric circuit satisfy the equations:

$$I_1 - 2I_2 + I_3 = -6$$
$$-I_1 + 2I_2 + I_3 = 4$$
$$2I_1 - I_2 + I_3 = -2$$

Use Cramer's rule to determine the values of the currents. (10 marks)

3. (a) Given that $\sin A = \frac{12}{13}$ and $\cos B = \frac{7}{25}$, and that A and B are acute, determine the values of:

(i) $\sin(A+B)$;
(ii) $\cos(A-B)$. (6 marks)

(b) Prove the identities:

(i) $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$;

(ii) $\dfrac{\cos B + \cos C}{\sin B - \sin C} = \cot\left(\dfrac{B-C}{2}\right)$. (7 marks)

(c) Solve the equation $2\cos 2\theta + 7\sin\theta = 5$, for values of θ between 0° and 180° inclusive. (7 marks)

4. (a) Find a unit vector that is perpendicular to the vectors $\underset{\sim}{A} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}$ and $\underset{\sim}{B} = -3\underset{\sim}{i} + 2\underset{\sim}{j} + 6\underset{\sim}{k}$. (7 marks)

(b) Given that the three vectors $\underset{\sim}{A} = \underset{\sim}{i} + \underset{\sim}{j} + a\underset{\sim}{k}$, $\underset{\sim}{B} = 7\underset{\sim}{i} + 2\underset{\sim}{j} + 0\underset{\sim}{k}$ and $\underset{\sim}{C} = 0\underset{\sim}{i} + \underset{\sim}{j} + 7\underset{\sim}{k}$ are coplanar, determine the value of a. (5 marks)

(c) Given the scalar function $\phi(x, y, z) = xy^3 + y^2 z$, determine, at the point (1, 1, -1):

(i) $\nabla\phi$

(ii) $\nabla \cdot \nabla\phi$

(iii) $\nabla \times \nabla\phi$. (8 marks)

5. (a) Given that the determinant:

$$\begin{vmatrix} -3 & 6 & -1 \\ -1 & 1-x & 0 \\ 3 & 0 & 1-x \end{vmatrix} = 0$$, determine the possible values of x. (7 marks)

(b) Two resistors, when connected in parallel, gave a total resistance of 9.6 ohms. When connected in series, the total resistance was 40 ohms. If one of the resistances is R ohms,

(i) show that $R^2 - 40R + 384 = 0$;

(ii) determine the values of the resistors. (6 marks)

(c) Application of Kirchoff's laws to a resistive network yielded the simultaneous equations:

$$I_1 + I_2 - I_3 = 0$$
$$-I_1 + 2I_2 + I_3 = 9$$
$$2I_1 - I_2 + 3I_3 = 1$$

Use the substitution method to determine the values of the currents. (7 marks)

6. (a) Find $\frac{dy}{dx}$ from first principles, given that $y = \frac{1}{x+4}$. (5 marks)

(b) Use implicit differentiation to determine the equation of the:

(i) tangent;
(ii) normal

to the curve $9x + x^3y^2 - 2xy^2 + 3y = 6$, at the point (1, -1). (10 marks)

(c) The power developed in a resistor of resistance R ohms in series with an emf source of V volts with an internal resistance r ohms is given by $P = \frac{V^2 R}{(R+r)^2}$

Show that the power developed is a maximum when R = r. (5 marks)

7. (a) Given $u = \dfrac{x+y}{x-y}$, show that $\dfrac{\partial^2 u}{\partial y \partial x} = \dfrac{\partial^2 u}{\partial x \partial y}$. (9 marks)

(b) Locate the stationary points of the function $z = x^2 + 3y^2 - 4xy + 6x - 2y$ and determine their nature. (8 marks)

(c) Evaluate the integral $\int_0^{\frac{\pi}{2}} (3\sin 2x + \cos x - 2)\, dx$. (3 marks)

8. (a) Given the trigonometric identities:

(i) $\cos 2\theta = 1 - 2\sin^2\theta$;

(ii) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$;

(iii) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$, use Osborne's rule to derive the corresponding hyperbolic identities. (3 marks)

(b) (i) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$;

(ii) Hence solve the equation $\cos 3\theta = \cos\theta$ for values of θ from 0° to 180° inclusive. (11 marks)

(c) Solve the equation $3\cosh\theta + 7\sinh\theta = 3$. (6 marks)

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