1521/203
1601/203
1602/203
MATHEMATICS II
June/July 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

CRAFT CERTIFICATE IN ELECTRICAL AND ELECTRONIC TECHNOLOGY (POWER OPTION) (TELECOMMUNICATION OPTION)

MODULE II

MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Non-programmable scientific calculator.

This paper consists of ***EIGHT*** *questions.*

Answer any ***FIVE*** *questions in the answer booklet provided.*

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1\. (a) Three currents in an electric circuit satisfy the simultaneous equations.

$$2I_1 + 3I_2 + 6I_3 = 25$$
$$6I_1 + 2I_2 - 3I_3 = -9$$
$$3I_1 - 6I_2 + 2I_3 = 34$$

Use the substitution method to determine the three currents. (12 marks)

(b) Determine the values of x which satisfy the equation $5(3^{2x}) + 6 = 31(3^x)$
Give answers correct to four decimal places. (8 marks)

2\. (a) Given the matrices $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

Determine:

(i) $C = 3A^2 - 4B$ $\sqrt{AA + BB - (A \times B)}$

(ii) $(AB)^{-1}$. (13 marks)

(b) Solve the equation $\cos(\theta + 20^\circ) - \cos(\theta + 38^\circ) = 0.3$ for $0^\circ \leq \theta \leq 360^\circ$. (7 marks)

3\. (a) Given the vectors $\underset{\sim}{A} = \underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{B} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$ determine:

(i) $|\underset{\sim}{A} \times \underset{\sim}{B}|$ $\sqrt{AA + BB - (A \cdot B)}$

(ii) the angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$. Dot product (10 marks)

(b) Determine the directional derivative of $V = x^2z^2 + 2xy^3 + y^2z$ at the point (1, 2, 1) in the direction of the vector $\underset{\sim}{A} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$. $\frac{A \cdot \nabla\phi}{|A|}$ (10 marks)

4\. (a) Expand $(1 - 3x)^{-\frac{1}{2}}$ as far as the term in x^2 and determine the range of values of x for which the expansion is valid. (4 marks)

(b) Determine the middle term in the binomial expansion of $(2x + 3)^8$ and determine it's value when $x = \frac{1}{12}$. (6 marks)

(c) (i) Determine the first three terms in the binomial expansion of $(1 + 8x)^{\frac{1}{3}}$.

(ii) By putting $x = \frac{1}{27}$ in the result in (i), determine the values $\sqrt[3]{35}$ correct to 4 decimal places. (10 marks)

$-\frac{1}{2}-2$

$-2-3$

5. (a) Given that $6e^{x}-4e^{-x}=A\sinh x+B\cosh x$, determine the values of A and B. **(6 marks)**

(b) Prove the hyperbolic identities

(i) $\tanh(x+y)=\dfrac{\tanh x+\tanh y}{1+\tanh x\tanh y}$

(ii) $\sinh 3\theta=3\sinh\theta+4\sinh^{2}\theta$

(iii) $\cosh 2\theta=1+\sinh^{2}\theta$ **(8 marks)**

(c) The energy W stored in an electric circuit is given by $W=Kr^{5}N^{2}$ where K is a constant, r is the radius and N is the number of revolutions. Use the binomial theorem to determine the approximate percentage change in W if r is increased by 1.4% and N is decreased by 1.8%. **(6 marks)**

6. (a) Determine the value of $\tan A$ given that $\tan(A-45^{\circ})=\dfrac{1}{4}$. **(3 marks)**

(b) Given that $7\cos\theta+20\sin\theta=R\cos(\theta-\alpha)$ where $R>0$ and $0^{\circ}\leq\alpha\leq 90^{\circ}$.

(i) determine the values of R and α and hence;
(ii) solve the equation $7\cos\theta+20\sin\theta=17$ for $0^{\circ}\leq\theta\leq 360^{\circ}$. **(9 marks)**

(c) Evaluate the integrals

(i) $\int(3x-5)^{4}\,dx$

(ii) $\int\sin^{2}3x\;dx$

(iii) $\int_{0}^{1}(2x^{3}+3x+2)dx$ **(8 marks)**

7. (a) Determine $\dfrac{dy}{dx}$, given

(i) $y=\dfrac{x-2}{x^{2}+3}$

(ii) $y=x^{3}\sin 2x$

(iii) $x^{2}+3y^{2}=2x^{4}+5$ **(8 marks)**

$x^2+3y^2-2x^4+5$

(b) Locate the stationary points of the function $z = x^2 + y^2 + xy + 6x - 6y + 5$ and determine their nature. (9 marks)

(c) The lengths of the three sides of a triangle are 6 cm, 8 cm and 10 cm. Use the Heron's formula to calculate the area of the triangle. (3 marks)

8. (a) Solve the equation $3^{2x} + 243 = 36(3)^x$. (4 marks)

(b) Solve the equation

$$\begin{vmatrix} x & 2 & 3 \\ 4 & 1 & 5 \\ 6 & 0 & x+1 \end{vmatrix} = 28$$

(4 marks)

(c) Use Cramer's rule to solve the simultaneous equations

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 3 \\ 2x_1 - x_2 - x_3 &= 11 \\ 3x_1 + 2x_2 + x_3 &= -5 \end{aligned}$$

(12 marks)

$x \quad 2$

$$x\begin{vmatrix} 1 & 5 \\ 0 & x+1 \end{vmatrix} - 2\begin{vmatrix} 4 & 5 \\ 6 & x+1 \end{vmatrix} + 3\begin{vmatrix} 4 & 1 \\ 6 & 0 \end{vmatrix}$$

$4x+4-30 \qquad 0-6$

$x(x+1-5) - 2(4x-26) + 3(-6)$.

$x^2 + 4x + 8x + 52 - 18$

$x^2 - 12x + 34 = 28$.

$x^2 - 12x + 6 = 0$.

THIS IS THE LAST PRINTED PAGE.