1601/203
1602/203
MATHEMATICS II
June/July 2023
Time: 3 Hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

CRAFT CERTIFICATE IN ELECTRICAL AND ELECTRONIC TECHNOLOGY (POWER OPTION) (TELECOMMUNICATION OPTION)

MODULE II

MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
This paper consists of ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing

1\. (a) Use the binomial theorem to expand $\left(1+\frac{1}{5}x\right)^{\frac{1}{3}}$ up to the term in x^3, hence evaluate $(1.04)^{\frac{1}{3}}$ correct to 4 decimal places. (10 marks)

(b) Three forces F_1, F_2 and F_3 in newtons necessary to keep a certain system in equilibrium satisfy the simultaneous equations.

$$2F_1 + 3F_2 - 4F_3 = -4$$
$$-F_1 + 2F_2 + 5F_3 = 18$$
$$3F_1 - 4F_2 + F_3 = -2$$

$-1\,(2F_1 + 3F_2 - 4F_3 = -4)$
$2\,(-F_1 + 2F_2 + 5F_3 = 18)$
$-2F_1 + 3F_2 - 4F_3 =$
$-2F_2 \quad 4F_2 + 10F_3$
$-4F_1$
$F_2 - F_3 =$
$-F_1 + 2F_2 + 5F_3 = 18.$

Use the elimination method to solve the equations. (10 marks)

2\. (a) Prove the trigonometric identities.

(i) $\dfrac{\cos\theta - \cos^3\theta}{\sin\theta} = \sin\theta\cos\theta$

(ii) $\dfrac{\cos 4\theta + \cos 2\theta}{\sin 4\theta + \sin 2\theta} = \cot 3\theta$

(iii) $\dfrac{\sin^2\theta(\sec\theta + \operatorname{cosec}\theta)}{\cos\theta\tan\theta} = 1 + \tan\theta$

(10 marks)

(b) $2(2^{2x}) - 5(2^x) + 2 = 0$ (6 marks)

(c) Prove the identity $1 + \cot^2\theta = \operatorname{cosec}^2\theta$. (4 marks)

3\. (a) Given the vectors: $\underset{\sim}{A} = 3\underset{\sim}{i} + 2\underset{\sim}{k},\ \underset{\sim}{B} = 4\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k},\ \underset{\sim}{C} = 3\underset{\sim}{i} + 5\underset{\sim}{j} - 4\underset{\sim}{k}$. 10

Determine:

(i) $-\underset{\sim}{B}$

(ii) $2\underset{\sim}{A} + 3\underset{\sim}{C}$ (5)

(iii) $4\underset{\sim}{B} - 6\underset{\sim}{A}$ (8 marks)

(b) Given that $\sin\theta = \dfrac{1}{\sqrt{5}}$ where θ is an acute angle, calculate the values of:

(i) $\cos\theta$

(ii) $\tan\theta$. (5 marks)

(c) $2^{2x+2} - 5(2^{x+1}) + 6 = 0$. (7 marks)

4. (a) Given that $z=\cos(2x+y^2)$, show that $y^2\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=0$. (4 marks)

(b) Given that $x=3\sec\theta$ and $y=12\tan\theta$, determine $\frac{d^2y}{dx^2}$ at the point $\theta=\frac{\pi}{4}$. (8 marks)

(c) Given that $f(x)=3x^3-28x^2+12x+4$, determine:

(i) the co-ordinates of the stationary points.

(ii) the nature of the points in (i). (8 marks)

5. (a) Given that the matrix $A=\begin{bmatrix} x-4 & -3 \\ x & x+1 \end{bmatrix}$ is a singular matrix, determine the:

(i) possible values of x; $(x-4 \times x+1)-(x\times(-3))$ $-3x$

(ii) two forms of matrix A. (7 marks)

(b) Solve by completing the square method $3x^2-7x-11=0$. (7 marks)

(c) Given $y=\frac{1}{x^2}$ find $\frac{dy}{dx}$, from first principles. (6 marks)

6. (a) Three currents I_1, I_2 and I_3 in amperes flowing in a direct current circuit satisfy the simultaneous equation.

$$2I_1+4I_2+I_3=6$$
$$3I_1+5I_2+2I_3=11$$
$$2I_1+5I_2-I_3=1$$

10 15 12 27 15 12

Use substitution method to solve the equations. (12 marks)

(b) Use implicit differentiation to determine the:

(i) value of $\frac{dy}{dx}$

(ii) the equation of the normal to the curve $x^3+2y^2-6xy+3x+2y=10$ at the point $(1,-1)$. (8 marks)

7. (a) The volume of a rectangular box of dimensions a, b and c is given by $v = abc$. Use partial differentiation to determine the approximate change in volume if a increases from 4 cm to 4.1 cm, b decreases form 3 cm to 2.6 cm while c increases form 6 cm to 6.5 cm. (4 marks)

(b) Evaluate the integrals

(i) $\int_0^1 \left(\frac{x^3 - x^2 + x}{x^{-1}}\right) dx$

(ii) $\int_0^\pi (x + \sin^2 x) dx$ (8 marks)

(c) Solve the equation $2\sin^2\theta = 2 - \cos\theta$ for $0^\circ \leq \theta \leq 360^\circ$. (8 marks)

8. (a) Application of Kirchoff's laws to a resistive network yielded the simultaneous equations.

$$I_1 + I_2 - I_3 = 0$$
$$-I_1 + 2I_2 + I_3 = 9$$
$$2I_1 - I_2 + 3I_3 = 1$$

(15)

Use Cramer's rule to determine the values of currents I_1, I_2, and I_3. (13 marks)

(b) Given the matrices:

15 20 18

$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 3 & 2 \\ 3 & 2 & 0 \end{bmatrix}$ determine $(AB)^T$. (5 marks)

(c) Solve the equation $3^{2x+1} = 7^{x+2}$ correct to 4 d.p. (2 marks)

$[1\times2 + 0\times1 + 3\times3]$ $[1\times2 + 0\times3 + 3\times2]$ $[1\times0 + 0\times2 + 3\times0]$

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