2405/301
MATHEMATICS
Oct./Nov. 2016
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN APPLIED STATISTICS

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Scientific calculator
*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions in the answer booklet provided.*
***ALL** questions carry equal marks.*
Maximum marks for each part of a question are indicated.
Candidates should answer all questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the matrices

$$x=\begin{pmatrix}1 & -2 & 1\\ 0 & 1 & 3\\ -4 & -1 & 2\end{pmatrix},\ y=\begin{pmatrix}2 & 4 & -3\\ 5 & 8 & 1\\ 3 & -2 & 7\end{pmatrix}\ and\ z=\begin{pmatrix}1 & 2 & -4\\ -6 & 3 & 5\\ 7 & 2 & -9\end{pmatrix}$$

Find

(i) y^2

(ii) zyx^T

$30 = 8a + 4b + 2c$

$30 = 4a + 3b + c$

$10 = 2a + b + 3c$

(7 marks)

(b) In the production of various cereals, fertilizers a, b and c are mixed in various combinations. To produce 30 tonnes of barley 8 units of a, 4 units of b and 2 units of c are used. For wheat, 4 units of a, 3 units of b and 1 unit of c are applied to produce 30 tonnes while for maize, 2 units of a, 1 unit of b and 3 units of c produce 10 tonnes. Using the inverse matrix method, find the quantity of each fertilizer that is required to produce the combined harvest. (13 marks)

2. (a) Find the differential equation with the given form of general solution

$y = Ae^{3x} + Bxe^{3x}$ (7 marks)

(b) Solve the following ordinary differential equations

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = e^{-2t}$$

given that at $t = 0$, $x = 2$ and $\frac{dx}{dt} = -2$

(13 marks)

3. (a) Given that $U = l_n(x^2 + y^2)$ Show that

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$$

(7 marks)

(b) Given that $z = e^u \sin 2v$, if u is increasing at 3 units per second and v is decreasing at 5 units per second, determine the rate at which z is changing when $u = 0.1$ unit and $v = 0.5$ unit. (7 marks)

(c) The radius of a right cone is increased from 6 cm to 6.2 cm and its height is reduced from 10 cm to 9.9 cm. Use partial derivatives to determine the change in volume of the cone. (6 marks)

$\frac{\partial v}{\partial r}(\partial r) + \frac{\partial v}{\partial h}(\partial h).$ =

$\frac{1}{3}\pi r^2 h.$

4. (a) Given that $z^4 = -16$, find all possible values of z in the form $a + bj$. (8 marks)

(b) Use Demoivre's theorem to show that $Sin^6\theta = \frac{1}{32}(10 - 15\cos 2\theta + 6\cos 4\theta - \cos 6\theta)$ (6 marks)

(c) Given that

$z_1 = 10 - 5j$
$z_2 = 4 + 6j$
$z_3 = 2 - 7j$

Find

(i) $z_2 z_3$

(ii) $w = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}$

in the form $a + bj$

(6 marks)

$(a+bj)^4$

$j^2 = -1$

5. (a) (i) Use Taylor's theorem to expand $Sin(\pi/4 + h)$ in ascending powers of h up to and including h^4.

(ii) Hence determine the value of $Sin\ 46°$ correct to five decimal places. (11 marks)

(b) (i) Determine the first four terms of the Maclaurin's series expansion of $f(x) = l_n(1 + 4x)$.

(ii) Hence evaluate

$$\int_0^1 \frac{1}{x} l_n(1 + 4x)\, dx$$

giving the answer correct to four decimal places. (9 marks)

$-107j^2 - 36j + 68$

6. (a) Evaluate

$$\int_0^1 \int_0^{\sqrt{1+x^2}} (x^2y^3 + 3y)\, dy\, dx$$

(9 marks)

$(1+4x)^{-1}$

(b) Change the order of integration and evaluate

$$\int_0^2 \int_0^x (3x + 2x^2y^2)\, dy\, dx$$

(7 marks)

$-4(1+4x)^{-2}$

$8(1+4x)^{-3}$

$= \frac{8}{(1+4)^3}$

(c) Use double integrals to determine the area bounded by the graphs $y = 0$, $x = 0$ and $y = 2 - x$. (4 marks)

7. (a) Prove that

$$\frac{\tan x + Sec\, x}{Sec\, x(1 + \frac{\tan x}{Sec x}} = 1$$

(5 marks)

(b) Given a trigonometrical function $y_1 = 2\cos 3\theta \sin\theta$,

(i) rewrite the function as difference of two trigonometric ratios hence draw its graph for $-90° \leq \theta \leq 90°$.

(ii) solve for $y_1 = Cos\theta$, using the graph. -0.4054

(15 marks)

8. (a) Show that $x = 2$ is an approximate solution of the equation $e^x = 4x$. Hence apply the Newton Raphson method to calculate the root correct to four decimal places. (9 marks)

(b) The function f(x) is tabulated in table 1.

$e^x = 4x$

$4x - e^x = 0$

Table 1

x	-8	-6	-4	-2	0	2
$f(x)$	-2167	-927	-279	-31	9	33

Use Newton-Gregory formulae to calculate

(i) $f(-7.5)$

(ii) $f(1)$

forward

$fx = f_p + P f x + \frac{P(P-1)}{2!}$

(11 marks)

Back

$fx = f_p + P f x + \frac{P(P+1)}{2!} f\nabla^2 + \frac{P(P+1)(P+2)}{3!}$

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$-90 + 80$

$-7.5 + 6$