2305/301 2308/301
2306/301 2309/301
2307/301
MATHEMATICS
Oct./Nov. 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN BUILDING
DIPLOMA IN QUANTITY SURVEYING
DIPLOMA IN CIVIL ENGINEERING
DIPLOMA IN HIGHWAY ENGINEERING
DIPLOMA IN ARCHITECTURE

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Mathematical table / calculator;
Drawing instruments;
Answer booklet.
This paper consists of ***EIGHT*** *questions.*
Answer ***FIVE*** *questions in the answer booklet provided.*
All *questions carry equal marks.*
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

 Turn over

1. (a) Prove the identities:

(i) $\frac{1-\sin\theta}{1+\sin\theta}=\frac{1}{(\sec\theta+\tan\theta)^2}$

(ii) $\tan 3A=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$

(9 marks)

(b) Prove that $\sin\theta+\sin(\theta+120^\circ)+\sin(\theta+240^\circ)=0$

(4 marks)

(c) (i) Express $12\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$.

(ii) Hence solve the equation $12\cos\theta + 5\sin\theta = 6.5$, for values of θ between 0° and 360° inclusive.

(7 marks)

2. (a) Write down the middle term in the binomial expansion of $(x + 2y)^{12}$, and determine its value when $x=\frac{1}{2}$ and $y=\frac{2}{3}$.

(7 marks)

(b) Determine the first four terms in the binomial expansion of $\left(1-\frac{2}{3}x\right)^{-\frac{1}{3}}$, and state the values of which the expansion is valid.

(4 marks)

(c) (i) Use the binomial theorem to show that, for very small value of x,

$\sqrt{\left(\frac{1+2x}{1-2x}\right)}=1+2x+2x^2+4x^3$

approximately.

(ii) By setting $x=\frac{1}{100}$ in (i) above, determine the approximate value of $\sqrt{51}$, correct to four decimal places.

(9 marks)

3. (a) Given the matrices $A=\begin{bmatrix}-1 & 1 & -1\\ 1 & 2 & -1\\ 1 & 0 & 2\end{bmatrix}$,

and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & 1\\ 0 & 1 & -1\end{bmatrix}$, determine $(AB)^{-1}$.

(10 marks)

(b) Three forces F_1, F_2 and F_3 in newtons, necessary for the stability of a structure satisfy the simultaneous equations

$$F_1 - F_2 + F_3 = 5$$
$$F_1 - 2F_2 + F_3 = 2$$
$$F_1 + F_2 - F_3 = -1$$

Use Cramer's rule to solve the equations.

(10 marks)

4. (a) Show that the general solution of the differential equation $(x^2 + y^2)\,dx - xy\,dy = 0$ may be expressed in the form $x = cy$, where c is an arbitrary constant.

(7 marks)

(b) Solve the differential equation

$$6\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = e^{-2x}$$, that when $x = 0$, $y = 0$ and $\frac{dy}{dx} = -1$

(13 marks)

5. (a) Find $\frac{dy}{dx}$ from first principles, given that $y = \frac{3}{x^2}$.

(6 marks)

(b) Use implicit differentiation to determine the equation of the tangent to the curve $x^2 - 2y^2 + 3xy - 2x + 6y = 4$, at the point (0,2).

(6 marks)

(c) A curve has the parametric equations $x = \theta - \cos\theta$, $y = \sin\theta$. Determine the radius of curvature at $\theta = \pi$.

(8 marks)

6. (a) (i) Evaluate the integral:

$$\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx$$

(ii) Show that $\int_0^1 \frac{dx}{(x+1)(x^2+x+1)} = \ln\left(\frac{2}{\sqrt{3}}\right) + \frac{\pi}{6\sqrt{3}}$

(13 marks)

(b) Use integration to determine the length of the curve $y = \frac{1}{3}x^{\frac{3}{2}}$ between the points $x = 1$ and $x = 2$.
Correct two decimal places.

(7 marks)

7. (a) Given the complex numbers $z_1 = 2 - j$, $z_2 = 1 - 2j$ and $z_3 = 1 + j$, express

$$z = z_1 + \frac{z_1 + z_3}{z_1 z_3}$$ in polar form.

(8 marks)

(b) Find all the roots of the equation

$$z^3 + 3z^2 + 4z + 2 = 0.$$

(6 marks)

(c) Use De Moivre's theorem to prove that

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

(6 marks)

8. (a) **Table 1** shows data obtained from an experiment to determine the relationship between the force (x) and extension (y) on a cable.

Table 1

Force N(x)	10	20	30	40	50	60	70
Extension mm (y)	0.12	0.30	0.51	0.75	1.10	1.35	1.60

Calculate Karl Pearson's correlation coefficient.

(10 marks)

(b) A continuous random variable X has a probability density function defined by

$$f(x) = \begin{cases} k, & 0 \le x \le 1 \\ k(4x - 2), & 1 \le x \le 2 \\ 0, & elsewhere \end{cases}$$

Determine the:

(i) value of the constant k,
(ii) mean,
(iii) $P(x \ge 1.5)$

(10 marks)

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