2305/301 2308/301
2306/301 2309/301
2307/301
MATHEMATICS
Oct./Nov. 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN BUILDING
DIPLOMA IN QUANTITY SURVEYING
DIPLOMA IN CIVIL ENGINEERING
DIPLOMA IN HIGHWAY ENGINEERING
DIPLOMA IN ARCHITECTURE

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/calculator;
Drawing instruments.
*Answer **FIVE** questions of the **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are shown.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the vectors $\underset{\sim}{a} = 7\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = 8\underset{\sim}{i} - 4\underset{\sim}{j} + 9\underset{\sim}{k}$, determine the:

a) b
7i +2i -k

(i) unit vector perpendicular to both vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

b
8i-4j9k

(ii) area of the parallelogram spanned by the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(7 marks)

$\begin{vmatrix} i & j & k \\ 7 & 2 & -1 \\ 8 & -4 & 9 \end{vmatrix}$

(b) Use De Moivre's theorem to show that

i(18-4) - j(8-63) + k(-28 -16)
i14 + j71 + k-44
= 14i + 71j - 44k

$$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3).$$

(5 marks)

unit vector

(c) Solve the equations

$\dfrac{14i + 71j - 44k}{\sqrt{(14)^2 + (71)^2 - (44)^2}}$

(i) $4 + aj + b = 2a + 8j$ → 4aj + b = 2a + 8j
= 4aj + b = 16aj
= 4 + b - 15aj

Area = 1/2 (14 + 71 - 44)
= 20.5

(ii) $z^3 + 1 = 0$, giving the answer in the form of $a + bj$.

(8 marks)

2. (a) Evaluate the integrals:

(i) $\int \cos^5 x \, dx$

(ii) $\int_1^3 x^4 \ln x \, dx$

(9 marks)

(b) A region is enclosed between the x-axis and the curve $y = 4 - x^2$. Determine:

(i) its area;

(ii) the volume of the solid generated by rotating the area through 360°.

(11 marks)

3. (a) Given the matrices

$A = \begin{pmatrix} 4 & -1 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 5 \\ -6 & 2 \end{pmatrix}$,

show that $A^T B^T = (BA)^T$

(4 marks)

(b) Solve the equation:

$$\begin{vmatrix} 6 & 3 & 9 \\ 7 & \lambda & 11 \\ -1 & 5 & \lambda \end{vmatrix} = 0$$

6 [(x − x)

(5 marks)

(c) Three torques T_1, T_2 and T_3 in MNm acting on an arm of a construction plant satisfy the simultaneous equations:

$$T_1 - T_2 + T_3 = 7$$
$$5T_1 - 9T_2 + 3T_3 = -7$$
$$3T_1 - 3T_2 + T_3 = 3$$

Use Cramer's rule to solve the equations. (11 marks)

4. (a) Determine the value of the constant term in the binomial expansion of

$$\left(4x + \frac{1}{3x}\right)^8$$

(4 marks)

(b) (i) Using the binomial theorem, expand $\left(\frac{1+2x}{1+x}\right)^{\frac{1}{2}}$ as far as the term in x^3.

(ii) Hence determine $\sqrt{\frac{12}{11}}$, correct to five decimal places.

(10 marks)

(c) A welfare group consisting of 10 men and 18 women is to elect 3 of the members as executives. If the three positions are distinct, determine the number of ways of forming the executive with the condition that there has to be at least one from each gender.

(6 marks)

5. (a) A random variable x has a probability density function $f(x)$ given by:

$$f(x) = \begin{cases} K(x^2 - 4x + 5), & 0 \leq x < 2 \\ K(5 - 2x), & 2 < x \leq 5 \\ 0, & elesewhere \end{cases}$$

Determine the:

(i) value of constant K.
(ii) mean x.
(iii) standard deviation.

(13 marks)

(b) The mean and the standard deviation of the masses of a sample of 17 pieces of timber are 4.0 kg and 0.1 kg respectively.

Determine the:

(i) unbiased estimate of the population standard deviation.

(ii) 99% confidence interval of the mean.

(7 marks)

6. (a) Newton's law of cooling states that the rate of cooling of a body is directly proportional to the temperature, θ of the surroundings.
A body initially at 300°C cooled to 200° in a time of 4 minutes. Determine the temperature of the body after cooling for 10 minutes.

(8 marks)

(b) Use the method of undetermined co-efficients to solve the differential equations:

$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+13y=x$, given that $x=0, y=0$ and $\frac{dy}{dx}=0$. (12 marks)

$m^2+4m+13=0$

7. (a) Prove the identities:

$P=13$ $S=4$ $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\frac{-4 \pm \sqrt{16-4\times1\times13}}{2\times1}$ $\frac{-4 \pm 36}{2}$ 16 −20

(i) $\frac{2\cos\theta+2\sin\theta}{\sin 2\theta}=\operatorname{cosec}\theta+\sec\theta$

(ii) $\cos 3\theta=\cos^3\theta-3\cos\theta\sin^2\theta$ (7 marks)

(b) (i) Write $4\cos\theta-9\sin\theta$ in the form $R\cos(\theta+\alpha)$, where R is a positive constant and $0°\leq\alpha\leq 90°$

(ii) Hence solve the equation:

$4\cos\theta-9\sin\theta=7$ for values of θ between 0° and 360° inclusive.

(8 marks)

(c) The frustum of a solid right cone has height 39 cm. The radii of the plane surface are 40 cm and 14 cm. Determine its volume. (5 marks)

8. (a) Determine the equation of the normal to the curve:

$x^2y^3-5xy^2+4y=0$ at the point $(1,1)$. (8 marks)

(b) An open cylindrical tank is designed to hold $400\pi\, dm^3$ of liquid.
Determine the dimensions of the tank if its surface area is to be a minimum.

(12 marks)

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