2601/103
2602/103
2603/103
ENGINEERING MATHEMATICS I
Oct./Nov. 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Drawing instruments;
Mathematical tables/Non-programmable scientific calculator.
This paper consists ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. ✓ (a) Determine the fifth term of the binomial expansion of $(3x+4y)^{16}$ and evaluate its value at $x=\frac{1}{3}$ and $y=\frac{1}{4}$. (6 marks)

(b) Obtain the first three terms in the binomial expansion of $(8-x)^{\frac{1}{3}}$. State the range of x for which the expansion is valid. (6 marks)

(c) The resonant frequency of a series circuit is given by $f=\frac{1}{2\pi}\frac{1}{\sqrt{LC}}$, where L is inductance, C is capacitance. Use binomial theorem to determine the approximate change in f if L increases by 1% and C decreases by 2%. (8 marks)

2. ✓ (a) Given that α and β are the roots of the equation $ax^2+bx+c=0$ where a, b and c are constants. Express in terms of a, b and c.

1

1 1

(i) $\alpha^2+\beta^2$; $(3x+4y)^{16}$

1 3 3 1

1 4 6 4 1

(ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$. 1 5 10 10 5 1

(5 marks)

(b) Solve the equation by using formula method:

$$\frac{1}{(x-3)}+\frac{4}{(x-1)}=2$$ (7 marks)

(c) By applying Kirchoff's law to a d.c network, the following simultaneous equations are obtained:

$2I_1-3I_2+I_3=4$ ----- (i)

$3I_1+2I_2-2I_3=2$ ---- (ii)

$4I_1-I_2+3I_3=16$ ---- (iii)

Use substitution method to determine the values of the currents, correct to 2 decimal places. (8 marks)

3. (a) Simplify:

(i) $\left(\frac{1}{x}-\frac{1}{y}\right)\div\frac{(y^2-x^2)}{x^2y^2}$

(ii) $\log_{\frac{1}{2}}8+\log_2\left(\frac{1}{8}\right)+\log_3\left(\frac{1}{9}\right)$.

(7 marks)

(b) Solve the equations:

(i) $5(5^{\log_{10}x}) + 5^{(2-\log_{10}x)} = 30$;

(ii) $\log_4 x - 2\log_x 4 = 1$.

(13 marks)

4. (a) Given that $\text{Sin A} = \frac{24}{25}$ and $\text{Cos B} = -\frac{5}{13}$ where A is acute and B is obtuse. Determine:

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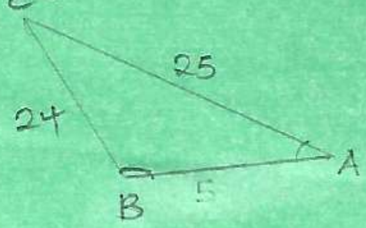
(i) $\text{Sin}(A - B)$;

(ii) $\text{Cos}(A + B)$.



(7 marks)

(b) Prove the identities:

(i) $\dfrac{1}{(1-\text{Sin}\,\theta)} - \dfrac{1}{(1+\text{Sin}\,\theta)} = 2Tan\theta\,\text{Sec}\,\theta$;

(ii) $\dfrac{2\,\text{Sin 4A} + \text{Sin 6A} + \text{Sin 2A}}{2\,\text{Sin4A} - \text{Sin 6A} - \text{Sin 2A}} = \text{Cot}^2\text{A}$.

(6 marks)

(c) Express the function:

(i) $7\,\text{Sin}\,\phi + 4\,\text{Cos}\,\phi$ in the form $\text{R Cos}(\phi - \alpha)$ where $\text{R} > 0$ and $0 \leq \alpha \leq 90°$.

(ii) Hence solve

$7\,\text{Sin}\,\phi + 4\,\text{Cos}\,\phi = \sqrt{65}$ for $0 \leq \phi \leq 360°$. (7 marks)

5. (a) Given the complex numbers

$Z_1 = 2 + 3J$, $Z_2 = 4 - 3j$ and $Z_3 = 1 + 4j$, express

$Z = \dfrac{Z_1 Z_2 Z_3}{Z_1 + Z_2 + Z_3}$ in the form $a + jb$.

(6 marks)

(b) Find all the roots of the equation $Z^3 - 1 - j\sqrt{3} = 0$ in polar form. (6 marks)

(c) Given that $Z = -2 - j$ is a root of the equation $Z^4 + 10Z^3 + 39Z^2 + 70Z + 50 = 0$ determine the other roots. (8 marks)

6. (a) Given that $MCosh3x + NSinh3x \equiv 7e^{3x} + 6e^{-3x}$ determine the values of M and N. (5 marks)

(b) (i) Prove that

$$Sinh^{-1}(x) = \ln\left(x + \sqrt{x^2+1}\right);$$

(ii) Hence evaluate $Sin\,h^{-1}(3)$ to three decimal places. (5 marks)

(c) Show that:

(i) $\dfrac{Sinhx}{(Coshx-1)} = Cothx + Co\sec hx$

(ii) $\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$ (10 marks)

7. (a) Find $\boxed{\dfrac{dy}{dx}}$ of $\boxed{g = e^x}$ from first principles. (6 marks)

(b) Given the following functions, find $\boxed{\dfrac{dy}{dx}}$

(i) $y = x^2 Cos^3(4x)$

(ii) $y = \ln\left(\dfrac{2x}{3x^2+4}\right)$

(iii) $y = \dfrac{(x-1)^2}{x+4}$ (9 marks)

(c) A function $z = f(x,y) = e^{xy}$, show that:

$$\frac{1}{y}\frac{\partial z}{\partial x} + \frac{1}{x}\frac{\partial z}{\partial y} = 2z$$

(5 marks)

8. (a) Evaluate the integrals:

(i) $\int_0^{\frac{\pi}{12}} \tan^2(3x)dx$

(ii) $\int \dfrac{x+15}{(x-2)(x+3)}dx$

(iii) $\int_1^4 \sqrt{5x-4}\,dx$ (12 marks)

(b) (i) Sketch the region bounded by $y = x^2$ and $x = y^2$ and hence find the area.

(ii) Determine the mean value of the current function $I = 50\ Sin(\omega t)$ over the interval $t = 0$ and $t = \pi$ in terms of ω and π. (8 marks)

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