2521/102 2602/103
2601/103 2603/103
ENGINEERING MATHEMATICS I
June/July 2023
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (POWER OPTION) (TELECOMMUNICATION OPTION) (INSTRUMENTATION OPTION)

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Drawing instruments;

Mathematical tables/Non-programmable scientific calculator.

*This paper consists **EIGHT** questions.*

*Answer any **FIVE** questions.*

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Solve the equations:

(i) $9\times 27^{2x+1}=81^{x-1}$

(ii) $7^{3x-1}=4(10^{x+2})$ (10 marks)

(b) Solve the equations:

(i) $\log_2(x+1)-\log_2 x=\log_2(x-1)$

(ii) $3\log_2 x+\log_x 64=11$

(10 marks)

2. (a) Three electric charges Q1, Q2 and Q3 in coloumbs in a d.c circuit satisfy the simultaneous equations:

$$3Q_1-2Q_2+Q_3=-1$$
$$Q_1+Q_2+2Q_3=8$$
$$2Q_1+3Q_2-4Q_3=12$$

Use elimination method to solve the equations. (12 marks)

(b) Solve the equation:

$4^{2x}-6(4^x)+8=0$ (8 marks)

3. (a) Prove the identities:

(i) $\dfrac{\cos\theta}{1+\sin\theta}=\sec\theta-\tan\theta$

(ii) $\cos(x+90^\circ)+\cos(x-90^\circ)=-2\sin x$

(9 marks)

(b) Solve the equation:

$6\cos^2\theta+\sin\theta-5=0$ for $0^\circ\leq\theta\leq 360^\circ$ (11 marks)

4. (a) Given that $f(x)=\dfrac{3x+5}{5x-7}$, determine:

(i) $f^{-1}(x)$

(ii) $f^{-1}(2)$

(6 marks)

(b) Solve the equation:

$$x = \sin^{-1}\frac{1}{2} \quad \text{for } 0 \leq x \leq 720°$$ (4 marks)

(c) Solve the equations:

(i) $\cosh x + 2\sinh x = 0$

(ii) $\sinh^2 x - \sinh x - 2 = 0$

(10 marks)

5. (a) Given the complex numbers $z_1 = 5 + j2$, $z_2 = 3 - j4$ and $z_3 = 1 + j2$, determine in the form $a + jb$:

(i) $2z_1 + z_2 - z_3$

(ii) $\dfrac{z_1 + z_3}{z_2 - z_1}$

(8 marks)

(b) (i) Show that $z = 2$ is a root of the equation $z^3 - 2z^2 + z - 2 = 0$.

(ii) Hence solve the equation for the other roots.

(8 marks)

(c) Convert the equation $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ to polar form giving the answer in the form $r = f(\theta)$ (4 marks)

6. (a) Determine the number of six digit codes which can be generated using the digits 1, 2, 3, 4, 5 and 6 if it must end with an even digit. (6 marks)

(b) Use the binomial theorem to expand $(1+2x)^{\frac{1}{3}}$ as far as the term in x^2 and state the values of x for which the expansion is valid. (5 marks)

(c) (i) Expand using binomial theorem the function $\left(\dfrac{1+x}{1-x}\right)^{\frac{1}{3}}$ up to the term in x^2

(ii) Hence evaluate $\sqrt[3]{\dfrac{3}{2}}$.

(9 marks)

7. (a) Differentiate $f(x)=\frac{1+x}{1-x}$ from first principles. (6 marks)

(b) Determine the stationary points of the function $f(x)=4x^3+3x^2-6x+11$ and classify them. (9 marks)

(c) The resistance R of a cable depends on its radius r and its length x such that:

$R=\frac{kx}{r^2}$ where k is a constant. Determine the percentage change in R if x is increased by 2% and r is decreased by 0.5%. (5 marks)

8. (a) Evaluate the integrals:

(i) $\int \frac{x^2+2x}{(x-1)(x^2+2)}dx$

(ii) $\int_0^{\pi} x \cos 2x \, dx$

(14 marks)

(b) Determine the x-co-ordinate of the centroid of the region bounded by the x-axis, the y-axis and the line $y=2-x$. (6 marks)

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