CHAPPTER 1: ENGINEERING MATHEMATICS

Unit of learning code: ENG/CU/EI/CC/1/5

Related Unit of Competency in Occupational Standard: Apply Engineering Mathematics

1.1 Introduction to the unit of learning

This unit describes the competencies required by an automotive technician in order to apply engineering mathematics. It involves use concepts of arithmetic in solving work problems, applying algebra, applying trigonometry and hyperbolic functions, applying complex numbers, applying coordinate geometry, carrying out binomial expansion, applying calculus, solving ordinary differential equations, carrying out mensuration, applying power series, applying statistics, applying numerical methods, applying vector theory, applying matrix, solving partial differential equations, applying Laplace transforms and applying Fourier series.

1.2 Summary of Learning Outcomes

1. Use concepts of arithmetic in solving work problems
2. Use common formula and algebraic expressions for work
3. Use trigonometry to solve practical engineering problems
4. Perform estimations, measurements and calculations
5. Apply matrices in work
6. Apply vectors in work
7. Collect, organize and interpret statistical data
8. Apply concepts of probability for work
9. Perform commercial calculation

Learning Outcome 1: Use Concepts of Arithmetic in Solving Work Problems

1.2.1.1 Introduction to the Learning Outcome

This learning outcome covers algebra and the learner should be able to: carry out fundamental operations, addition, subtraction, multiplication and division of positive and negative numbers, fractions and decimals operations and conversions, indices, ratios and proportions,

1.2.1.2 Performance Standard

1.2.1.2.1 Fundamental operations

1.2.1.2.2 Addition, subtraction, multiplication and division of positive and negative numbers

1.2.1.2.3 Fractions and decimals operations and conversions

1.2.1.2.4 Indices

1.2.1.2.5 Ratios and proportions

1.2.1.2.6 Meaning

1.2.1.2.7 Conversions into percentages

1.2.1.2.8 Direct and inverse proportions determination

1.2.1.2.9 Use of scientific calculator

1.2.1.3 Information Sheet

Fundamental operations

Brackets and the precedence rules are used to remove ambiguity in a calculation. In any calculation involving all four arithmetic operations as we proceed follows:

a. Working from the left evaluate divisions and multiplications as they are encountered

This leaves a calculation involving just addition and subtraction.

b. Working from the left evaluate additions and subtraction as they are

The four basic arithmetic operations are: addition and subtraction, multiplication and division

The order of precedence of operations for problems containing fractions is the same as that for integers, i.e. remembered by BODMAS (Brackets, Of, Division, Multiplication, Addition and Subtraction).

<u>Rules that govern basic laws of arithmetic:</u>

1. Commutativity

Two integers can be added or multiplied in either order without affecting the result. For example:

$$5 + 8 = 8 + 5 = 13 \ and \ 5 \times 8 = 8 \times 5 = 40$$

Addition and multiplication are commutative operations, whereas subtraction and division are not commutative operations.

$$4 - 2 \neq 2 - 4 \ as \ 4 - 2 = 2 \ and \ 2 - 4 = -2$$

$$4 \div 2 \neq 2 \div 4 \ as \ 4 \div 2 = 2 \ and \ 2 \div 4 = 0.5$$

2. Associativity

The way in which three or more integers are associated under addition or multiplication does not affect the result.

3. Distributivity

Multiplication is distributed over addition and subtraction from both the right and left.

Division is distributed over addition and subtraction from the right but not from the left.

Addition, subtraction, multiplication and division of positive and negative numbers

Adding two numbers gives their sum and subtracting two numbers gives their difference. For example:

$$7 + 3 = 10$$

$$7 - 3 = 4$$

$$3 - 7 = -4$$

Adding a negative number is the same as subtracting its positive counterpart. For example:

$$7 + (-2) = 5$$

Subtracting a negative number is the same as adding its positive counterpart. For example:

$$7 - (-2) = 9$$

Multiplication and division

Multiplying two numbers gives their product while dividing two positive and negative numbers gives their quotient.

Multiplying and dividing two positive or two negative numbers gives a positive number.

$$12 \times 2 = 24 \; and \; (-12) \times (-2) = 24$$

$$12 \div 2 = 6 \; and \; (-12) \div (-2) = 6$$

Multiplying or dividing a positive number by a negative number gives a negative number.

$$12 \times (-2) = -24; 12 \div (-2) = -6$$

Fractions and decimals operations and conversions

Fractions

When 2 is divided by 3, it may be written as $\frac{2}{3}$is called a fraction. The number above the line, i.e. 2, is called the numerator and the number below the line, i.e. 3, is called the denominator. When the value of the numerator is less than the value of the denominator, the fraction is called a proper fraction; thus $\frac{2}{3}$ is a proper fraction. When the value of the numerator is greater than the denominator, the fraction is called an improper fraction. Thus $\frac{7}{3}$ is an improper fraction and can also be expressed as a mixed number, that is, an integer and a proper fraction. Thus the improper fraction $\frac{7}{3}$ is equal to the mixed number $2\frac{1}{3}$

When a fraction is simplified by dividing the numerator and denominator by the same number, the process is called cancelling. Cancelling by 0 is not permissible.

Simplify: $\frac{1}{3} + \frac{2}{7}$

The lowest common multiple (i.e. LCM) of the two denominators is 3 d 7, i.e. 21

Expressing each fraction so that their denominators are 21, gives:

$$\frac{1}{3} + \frac{2}{7} = \frac{1}{3} \times \frac{7}{7} + \frac{2}{7} \times \frac{3}{3} = \frac{7}{21} + \frac{6}{21} = \frac{13}{21}$$

Simplify: $\frac{1}{3} - \left(\frac{2}{5} + \frac{1}{4}\right) \div \left(\frac{3}{8} \times \frac{1}{3}\right)$

Solution $\frac{1}{3}-\left(\frac{2}{5}+\frac{1}{4}\right)\div\left(\frac{3}{8}\times\frac{1}{3}\right)=\frac{1}{3}-\left(\frac{4\times2+1\times5}{20}\right)\div\left(\frac{3}{24}\right)$

$$\frac{1}{3}-\frac{13}{20}\div\frac{1}{8}=\frac{1}{3}-\frac{13}{20}\times\frac{8}{1}$$

$\frac{1}{3}-\frac{26}{5}=\frac{(1\times5)-(26\times3)}{15}=\frac{-73}{15}=-4\frac{13}{15}$

Decimals

The decimal system of numbers is based on the digits 0 to 9. A number such as 53.17 is called a decimal fraction, a decimal point separating the integer part, i.e. 53, from the fractional part, i.e. 0.17

A number which can be expressed exactly as a decimal fraction is called a terminating decimal and those which cannot be expressed exactly as a decimal fraction are called non-terminating decimals. Thus, $\frac{3}{2}=1.5$ is a terminating decimal, but $\frac{4}{3}=1.3333$ is a non-terminating decimal. 1.33333…can be written as 1.P3, called 'one point three recurring'.

The answer to a non-terminating decimal may be expressed in two ways, depending on the accuracy required:

(i) Correct to a number of significant figures, that is, figures which signify something, and

(ii) Correct to a number of decimal places, that is, the number of figures after the decimal point.

The last digit in the answer is unaltered if the next digit on the right is in the group of numbers 0, 1, 2, 3 or 4, but is increased by 1 if the next digit on the right is in the group of numbers 5, 6, 7, 8 or 9.

Evaluate

$$42.7+3.04+8.7+0.06=54.50$$

Indices

The lowest factors of 2000 are$2\times2\times2\times2\times5\times5\times5$. These factors are written as$2^4\times5^3$, where 2 and 5 are called bases and the numbers 4 and 3 are called indices.

When an index is an integer it is called a power. Thus, 24 is called 'two to the power of four', and has a base of 2 and an index of 4.

Special names may be used when the indices are2 and 3, these being called 'squared' and 'cubed', respectively. Thus 7^2 is called 'seven squared' and 9^3 is called 'nine cubed'. When no index is shown, the power is 1, i.e. 2 means2^1.

Reciprocal

The reciprocal of a number is when the index is-1 and its value is given by 1, divided by the base. Thus the reciprocal of 2 is $2^{-1}$and its value is $\frac{1}{2}$ or 0.5.

Square root

The square root of a number is when the index is$\frac{1}{2}$, and the square root of 2 is written as $2^{\frac{1}{2}}$21/2 or$\sqrt{2}$.

There are always two answers when finding the square root of a number and this is shown by putting both a + and a - sign in front of the answer to a square root problem.

Ratios and proportions

The ratio of one quantity to another is a fraction, and is the number of times one quantity is contained in another quantity of the same kind. If one quantity is directly proportional to another, then as one quantity doubles, the other quantity also doubles. When a quantity is inversely proportional to another, then as one quantity doubles, the other quantity is halved.

An alloy is made up of metals A and B in the ratio 2.5: 1 by mass. How much of A has to be added to 6 kg of B to make the alloy?

Ratio A: B: 2.5: 1 (i.e. A is to B as 2.5 is to 1) or $\frac{A}{B} = \frac{2.5}{1} = 2.5$

When B = 6 kg, $\frac{A}{6} = 2.5$ from which,

$$A = 6 \times 2.5 = 15kg$$

Conversions into percentages

Percentages are used to give a common standard and are fractions having the number 100 as their denominators. For example, 25 per cent means $\frac{25}{100}$ i.e. $\frac{1}{4}$ and is written 25%.

A decimal fraction is converted to a percentage by multiplying by 100. Thus,

1.875 corresponds to $1.875 \times 100\% = 187.5\%$

Direct and inverse proportions determination

Directly proportional: as one amount increases, another amount increases at the same rate. The symbol for "directly proportional" is $\propto$

Inversely proportional: when one value decreases at the same rate the other increases.

Use of scientific calculator

A scientific calculator is a special electronic calculator that aids the calculation involving problems in mathematics, engineering and science. It can be used in the following ways;

- Basic operations i.e. addition subtraction, multiplication and division.
- Calculation of exponents.
- Solving problems with an order of operations.
- Working out squares and square roots of numbers.
- Solving logarithmic problems.
- Solving of problems in statistics e.g. find the mean, variance, standard deviation etc.

Example: The "log" function in the calculator is the key that allows you to work with the logarithm.

The steps in using the log functions are;

i. Type the number you are to work with in your scientific calculator.
ii. Press the "log" button on the calculator.

iii. From your screen, obtain the exponent from the original number you entered.
iv. Check your results.

1.2.1.4 Learning Activities

1. Create an instruction sheet with the following five fractions: $\frac{1}{3}, \frac{1}{8}, \frac{2}{3}, \frac{1}{4}\, and\, \frac{1}{5}$

   Create a pizza (circular cut outs), using construction paper and decorate the toppings to represent each fraction.

   For example, if they had a quarter (fourth), they should cover one-quarter of the pizza with a specific pattern.

2. Using your calculator check that you agree with the answers to the following problems:

   Evaluate the following, correct to 4 significant figures:

   a. $4.7826 + 0.02713 = 4.80973 = 4.810$
   b. $21.93 \times 0.012981 = 0.2846733 = 0.2847$

   Evaluate the following, correct to 3 decimal places:

   a. $\sqrt{0.007328}$
   b. $\sqrt{52.91} - \sqrt{31.76}$

1.2.1.5 Self-Assessment

1. Place the appropriate symbol $< or >$ between each of the following pairs of numbers:
   a. 3 _ 2
   b. 8 _ -13
   c. -25 _ 0
2. Find the value of each of the following:
   a. $\mathbf{13 + 9 \div 3 - 2 \times 5}$
   b. $\mathbf{(13 + 9) \div (3 - 2) \times 5}$
3. Complete the following:
   a. $\mathbf{\frac{4}{5}} =$
   b. $\mathbf{48\%\ \boldsymbol{of}\ 50} =$
4. Write the following proportions as ratios:
   a. $\mathbf{\frac{1}{2}\ \boldsymbol{of\ A}, \frac{1}{5}\ \boldsymbol{of\ B}, \frac{3}{10}\ \boldsymbol{of\ C}}$
5. Reduce each of the following to their lowest form:
   a. $\mathbf{\frac{12}{18}}$
   b. $\mathbf{\frac{144}{21}}$
6. Round each of the following decimal numbers, first to 3 significant figures and to 2 decimal places:
   a. 21.355

b. 0.02456

1.2.1.6 Tools, Equipment, Supplies and Materials

- Calculator
- Paper and pencil
- Computer
- Ruler and graph paper
- Marker pen

1.2.1.7 References

Stroud K. A. (2001). Engineering Mathematics. Fifth Edition. North America: Industrial Press, Inc.

John Bird. (2003). Engineering Mathematics. Fourth Edition. London, UK: Newnes Elsevier Science

1.2.1.8 Answers to Self-Assessment

1. Place the appropriate symbol $< or >$ between each of the following pairs of numbers:
   a. 3>2
   b. 8>-13
   c. -25 < 0
2. Find the value of each of the following:
   a. $\mathbf{13 + 9 \div 3 - 2 \times 5}$

$$13 + (9 \div 3) - (2 \times 5)$$

$$13 + 3 - 10 = (13 + 3) - 10 = 6$$

   b. $(\mathbf{13 + 9}) \div (\mathbf{3 - 2}) \times \mathbf{5}$

$$(13 + 9) \div (3 - 2) \times 5 = 22 \div 1 \times 5 = (22 \div 1) \times 5 = 110$$

3. Complete the following:
   a. $\frac{4}{5}\ as\ a\ \% =$

$$\frac{4}{5} \times 100 = 80\%$$

   b. $\mathbf{48\%\ of\ 50} =$

$$\frac{48}{100} \times 50 = 24$$

4. Write the following proportions as ratios:
   a. $\frac{1}{2}\ \boldsymbol{of\ A}, \frac{1}{5}\boldsymbol{of\ B}, \frac{3}{10}\boldsymbol{of\ C}$

The LCM of the denominators 2, 5 and 10 is 10 then;

$$\frac{1}{2} of\ A\ is\ \frac{1 \times 5}{2 \times 5} = \frac{5}{10}, \frac{1}{5} of\ B\ is\ \frac{1 \times 2}{5 \times 2} = \frac{2}{10} and\ \frac{3}{10} of\ C$$

Hence: A: B: C = 5:2:3

5. Reduce each of the following to their lowest form:

a. $\frac{\mathbf{12}}{\mathbf{18}}$

$$\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$$

b. $\frac{\mathbf{144}}{\mathbf{21}}$

$$\frac{144}{21} = \frac{144 \div 3}{21 \div 3} = \frac{48}{7} = 4\frac{6}{7}$$

6. Round each of the following decimal numbers, first to 3 significant figures and to 2 decimal places:

a. 21.355

21.4 3sf

21.36 2dp

b. 0.02456

0.025 2sf

0.03 2dp

1.2.2 Learning Outcome 2: Use formulae and algebraic expressions for work

1.2.2.1 Introduction to the learning outcome

This learning outcome covers algebra and the learner should be able to: perform calculations involving Indices as per the concept; perform calculations involving Logarithms as per the concept; use scientific calculator is used mathematical problems in line with manufacturer's manual; perform simultaneous equations as per the rules. Algebra is used throughout engineering, but it is most commonly used in mechanical, electrical, and civil branches due to the variety of obstacles they face. Engineers need to find dimensions, slopes, and ways to efficiently create any structure or object.

1.2.2.2 Performance Standard

1.2.2.3 Algebraic linear equation

1.2.2.4 Simultaneous

1.2.2.5 Quadratic

1.2.2.6 Linear graphs

1. Plotting
2. Interpretation
3. Applications of linear graphs

4. Curves of first and second degree
5. Plotting
6. Interpretation
7. Applications

1.2.3 Information Sheet

1.2.3.1 Definitions of terms

Algebra is the study of mathematical symbols and the rules for manipulating these symbols; it is a unifying thread of almost all of mathematics. It includes everything from elementary equation solving to the study of abstractions such as groups, rings, and fields.

Logarithms: This is the power to which a number must be raised in order to get some other number which means that logarithms is the inverse of the exponential.

Linear equations: A linear equation is any equation of the form $\boldsymbol{a} + \boldsymbol{bx}$=0 where a and b are real numbers and $\boldsymbol{x}$ is a variable.

Quadratic equation: It is any equation of the form a$\boldsymbol{x^2} + \boldsymbol{bx} + \boldsymbol{c}$.It is any equation of the second degree meaning that one of its terms is a squared.

Performance of calculations involving indices

The lowest factors of 2000 are 2×2×2×2×5×5×5. These factors are written as $2^4 \times 5^3$, where 2 and 5 are called bases and the numbers 4 and 3 are called indices. When an index is an integer it is called a power. Thus, 2^4 is called ‘two to the power of four’, and has a base of 2 and an index of 4.

Given$\boldsymbol{b^x}$, b is known as the base and x is known as the power or index to which b is raised. It can be interpreted as:

$$\boldsymbol{b^x} = \boldsymbol{b} \times \boldsymbol{b} \times \boldsymbol{b} \ldots\ldots\ldots\ldots\ldots\ldots (\boldsymbol{x\ times})$$

$\boldsymbol{x}$ Can be real number.

The reciprocal of a number is when the index is _1 and its value is given by 1, divided by the base.

The square root of a number is when the index is$\boldsymbol{\frac{1}{2}}$, and the square root of 2 is written as $\boldsymbol{2^{\frac{1}{2}}}$ or$\boldsymbol{\sqrt{2}}$. The value of a square root is the value of the base which when multiplied by itself gives the number.

Laws of indices

When simplifying calculations involving indices, certain basic rules or laws can be applied, called the laws of indices.

For any number $\boldsymbol{x}$ and $\boldsymbol{y}$ the following basic laws hold;

$a^x\ a^x = a^{x+y}$

$\frac{a^x}{a^y} = a^{x-y}$

$(a^x)^y = a^{xy}$

$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

$a^{-y} = \frac{1}{a^y}$

$a^0 = 1$

Note: The above law only applies if bases are similar e.g. $2^3.2^4 = 2^7 = 128$

$$\frac{a^x}{a^x} = a^{x-y}$$

Example:

$\frac{2^4}{2^3} = 2^{4-3} = 2^1 = 2.$ If x and y equals one, $x = 1, y = 1$ then $\frac{a^1}{a^1} = a^{1-1} = a^0 = 1$

$(a^x)^y = a^{xy} = (a^y)^x$]jh

Example:

$(a^3)^2 = a^6 = (a^2)^3$

$(ab)^x = a^x a^x$

Example:

$(ab)^4 = a^4 b^4$

NB: The expression $a^{-x}$is equivalent to $\frac{1}{x}$

$\frac{1}{a^x} = \frac{a^0}{a^x} = a^{0-x} = a^{-x}$

Calculating the indicial equations

Given $3(2^{2x}) + 12(2^x) - 96 = 0, solve\ for\ x$

Steps

Let $2^x = p$

Substitute p in the equation

$\boldsymbol{i.e}\ \mathbf{3}\boldsymbol{p}^{\mathbf{2}} + \mathbf{12}\boldsymbol{p}\ - \mathbf{96} = \mathbf{0}$, simplifying the equation, it becomes$\boldsymbol{p}^{\mathbf{2}} + \mathbf{4}\boldsymbol{p}\ - \mathbf{32} = \mathbf{0}$.

Factorize $\boldsymbol{p}^{\mathbf{2}} + \mathbf{4}\boldsymbol{p}\ - \mathbf{32} = \mathbf{0}$

$\boldsymbol{p}^{\mathbf{2}} + \mathbf{8}\boldsymbol{p}\ - \mathbf{4}\boldsymbol{p} - \mathbf{32} = \mathbf{0}$

$\boldsymbol{P}(\boldsymbol{P} + \mathbf{8}) - \ \mathbf{4}(\boldsymbol{P} + \mathbf{8}) = \ \mathbf{0}$

$(\boldsymbol{P} - \mathbf{4})\ (\boldsymbol{p} + \mathbf{8}) = \ \mathbf{0}$

$\boldsymbol{P} = \mathbf{4}\ \mathbf{0}\boldsymbol{r}\ \boldsymbol{p} = -\mathbf{8}$

Since $\mathbf{2}^{\boldsymbol{x}} = \mathbf{2}^{\mathbf{2}}\quad \boldsymbol{x} = \mathbf{2}$ then the other solution has no real roots

Performance of calculations involving logarithms

Laws of logarithms

$\log \boldsymbol{A} + \log \boldsymbol{B}\ = \ \log \boldsymbol{AB}$

Example

Solve $\boldsymbol{log}_{\mathbf{10}}\mathbf{5}$ + $\boldsymbol{log}_{\mathbf{10}}\mathbf{4}$

$\boldsymbol{log}_{\mathbf{10}}\mathbf{5}$ + $\boldsymbol{log}_{\mathbf{10}}\mathbf{4}$= $\boldsymbol{log}_{\mathbf{10}}(\mathbf{5}\boldsymbol{x}\mathbf{4})$= $\boldsymbol{log}_{\mathbf{10}}$20

$\boldsymbol{Log\ A} - \ \boldsymbol{Log\ B} = \mathrm{Log}\frac{\boldsymbol{A}}{\boldsymbol{B}}$

Example:

Use the second law to simplify:

$\boldsymbol{log}_{\mathbf{10}}\mathbf{6} - \boldsymbol{log}_{\mathbf{10}}\mathbf{3}$

$\boldsymbol{log}_{\mathbf{10}}\mathbf{6} - \boldsymbol{log}_{\mathbf{10}}\mathbf{3}$ =$\boldsymbol{log}_{\mathbf{10}}\ \frac{\mathbf{6}}{\mathbf{3}} = \boldsymbol{log}_{\mathbf{10}}\mathbf{2}$

Log $\boldsymbol{B}^{\boldsymbol{n}} = \boldsymbol{n}\log \boldsymbol{B}$

Example:

Use the third law to write the alternative form of

3 $\boldsymbol{log}_{\mathbf{10}}\mathbf{5} = \boldsymbol{log}_{\mathbf{10}}\mathbf{5}^{\mathbf{3}}$ =$\boldsymbol{log}_{\mathbf{10}}\mathbf{125}$

1.2.3.1.1 1.2.2.3.5 Solving logarithmic equations

Given$\boldsymbol{log}_{\mathbf{2}}(\boldsymbol{x} + \mathbf{4}) + \boldsymbol{log}_{\mathbf{2}}(\mathbf{6}) = \boldsymbol{log}_{\mathbf{2}}\mathbf{54}$, Find x,

Steps

From laws of logarithms

Law 1, $\boldsymbol{log_2(x+4) + log_2(6) = log_2 54}$

$$\boldsymbol{log_2((x+4)\times(6)) = log_2 54}$$

Distribute $\boldsymbol{(x+4)(6) = 6x + 24}$

$$\boldsymbol{log_2(6x+24) = log_2 54}$$

Drop the logs

$$\boldsymbol{6x + 24 = 54}$$

Solve the linear equation

$$\boldsymbol{6x = 30}$$

$$\boldsymbol{x = 5}$$

Use of scientific calculator in solving mathematical problems

A scientific calculator is a special electronic calculator that aids the calculation involving problems in mathematics, engineering and science. It can be used in the following ways;

- Basic operations i.e. addition subtraction, multiplication and division.
- Calculation of exponents.
- Solving problems with an order of operations.
- Working out squares and square roots of numbers.
- Solving logarithmic problems.
- Solving of problems in statistics e.g. find the mean, variance, standard deviation etc.

Example: The "log" function in the calculator is the key that allows you to work with the logarithm.

The steps in using the log functions are;

v. Type the number you are to work with in your scientific calculator.
vi. Press the "log" button on the calculator.
vii. From your screen, obtain the exponent from the original number you entered.
viii. Check your results.

Obtainment of solution to system of linear equations involving three unknowns

This is how one should solve a system with three unknowns.

Steps

i. Take a pair of the equation from the system.
ii. Using addition/subtraction, eliminate the same variables from each pair.
iii. Using additional/subtraction, solve the system of two new equations.

iv. Substitute the solution back into one of the equations and solve for the third variable.

v. Confirm your solution by the use of one of the equations.

Example:

Solve the following system of equations.

$$\mathbf{8x - 6y + 2z = -20}$$

$$\mathbf{4x + 2y + 6z = 0}$$

$$\mathbf{-2x + 4y - 10Z = 34}$$

Pick two pairs

$$\mathbf{8x - 6y + 2z = -20}$$

$$\mathbf{4x + 2y + 6z = 0}$$

and

$$\mathbf{4x + 2y + 6z = 0}$$

$$\mathbf{-2x + 4y - 10Z = 34}$$

Eliminate the same variables from each system

$$\mathbf{8x - 6y + 2z = -20}$$

$$\mathbf{4x + 2y + 6z = 0}$$

Simplify

$$\mathbf{4x - 3y + z = -10}$$

$$\mathbf{2x + y + 3z = 0}$$

$$\mathbf{4x - 3y + z = -10}$$

$$\underline{\mathbf{-4x - 2y - 6z = 0}}$$

$$\mathbf{-5y - 5z = -10}$$

$$\mathbf{4x + 2y + 6z = 0}$$

$$\mathbf{-2x + 4y - 10z = 34}$$

Simplify

$2x + y + 3z = 0$

$-x + 2y - 5z = 17$

$2x + y + 3z = 0$

$\underline{-2x + 4y - 10z = 34}$

$5y - 7z = 34$

Solve the system of the equations.

$-5y - 5z = -10$

$\underline{5y - 7z = 34}$

$-12z = 24$

Thus z $= -2$, solving for $y, y = 4$

Substitute in the original to obtain x

$-2x + 4y - 10z = 17$

$-2 + 4(4) - 10(-2) = 17$

$-2x = -2 \quad x = 1$

Performance of calculations involving Quadratic Equations

It is an equation of the dorm $ax^2 + bx + c = 0$

Solving the quadratic equation

Quadratic equations can be solved by factorization.

Quadratic equation can be solved by the use of the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Also, quadratic equations can be solved through the completing squares method.

By the use of the graphical method.

Example:

Solve using the quadratic formula;

$x^2 + 2x + 1 = 0$

Solution:

$\mathbf{a} = 1$

$\mathbf{b = 2}$

$\mathbf{c = 1}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 1}}{2 \times 1}$$

The discriminant is;

$$b^2 - 4ac = 2^2 - 4 \times 1 \times 1 = 0$$

$$x = \frac{-2 \pm \sqrt{0}}{2 \times 1}$$

$$= \frac{-2}{2 \times 1} = 1$$

Calculations involving sequence and series are performed as per the concept

A sequence are numbers or functions arranged in a special order while series is the sum of sequence of terms.

Types of sequence in mathematics

- Arithmetic sequence: is a set of the numbers that follows a pattern formed by adding a common difference to the proceeding number.
- Geometric sequence: is a set of numbers that are multiplied (or divided) by a constant called the common ratio.

Types of series

Arithmetic series is the sum of the terms of arithmetic sequence while geometric series is the sum of the terms of the geometric sequence. The n^{th} term of the arithmetic sequence is given by $a + (n-1)d$ while the sum of terms in the series is given by $\frac{n}{2}[2a + (n-1)d]$. The n^{th} term of a G.P is given by ar^{n-1}. The sum of the infinite terms of the series $s_n = \frac{a^n}{1-r}$

Where 0< r<1

a Is the first term.

n Is the number of terms?

d Is the common difference

r Is the common ratio

Example

Determine the type of the sequence

$\mathbf{1, 4, 7, 10, 13}$ ... It is an arithmetic sequence with a common difference of 3.

$\mathbf{2, 4, 8, 16, 32}$ ... Is a geometric sequence with a common ratio of $\mathbf{2}$

In the arithmetic sequence $\mathbf{1, 4, 7, 10, 13}$, ... find the sum of the first 10 terms.

$$\boldsymbol{a = 1}, \quad \boldsymbol{d = 3} \quad \boldsymbol{n = 10}$$

Use the formula $\boldsymbol{\frac{n}{2}[2a + (n-1)d\,]}$

$$\boldsymbol{\frac{10}{2}[2(1) + (10-1)3\,]}$$

$$\boldsymbol{= 5(2 + 27) = 145}$$

In the geometric sequence$\mathbf{2, 4, 8, 16, 32}$. Find the sum of the first 10 terms.

1.2.4 Learning Activities

1. In groups of two solve this hexagon puzzle as an algebra challenge. It involves rolling a dice to work out the value of an algebraic expression. Students to take turns rolling the dice and the number rolled becomes the value of ’n’. The player then gets one chance to choose a hexagon and solve the equation using the assigned value of ’n’. At the end of the game, the student with the most number of solved hexagons wins.

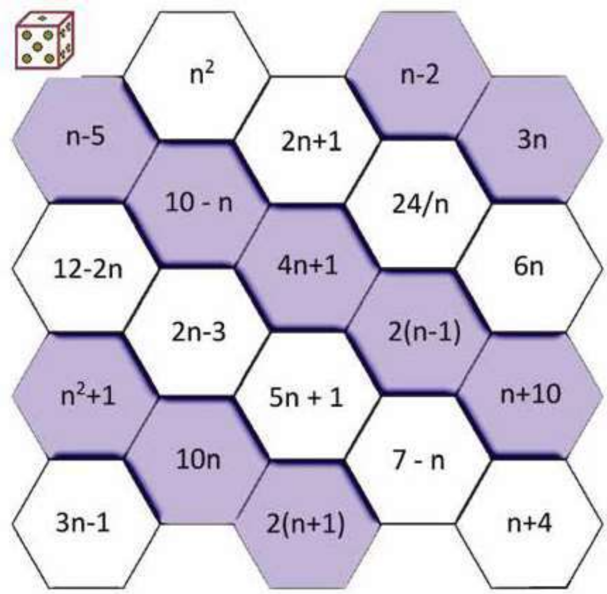

Figure 1: Hexagon Puzzle 1

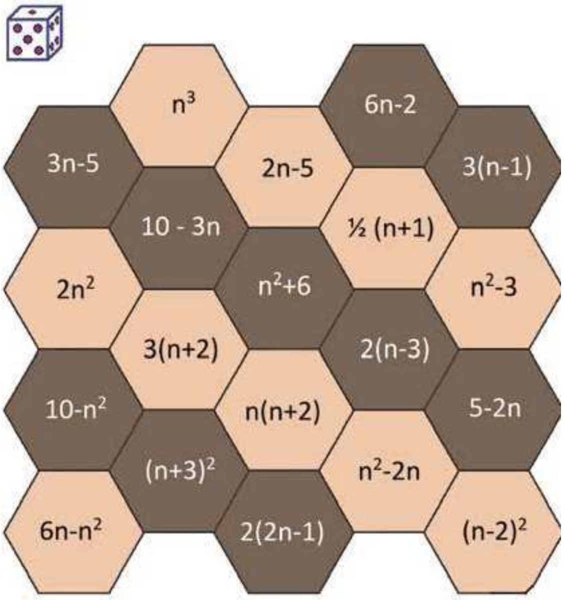

Figure 2: Hexagonal Puzzle 2

2. Perform a survey in your locality (in around 50 homesteads) and find out the total number of rabbits' population. It is believed that the population will increase by 2% every year. Predict the rabbit population after 3 years.

1.2.5 Self-Assessment

1. Find the value of $\boldsymbol{4p^2qr^3}$, given that $\boldsymbol{p = 2, q = \frac{1}{2}\ and\ r = 1\frac{1}{2}}$
2. Find the sum of: $\boldsymbol{5a - 2b, 2a + c, 4b - 5d\ and\ b - a + 3d - 4c}$
3. Subtract $\boldsymbol{2x + 3y - 4z\ from\ x - 2y + 5z}$
4. Multiply $\boldsymbol{3x - 2y^2 + 4xy}$ by $\boldsymbol{2x - 5y}$
5. Simplify: $\boldsymbol{2p \div 8pq}$
6. Simplify: $\boldsymbol{\frac{x^2y}{xy^2 - xy}}$
7. Simplify:

$\boldsymbol{(a^3\sqrt{b}\sqrt{c^5})(\sqrt{a}\sqrt[3]{b}c^3)}$ And evaluate when $\boldsymbol{a = \frac{1}{4}, b = 6\ and\ c = 1}$

8. Factorize: $\boldsymbol{2ax - 3ay + 2bx - 3by}$

1.2.6 Tools, Equipment, Supplies and Materials

- Calculator
- Paper and pencil
- Computer
- Ruler and graph paper
- Dice

1.2.7 References

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1.2.8 Answers to the Self-Assessment

1. Find the value of$\boldsymbol{4p^2qr^3}$, given that $\boldsymbol{p = 2, q = \frac{1}{2}\ and\ r = 1\frac{1}{2}}$

Solution:

Replacing p, q and r with their numerical values gives:

$$\boldsymbol{4p^2qr^3} = \boldsymbol{4(2)} \times (\boldsymbol{2^2}) \times \boldsymbol{\frac{1}{2}} \times \left(\left(\frac{3}{2}\right)^3\right) = \boldsymbol{27}$$

2. Find the sum of: $\boldsymbol{5a - 2b, 2a + c, 4b - 5d\ and\ b - a + 3d - 4c}$

Solution:

The algebraic expressions may be tabulated as shown below, forming columns for a’s, b’s, c’s and d’s.

Thus:

$$\boldsymbol{5a + (-2b) + 2a + c + 4b + (-5d) + b + (-a) + 3d + (-4c)}$$
$$\boldsymbol{= 6a + 3b - 3c - 2d}$$

3. Subtract $\boldsymbol{2x + 3y - 4z\ from\ x - 2y + 5z}$

$\boldsymbol{(x - 2y + 5z) - (2x + 3y - 4z)}$

$\boldsymbol{= -x - 5y + 9y}$

4. Multiply $\boldsymbol{3x - 2y^2 + 4xy}$ by $\boldsymbol{2x - 5y}$

$\boldsymbol{2x(3x - 2y^2 + 4xy) = 6x^2 - 4xy^2 + 8xy^2}$

$\boldsymbol{-5y(3x - 2y^2 + 4xy) = -15xy + 10y^3 - 20xy^2}$

$\boldsymbol{(6x^2 - 4xy^2 + 8xy) + (-15xy + 10y^3 - 20xy^2)}$

$\boldsymbol{6x^2 - 24xy^2 + 8xy^2 - 15xy + 10y^3}$

5. Simplify: $\boldsymbol{2p \div 8pq}$

$\boldsymbol{\frac{2p}{8pq} = \frac{1}{4q}}$

6. Simplify: $\boldsymbol{\frac{x^2y}{xy^2 - xy}}$

The highest common factor (HCF) of each of the three terms comprising the numerator and denominator is xy. Dividing each term by xy gives:

$$\boldsymbol{\frac{x^2y}{xy^2 - xy} = \frac{\frac{x^2y}{xy}}{\frac{x}{xy} - \frac{xy}{xy}} = \frac{x}{y - 1}}$$

7. Simplify:

$\boldsymbol{\left(a^3\sqrt{b}\sqrt{c^5}\right)\left(\sqrt{a}\sqrt[3]{b}c^3\right)}$ And evaluate when $\boldsymbol{a = \frac{1}{4}, b = 6\ and\ c = 1}$

Using the fourth law of indices, the expression can be written as:

$$\boldsymbol{\left(a^3b^{\frac{1}{2}}c^{\frac{5}{2}}\right)\left(a^{\frac{1}{2}}b^{\frac{2}{3}}c^3\right)}$$

Using the first law of indices gives:

$\boldsymbol{a^{3+\frac{1}{2}}b^{\frac{1}{2}+\frac{2}{3}}c^{\frac{5}{2}+3} = a^{\frac{7}{2}}b^{\frac{7}{6}}c^{\frac{11}{2}}}$

It is usual to express the answer in the same form as the question. Hence:

$\boldsymbol{a^{\frac{7}{2}}b^{\frac{7}{6}}c^{\frac{11}{2}} = \sqrt{a^7}\sqrt[6]{b^7}\sqrt{c^{11}} = \sqrt{\left(\frac{1}{4}\right)^7}\left(\sqrt[6]{6^7}\right)\left(\sqrt{1^{11}}\right)}$=1

8. Factorize: $\boldsymbol{2ax - 3ay + 2bx - 3by}$

a is a common factor of the first two terms and b a common factor of the last two terms

Thus: $\boldsymbol{2ax - 3ay + 2bx - 3by = a(2x - 3y) + b(2x - 3y)}$

$(\boldsymbol{2}x - \boldsymbol{3y})(\boldsymbol{a} + \boldsymbol{b})$

2 Learning Outcome 3: Use trigonometry to solve practical work problems

2.2.2 Introduction to the learning outcome

This learning outcome covers applications of trigonometric rules, identities the calculation of area, perimeter of shapes and solids .it also covers one to one relationships and calculation involving hyperbolic functions and identities.

2.2.3 Performance Standard

1. Meaning of trigonometry
2. Pythagoras theorem
3. Trigonometry ratios of angles
4. Trigonometric identities
5. Conversion of angles

2.2.4 Information Sheet

2.2.4.1 Definition of Terms

Trigonometry rule: This is a rule aimed at expressing the relationship between the angles of a triangle and the side lengths.

Hyperbolic function: This is a function of an angle that is defined for the hyperbola, not on the circle.

One to one function: A function said to be one to one if different elements in $\boldsymbol{a}$ map different element in $\boldsymbol{b}$.

Performance of calculations are performed using trigonometric rules

Consider a unit circle of radius 1

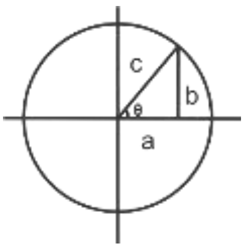

Figure 3: Circle

$$\boldsymbol{sin\theta = \frac{b}{c}}$$

$$\boldsymbol{cos\theta = \frac{a}{c}}$$

$$\boldsymbol{tan = \frac{b}{a}}$$

$$\boldsymbol{a^2 + b^2 = c^2}$$

$$\boldsymbol{\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1}$$

Some of the trigonometric rules are;

(Cos θ^2) + (sinθ^2) $= \mathbf{1}$

$\boldsymbol{cos^2\theta + sin^2\theta = 1} \ldots.(\mathbf{1})$

$\boldsymbol{dividing\ (1) by\ sin^2\theta}$

$\boldsymbol{cot^2\theta + 1 = cosec^2\theta}$

$\boldsymbol{cot^2\theta = cosec^2\theta - 1}$

Note:

$\boldsymbol{cot\theta = \frac{\cos\theta}{\sin\theta}}$

$\boldsymbol{cosec\theta = \frac{1}{sin\theta}}$

$\boldsymbol{sec\theta = \frac{1}{\cos\theta}}$

$\boldsymbol{tan\theta = \frac{\sin\theta}{\cos\theta}}$

Dividing $(\boldsymbol{i})$ by

$\boldsymbol{sin^2\theta}$

$\boldsymbol{1 + tan^2\theta = sec^2\theta - 1}$

$\boldsymbol{tan^2\theta = sec^2\theta - 1}$

The double angle formula can then be given as $\boldsymbol{\sin 2A = A\ cosA + \cos ASinA}$

$\boldsymbol{\sin 2A = \cos A \cos A - \sin A \sin A}$

Note: If values are different, it becomes a factor theorem with 2A=A+B.

From the above relationship, we derive the half angle formula; that is

$\boldsymbol{\cos 2A = Cos^2 A - Sin^2 A}$

$\boldsymbol{\cos 2A = 2\ \ cos^2}$=A-1

$\boldsymbol{\cos A = 2\ cos^2 (\frac{A}{2}) - 1}$

$\boldsymbol{\cos A = 1 - 2sin^2\ (A/_2)}$

Therefore, $2sin^2\left({}^{A}\!/_{2}\right) = 1 - cos\,B$

$\cos\left({}^{A}\!/_{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$ and $\sin\left({}^{A}\!/_{2}\right) = \pm\sqrt{\frac{1-COSa}{2}}$

Example

Without using calculator, find $cos\ 15°$

Solution

$cos15° = (cos45° - cos30°)$

$= cos45\ cos30 + sin45\ sin30$

$= \sqrt{\frac{2}{2}}.\sqrt{\frac{3}{2}} + \sqrt{\frac{2}{2}}.\sqrt{\frac{1}{2}}$

$= \sqrt{\frac{2}{4}}(\sqrt{3} + 1)$

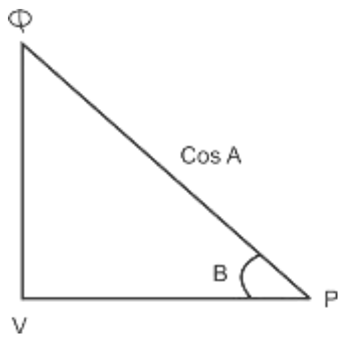

Figure 4: Triangle PQV

From triangle PQV

$sinB = \frac{QV}{CosA} = QV = CosASinB$

Therefore from (i) and (ii)

$\sin(A + B) = sinAcosB + cosAsinB$

Replacing B by (-B) in equation (i), we get:

$\sin(A - B) = sinAcos(-B) + \cos Asin(-B)$

$\sin(A - B) = \sin AcosB - cosAsinB$

Similarly;

$\cos(A+B) = cosAcosB - sinAsinB$

$\cos(A-B) = cosAcosB + sinAsinB$

Performance of calculations are performed using hyperbolic functions

These functions are expressed as exponential functions and are generally expressed as hyperbolic sine or hyperbolic cosine, where other relationships can be derived.

The hyperbolic identities

Hyperbolic identities are obtained from the hyperbolic trigonometric functions. i.e

$$\text{Tanh}\, x = \frac{sinhx}{coshx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$Sech\, x = \frac{2}{e^x + e^{-x}}$$

$$\coth x = \frac{e^x + e^{-x}}{e^x + e^{-x}}$$

$$cosech\, x = \frac{2}{e^x - e^{-x}}$$

The derived hyperbole identities then become:

$cosh^2 x - \ sinh^2 x = 1$

$1 - tanh^2 x = sech^2 x$

$coth^2 x - 1 = cosech^2 x$

$\sinh(x \pm y) = \sinh x\, cosh\, y \pm coshx \sinh y$

$\cosh(x \pm y) = \cosh x \cosh y \ \pm \sinh x \sinh y$

$\sinh(-x) = -\sinh x$

$\cosh x = \cosh(-x)$

Examples

Solve the following equation in terms of a natural logarithm.

$4cosh\, 2\, x\ + 20\, sinh\, 2x = 10$

Solution

$$\cosh 2x = \frac{e^{2x} + e^{-2x}}{2}$$

$\sinh 2x = \frac{e^{2x} - e^{-2x}}{2}$

Therefore $2e^{2x} + 2e^{-2x} + 10e^{2x} - 10e^{-2x} = 10$

$12^{2x} - 8e^{-2x} = 10$

$6e^{4x} - 5e^{2x} - 4 = 0$

$(3e^{2x} - 4)(2e^{2x} + 1) = 0$

$e^{2x} = \frac{4}{3}\ or\ e^{2x} = -\frac{1}{2}$

$x = \frac{1}{2}\ell n\ \frac{4}{3}\ since\ the\ real\ solution\ occurs\ when\ e^{2x} > 0$

$coshx = \frac{e^{x}+e^{-x}}{2}$

$sinhx = \frac{e^{x}-e^{-x}}{2}$

Most of the properties are similar tom properties of trigonometric functions. For example; cosh function is even while $sinh$ function is odd. The only difference in properties is,

$(coshx)^2 - (sinhx)^2$

Examples

Calculate $sinh\ 5$ using exponential functions.

Solution

$sinh5 = \frac{e^{5}-e^{-5}}{2}$

$= 74.2.3\ to\ 3\ d.p$

Calculate cosh 7 using the factor theorem.

$\text{Cosh } 7 = \cosh(3 + 4) = cosh3cosh4 - sinh3sinh4$

$= (10..68)(27.308) - (10.018)(27.290)$

$= 1.543$

Given the value of x, one is able to calculate inverse of the hyperbolic functions

2.2.5 Learning Activities

1. Prove the following hyperbolic identities, use the definition of $sinhx\ and\ coshx$ in terms of exponential functions

   a) $\cos h^2x - \sin h^2\, x = 1$

   b) $1 - tanh^2x = sech^2x$

   c) $coth^2x - 1 = cosech^2x$

2. On graph papers:

   a) Plot the graphs of $sinhx\ and\ coshx$

   b) Superimpose the graphs above, do the curves intersect?

   c) Predict the graphs of $y = cothx, y = cosechx\ and\ y = sechx$

2.2.6 Self-Assessment

1) Solve the following equation in terms of a natural logarithm.

$$\boldsymbol{4cosh^2\, x\ + 20\, sinh^2x = 10}$$

2) Indicate the sign needed to make the following identity true.

   a) $Sinh^2\, x$( )$\cosh^2$ x = 1

3) A tree casts a horizontal shadow $8\sqrt{3}\, m,$ if a line is drawn from the end of the shadow to the top of the tree an angle of$60°$. What is the height of the tree?

4) Prove that: $\boldsymbol{\frac{sinA+SinB}{Cos\, A+\cos B} = \tan\frac{A+B}{2}}$

5) With an aid of an example, analyze when a function is said to be injective?

6) Justify the relationship between the hyperbolic and the trigonometric functions.

7) Given;

$\boldsymbol{a, b, c\ > 0}$, show that when $\boldsymbol{a > b}$,then $\boldsymbol{acosh\, x + b\, sinhx}$ can be written in the form R $\boldsymbol{cosh(x + \alpha)}$.Therefore determine a further condition for which the following equation has real solutions. $\boldsymbol{aCosh\, x +\ bsinh\, x = c}$

2.2.7 Tools, Equipment, Supplies and Materials

- Calculator

- SMP tables
- Measuring tables
- Graph Papers
- Pens and Pencils

2.2.8 References

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2.2.9 Answers to Self-Assessment

1. Solve the following equation in terms of a natural logarithm.

$$\boldsymbol{4cosh\,2\,x\ +20\ sinh\ 2x = 10}$$

Solution

$$\mathbf{cosh\ 2}\boldsymbol{x} = \frac{\boldsymbol{e^{2x} + e^{-2x}}}{\mathbf{2}}$$

$$\mathbf{sinh\ 2}\boldsymbol{x} = \frac{\boldsymbol{e^{2x} - e^{-2x}}}{\mathbf{2}}$$

Therefore $\boldsymbol{2e^{2x} + 2e^{-2x} + 10e^{2x} - 10e^{-2x} = 10}$

$$\boldsymbol{12^{2x} - 8e^{-2x} = 10}$$

$$\boldsymbol{6e^{4x} - 5e^{2x} - 4 = 0}$$

$$\boldsymbol{(3e^{2x} - 4)(2e^{2x} + 1) = 0}$$

$$\boldsymbol{e^{2x} = \frac{4}{3}\ or\ e^{2x} = -\frac{1}{2}}$$

$$\boldsymbol{x = \frac{1}{2}\ell n\ \frac{4}{3}\ since\ the\ real\ solution\ occurs\ when\ e^{2x} > 0}$$

$$\boldsymbol{coshx = \frac{e^{x}+e^{-x}}{2}}$$

$$\boldsymbol{sinhx = \frac{e^{x}-e^{-x}}{2}}$$

2. Indicate the sign needed to make the following identity true.
   $Sinh^2\ x(\ +)Cosh^2x = 1$
3. A tree casts a horizontal shadow $8\sqrt{3}\,m,$ if a line is drawn from the end of the shadow to the top of the tree an angle of$60°$. What is the height of the tree?

$$Tan\ 60° = \frac{height\ of\ tree}{length\ of\ shadow}$$

$$Tan\ 60° = \frac{h}{8\sqrt{3}}$$

$$h = 8\sqrt{3}\,Tan\ 60° = 24m$$

3 Learning Outcome 4: Perform estimations, measurements and calculations of quantities

3.2.2 Introduction to the learning outcome

This topic covers the units of measurement and their symbols, conversion of units of measurement, mensuration, measuring tools and equipment and performing measurements and estimations of quantities.

3.2.3 Performance Standard

1) Units of measurements and their symbols
2) Conversion of units of measurement
3) Calculation of length, width, height, perimeter, area and angles of figures
4) Measuring tools and equipment
5) Performing measurements and estimations of quantities

3.2.4 Information Sheet

3.2.4.1 Definition of terms

Units of measurements and their symbols

There are seven base units in the SI system:

- the kilogram (kg), for mass
- the second (s), for time
- the kelvin (K), for temperature
- the ampere (A), for electric current
- the mole (mol), for the amount of a substance
- the candela (cd), for luminous intensity
- the meter (m), for distance

Table 1: SI Units

QUANTIES	UNIT	SYMBOL
Length	• Millimeter • Meter • Kilometer	mm m km
Mass	• Gram • Kilogram • Tonne	g kg
Time	1 Second	s
Temperature	Degree Celsius	℃
Area	Square kilometer	km^2
Volume	Cubic Meter	m^3
Speed	2 Meter per second 3 Kilometer per hour	m/s km/hr
Density	Kilogram per cubic meter	$kg/_{m^3}$
Force	Newton	N

Conversion of units of measurement

Table 2: SI Prefix Symbols

Prefix	Symbol	Ordinary Notation	Factor
Giga	G	1000000000	10^9
Mega	M	1000000	10^6
Kilo	K	1000	10^3
Milli	m	0.001	10^{-3}
Micro	μ	0.000001	10^{-6}
Nano	N	0.000000001	10^{-3}

Table 3: Conversions of SI Units

QUANTITY	**UNITY**	**VALUE IN SI UNITS**
Length	3.2.4.1.1 Millimeter 3.2.4.1.2 Centimeter 3.2.4.1.3 Kilometer	3.2.4.1.4 1000mm = 1m 3.2.4.1.5 100cm = 1m 3.2.4.1.6 1km = 1000m
Time	3.2.4.1.7 Minute 3.2.4.1.8 Hour 3.2.4.1.9 Day	1minute = 60s 1hour = 60minute=3600s 1day = 24hrs

Calculation of length, width, height, perimeter, area and angles of figures

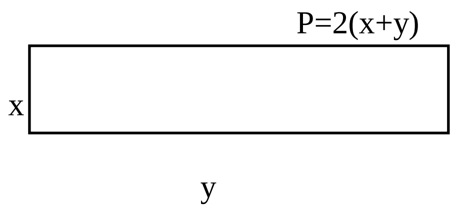
Perimeter

The perimeter is the length of the outline of a shape. To find the perimeter of a rectangle or square you have to add the lengths of all the four sides. X is in this case the length of the rectangle while y is the width of the rectangle.

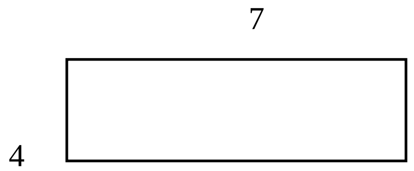
The perimeter, P, is:

$$\mathbf{P = x + x + y + y}$$

$$\mathbf{P = 2x + 2y}$$



Find the perimeter of this rectangle



$$\mathbf{P = 7 + 7 + 4 + 4}$$

$$\mathbf{P = 2 \cdot 7 + 2 \cdot 4}$$

$$\mathbf{P = 2 \cdot (7 + 4)}$$

$$\mathbf{P = 2 \cdot 11}$$

$$\mathbf{P = 22m}$$

Area

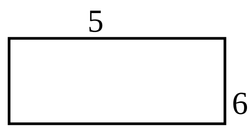
Area is the measurement of the surface of a shape. To find the area of a rectangle or a square you need to multiply the length and the width of a rectangle or a square.

Area, A, is x times y.

$\boldsymbol{A = x \cdot y}$

Examples:

Find the area of this square.



$$\mathbf{A = x \cdot y}$$

$$\mathbf{A = 5 \cdot 6}$$

$$\mathbf{A = 30m^2}$$

There are different units for perimeter and area. Perimeter has the same units as the length of the sides of rectangle or square whereas the area's unit is squared.

Obtain volume and surface area of solids

The surface area of a figure is defined as the sum of the areas of the exposed sides of an object.

The volume of an object is the amount of three-dimensional space an object takes up. It can be thought of as the number of cubes that are one unit by one unit by one unit that it takes to fill up an object.

Surface Area of a Rectangular Solid (Box)

$$\mathbf{SA = 2(lw + lh + wh)}$$

$$\mathbf{l = length\ of\ the\ base\ of\ the\ solid}$$

$$\mathbf{w = width\ of\ the\ base\ of\ the\ solid}$$

$$\mathbf{h = height\ of\ the\ solid}$$

Volume

Volume of a Solid with a Matching Base and Top

V=Ah

A = area of the base of the solid

h = height of the solid

Volume of a Rectangular Solid (specific type of solid with matching base and top)

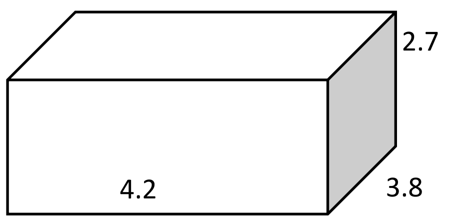
V= $l \times w \times h$

l = length of the base of the solid

w = width of the base of the solid

h = height of the solid

Example:



Based on the way our box is sitting, we can say that the length of the base is 4.2 m; the width of the base is 3.8 m; and the height of the solid is 2.7 m. Thus, we can quickly find the volume of the box to be:

$$\mathbf{V = lwh = 4.2 \times}$$
$$\mathbf{3.8 \times 2.7 = 43.092}\boldsymbol{m^3}$$

A cylinder

A cylinder is an object with straight sides and circular ends of the same size. The volume of a cylinder can be found in the same way you find the volume of a solid with a matching base and top. The surface area of a cylinder can be easily found when you realize that you have to find the area of the circular base and top and add that to the area of the sides. If you slice the side of the cylinder in a straight line from top to bottom and open it up, you will see that it makes a rectangle. The base of the rectangle is the circumference of the circular base, and the height of the rectangle is the height of the cylinder.

Volume of a cylinder

$V=Ah$

A = the area of the base of the cylinder

h = the height of the cylinder

Surface Area of a Cylinder

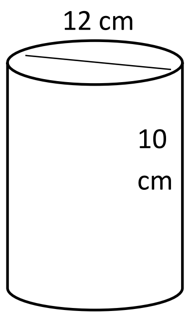
$SA=2(\pi r^2)+2\pi rh$

r = the radius of the circular base of the cylinder

h = the height of the cylinder

π = the number that is approximated by 3.141593

Find the area of the cylinder



$$\mathbf{SA = 2(\pi r2) + 2\pi rh}$$

$$\mathbf{SA = 2(\pi.62) + 2\pi(6)(10) = 603.18579}$$

Measuring tools and equipment

Vernier Caliper

Vernier Caliper is a widely used linear measurement instrument with a least count of 0.02 mm. It is used to measure linear dimensions like length, diameter and depth.

Micrometer

External Micrometer is also known as Outside Micrometer or External Micrometer.

It is used to check outside diameter of circle by the means of accuracy of 0.01 mm or up to 0.001 mm.

Steel Scale

Steel scale is single piece linear measuring instrument. Steel scale indicates two units that are cm and inches, cm division on one side and inches, on another side.

Measuring tape.

Ruler and Rule

Performing measurements and estimations of quantities

3.2.5 Learning Activities

1. In this particular activity, apples will be used to practice math skills of estimation and measurement of the circumference of an apple.

Materials you'll need:

- An apple
- A ruler
- Scissors
- Thread

a) You begin by placing the apple in front of your students. Let them hold the apple to ascertain its circumference. Encourage your students to cut the yarn to a length that they feel would properly wrap around the apple. This will require estimation on their part.

b) Use multiple apples of varying sizes, and have students estimate the amount of string that they'll need for each measurement.

c) Use the ruler to measure the string.

d) After determining whether they're too long or too short, have your students cut a second string to try to get closer to the circumference of the apple.

3.2.6 Self-Assessment

1. A rectangular tray is 820 mm long and 400 mm wide. Find its area in: (a) mm^2 (b) m^2 (c) cm^2

2. Calculate the areas of the following sectors of circles having:

(a) Radius 6 cm with angle subtended at center $50°$

Area of sector of a circle= $\frac{\theta^2}{360}(\pi r^2)$

(b) Diameter 80 mm with angle subtended at center $107°42'$

(c) Radius 8 cm with angle subtended at center 1.15 radians

3. A water tank is the shape of a rectangular prism having length 2 m, breadth 75 cm and height 50 cm. Determine the capacity of the tank in (a) m^3 (b) cm^3 (c) litres

4. Find the volume and total surface area of a cylinder of length 15 cm and diameter 8 cm

3.2.7 Tools, Equipment, Supplies and Materials

- Scientific Calculators
- Rulers, pencils, erasers
- Charts with presentations of data
- Graph books
- Dice
- Computers with internet connection
- Tape measure
- Thread
- Mathematical set

3.2.8 References

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3.2.9 Answers to Self-Assessment

1. A rectangular tray is 820 mm long and 400 mm wide. Find its area in: (a) mm^2 (b) m^2 (c) cm^2

$Area = length \times width = 820 \times 400 = 328000mm^2$Area D length ð width D 820 ð 400 D

$$1cm^2 = 100mm^2$$

Hence: $328000mm^2 = \frac{328000}{100}cm^2 = 3280cm^2$

$1m^2 = 10000cm^2$

Hence: $3280cm^2 = \frac{3280}{10000} = 0.3280m^2$

2. Calculate the areas of the following sectors of circles having:

(a) Radius 6 cm with angle subtended at center $50°$

Area of sector of a circle= $\frac{\theta^2}{360}(\pi r^2)$

$$= \frac{50}{360}(\pi 6^2) = \frac{50 \times \pi \times 36}{360} = 5\pi = 15.71cm^2$$

(b) Diameter 80 mm with angle subtended at center $107°42'$

If the diameter= $80mm$, then the radius, r = 40mm

$$= \frac{107\frac{42}{60}}{360}(\pi 40^2) = 1504mm$$

(c) Radius 8 cm with angle subtended at center 1.15 radians

3. A water tank is the shape of a rectangular prism having length 2 m, breadth 75 cm and height 50 cm. Determine the capacity of the tank in (a) m^3 (b) cm^3 (c) litres

Volume of rectangular prism= $l \times b \times h$

Volume of tank $= 2 \times 0.75 \times 0.5 = 0.75m^3$

$\boldsymbol{1m^3 = 10^6cm^3}$

$\boldsymbol{0.75m^3 = \frac{0.75}{10^6} = 750000cm^3}$

$\boldsymbol{1\ litre = 1000cm^3}$

Hence:

$750000cm^3 = \frac{750000}{1000} = 750litres$

4. Find the volume and total surface area of a cylinder of length 15 cm and diameter 8 cm

Volume of cylinder= $\pi r^2 h$

Since diameter= $8cm$, then radius $= 4cm$

Hence:

$$Volume = \pi \times 4^2 \times 15 = 754cm^3$$

Total surface area $= 2\pi rh + 2\pi r^2 = 477.5cm^2$

4 Learning Outcome 5: Apply Matrices in work

4.2.2 Introduction to the learning outcome

This learning outcome covers: Matrix definition, types, matrix operations, compatibility and determination of inverse of a matrix, solving simultaneous equations with two and three and application of the matrices.

4.2.3 Performance Standard

1. Meaning of matrix
2. Types of matrices
3. Matrix operations
4. Compatibility
5. Addition
6. Subtraction
7. Multiplication
8. Determination of inverse of a matrix
9. Solution of simultaneous equations with two and three unknowns
10. Applications of matrices

4.2.4 Information Sheet

Matrix: This is a set of real or complex numbers arranged in rows and columns to form a rectangular array and it is always denoted by capital letters.

Order: A matrix of order of $(\boldsymbol{m} \times \boldsymbol{n})$

Determinant: It is a physical quantity/value assigned to any square matrix.

E.g. given $\begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{bmatrix} = \boldsymbol{ad - cd}$

Inverse: The inverse of a matrix $\boldsymbol{Q^{-1}}$ is the matrix than when multiplied by the original matrix $\boldsymbol{Q}$ gives the identity matrix. i.e. $\boldsymbol{Q\, Q^{-1}}$=$\boldsymbol{I}$.

Matrices

The numbers within a matrix are called an array and the coefficients forming the array are called the elements of the matrix. The number of rows in a matrix is usually specified by m and the number of columns by n and a matrix referred to as an 'm by n' matrix. Thus, $\begin{pmatrix} \boldsymbol{2} & \boldsymbol{3} & \boldsymbol{6} \\ \boldsymbol{4} & \boldsymbol{5} & \boldsymbol{7} \end{pmatrix}$ is a '2 by 3' matrix?

Type of matrix

The numbers within a matrix are called an **array** and the coefficients forming the array are called the **elements** of the matrix. The number of rows in a matrix is usually specified by *m* and the number of columns by *n* and a matrix referred to as an '*m* by *n*' matrix.

Matrix Operations

To be added or subtracted matrices must be of the same order.

In describing the matrix, the number of rows the number of rows is stated first and the number of columns second. i.e.

$\begin{pmatrix} 6 & 3 \\ 2 & 4 \\ 5 & 1 \end{pmatrix}$ is a matrix of order of 3×2

Compatibility

A row matrix consists of 1 row only

A column matrix consists of 1 column only.

Addition

Corresponding elements in two matrices may be added to form a single matrix.

Problem 1. Add the matrices

$\begin{pmatrix} \boldsymbol{2} & \boldsymbol{-1} \\ \boldsymbol{-7} & \boldsymbol{4} \end{pmatrix} + \begin{pmatrix} \boldsymbol{-3} & \boldsymbol{0} \\ \boldsymbol{7} & \boldsymbol{-4} \end{pmatrix}$

$\begin{pmatrix} \boldsymbol{2 + -3} & \boldsymbol{-1+0} \\ \boldsymbol{-7+7} & \boldsymbol{4+-4} \end{pmatrix}$

$= \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix}$

Subtraction

If A is a matrix and B is another matrix, then (A_B) is a single matrix formed by subtracting the elements of B from the corresponding elements of A.

Problem 2. Subtract

$\begin{pmatrix} -3 & 0 \\ 7 & -4 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}$

To find matrix A minus matrix B, the elements of B are taken from the corresponding elements of A. Thus:

$\begin{pmatrix} -3 & 0 \\ 7 & -4 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}$

$\begin{pmatrix} -3-2 & 0--1 \\ 7--7 & -4-4 \end{pmatrix}$

$\begin{pmatrix} -5 & 1 \\ 14 & -8 \end{pmatrix}$

Multiplication

When a matrix is multiplied by a number, called scalar multiplication, a single matrix results in which each element of the original matrix has been multiplied by the number.

Problem 4.

If A $=\begin{pmatrix} -3 & 0 \\ -7 & -4 \end{pmatrix}$, B = $\begin{pmatrix} -2 & -1 \\ -7 & 4 \end{pmatrix}$ and C = $\begin{pmatrix} -1 & 0 \\ -2 & -4 \end{pmatrix}$

Find

2A – 3B + 4C

For scalar multiplication, each element is multiplied by the scalar quantity, hence

2A = 2 $\begin{pmatrix} -3 & 0 \\ -7 & -4 \end{pmatrix} = \begin{pmatrix} -6 & 0 \\ -14 & -8 \end{pmatrix}$

3B = 3$\begin{pmatrix} -2 & -1 \\ -7 & 4 \end{pmatrix} = \begin{pmatrix} -6 & -3 \\ -21 & 12 \end{pmatrix}$

4C = 4 $\begin{pmatrix} -1 & 0 \\ -2 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 0 \\ -8 & -16 \end{pmatrix}$

Hence 2A - 3B + 4C

$\begin{pmatrix} -6 & 0 \\ -14 & -8 \end{pmatrix} - \begin{pmatrix} -6 & -3 \\ -21 & 12 \end{pmatrix} + \begin{pmatrix} -4 & 0 \\ -8 & -16 \end{pmatrix}$

When a matrix A is multiplied by another matrix B, a single matrix results in which elements are obtained from the sum of the products of the corresponding rows of A and the corresponding columns of B. Two matrices A and B may be multiplied together, provided the number of elements in the rows of matrix A are equal to the number of elements in the columns of matrix B. In general terms, when multiplying a matrix of dimensions (m by n) by a matrix of dimensions (n by r), the resulting matrix has dimensions (m by r). Thus a 2 by 3 matrix multiplied by a 3 by 1 matrix gives a matrix of dimensions 2 by 1.

Problem 5. If A = $\begin{pmatrix} \mathbf{-2} & \mathbf{3} \\ \mathbf{1} & \mathbf{-4} \end{pmatrix}$ B = $\begin{pmatrix} \mathbf{-5} & \mathbf{7} \\ \mathbf{-3} & \mathbf{4} \end{pmatrix}$

Find A × B.

Let A × B = C where C = $\begin{pmatrix} \boldsymbol{C_{11}} & \boldsymbol{C_{12}} \\ \boldsymbol{C_{21}} & \boldsymbol{C_{22}} \end{pmatrix}$

C11 is the sum of the products of the first row elements of A and the first column elements of B taken one at a time, i.e. $\boldsymbol{C_{11}} = \left(\mathbf{-2} \times (\mathbf{-5})\right) + \left(\mathbf{3} \times (\mathbf{-3})\right) = \mathbf{-19}$

$\boldsymbol{C_{12}}$ is the sum of the products of the first row elements of A and the second column elements of B, taken one at a time, i.e. $\boldsymbol{C_{12}} = (\mathbf{2} \times \mathbf{7}) + (\mathbf{3} \times \mathbf{4}) = \mathbf{26}$

C21 is the sum of the products of the second row elements of A and the first column elements of B, taken one at a time, i.e. $\boldsymbol{C_{21}} = (\mathbf{1} \times \mathbf{-5}) + (\mathbf{-4} \times \mathbf{-3}) = \mathbf{7}$

Finally, C22 is the sum of the products of the second row elements of A and the second column elements of B, taken one at a time, i.e. $\boldsymbol{C_{22}} = (\mathbf{1} \times \mathbf{7}) + (\mathbf{-4} \times \mathbf{4}) = \mathbf{-9}$

Thus, A × B = $\begin{pmatrix} \mathbf{-19} & \mathbf{26} \\ \mathbf{7} & \mathbf{-9} \end{pmatrix}$

Determination of inverse of a matrix

The determinant of a 2 by 2 matrix

The determinant of a 2 by 2 matrix, $\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}$ is defined as (ad _ bc).

The elements of the determinant of a matrix are written between vertical lines. Thus, the determinant of $\begin{pmatrix} \mathbf{3} & \mathbf{-4} \\ \mathbf{1} & \mathbf{6} \end{pmatrix}$ is written as $\begin{vmatrix} \mathbf{3} & \mathbf{-4} \\ \mathbf{1} & \mathbf{6} \end{vmatrix}$ and is equal to $(\mathbf{3} \times \mathbf{6}) - (\mathbf{-4} \times \mathbf{1}) = \mathbf{22}$. Hence the determinant of a matrix can be expressed as a single numerical value, i.e. $\begin{vmatrix} \mathbf{3} & \mathbf{-4} \\ \mathbf{1} & \mathbf{6} \end{vmatrix} = \mathbf{22}$

The inverse or reciprocal of a 2 by 2 matrix

The inverse of matrix A is $\boldsymbol{A^{-1}}$ such that $\boldsymbol{A} \times \boldsymbol{A^{-1}} = \mathbf{1}$, the unit matrix.

Let matrix A be $\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}$ and let the inverse matrix, $\boldsymbol{A^{-1}}$ be $\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}$

Then, since $\boldsymbol{A} \times \boldsymbol{A^{-1}} = \mathbf{1}$

$$\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix} \times \begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

Multiplying the matrices on the left hand side, gives

$$\begin{pmatrix} \boldsymbol{a} + \mathbf{2}\boldsymbol{c} & \boldsymbol{b} + \mathbf{2}\boldsymbol{d} \\ \mathbf{3}\boldsymbol{a} + \mathbf{4}\boldsymbol{c} & \mathbf{3}\boldsymbol{b} + \mathbf{4}\boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

Equating corresponding elements gives:

$\boldsymbol{b} + \mathbf{2}\boldsymbol{d} = \mathbf{0}$

$\boldsymbol{b} = \mathbf{-2}\boldsymbol{d}$ And $\mathbf{3}\boldsymbol{a} + \mathbf{4}\boldsymbol{c} = \mathbf{0}$

$$\boldsymbol{a} = -\frac{\mathbf{4}}{\mathbf{3}}c$$

Substituting for a and b gives:

$$\begin{pmatrix} -\frac{4}{3}\boldsymbol{c} + 2\boldsymbol{c} & -2\boldsymbol{d} + 2\boldsymbol{d} \\ 3\left(-\frac{4}{3}\boldsymbol{c}\right) + 4\boldsymbol{c} & 3(-2\boldsymbol{d}) + 4\boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{2}{3}\boldsymbol{c} & \mathbf{0} \\ \mathbf{0} & -2\boldsymbol{d} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

Showing that $\frac{2}{3}\boldsymbol{c} = \mathbf{1}, \boldsymbol{c} = \frac{3}{2}\ \boldsymbol{and} - 2\boldsymbol{d} = \mathbf{1}\ \boldsymbol{i.e.d} = -\frac{1}{2}$

Since $\boldsymbol{b} = -2\boldsymbol{d}\ , \boldsymbol{b} = \mathbf{1}\ \boldsymbol{and\ since\ a} = -\frac{4}{3}\boldsymbol{c}\ , \boldsymbol{a} = -\mathbf{2}$

Thus the inverse of matrix $\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}$ is $\begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}$ that is $\begin{pmatrix} -\mathbf{2} & \mathbf{1} \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

Solution of simultaneous equations with two and three unknowns

The procedure for solving linear simultaneous equations in two unknowns using matrices is:

(i) Write the equations in the form $\boldsymbol{a_1x} + \boldsymbol{b_1y} = \boldsymbol{c_1}, \boldsymbol{a_2x} + \boldsymbol{b_2y} = \boldsymbol{c_2}$

(ii) Write the matrix equation corresponding to these equations, i.e. $\begin{pmatrix} \boldsymbol{a_1} & \boldsymbol{b_1} \\ \boldsymbol{a_2} & \boldsymbol{b_2} \end{pmatrix} \times \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} = \begin{pmatrix} \boldsymbol{c_1} \\ \boldsymbol{c_2} \end{pmatrix}$

(iii) Determine the inverse matrix of $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ i.e. $\frac{1}{a_1b_2 - b_1a_2}\begin{pmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{pmatrix}$

(iv) Multiply each side of (ii) by the inverse matrix

(v) Solve for x and y by equating corresponding elements.

The procedure for solving linear simultaneous equations in three unknowns using matrices is:

(i) Write the equations in the form

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

(ii) Write the matrix equation corresponding to these equations, i.e.

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

(iii) Determine the inverse matrix of

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

(iv) Multiply each side of (ii) by the inverse matrix

(v) Solve for x, y and z by equating the corresponding elements.

Applications of Matrices

- Cryptography

This is the process of hiding information for security purposes.

- Fourier analysis
- Gauss theorem
- Finding forces in the bridge
- Finding electric currents using matrix equation

4.2.5 Learning Activities

1. Adding and Subtracting Matrices

Use this kinesthetic activity to help students add and subtract matrices by creating human matrices.

Materials

- Masking/painter's tape
- Copy paper
- Markers

Teacher Directions

- Show students examples of matrices and demonstrate how to add and subtract them. If possible, use matrices that reflect real-life data, such as population numbers for a specific continent.
- Give each student a piece of paper and marker.
- Have students write a number of their choice on the paper.
- Divide the classroom in half using a piece of masking/painter's tape.
- Create a matrix on both sides of the tape using students as the numbers. Have students stand in rows and columns holding their number signs on either side of the tape.
- Have students not involved in creating the matrices add or subtract them.
- Review the correct answers with the class.
- Have students rotate between serving as the numbers in the matrices and figuring out the sums or differences.

Discussion Questions

- What steps do you follow to add and subtract matrices?
- In what real-world situations might you need to add and subtract matrices?

4.2.6 Self-Assessment

1. Add the matrices

$$\begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix} + \begin{pmatrix} -3 & 5 \\ 7 & 6 \end{pmatrix}$$

2. Subtract

$$\begin{pmatrix} 3 & 1 & -4 \\ 4 & 3 & 1 \\ 1 & 4 & -3 \end{pmatrix} - \begin{pmatrix} 2 & 7 & -5 \\ -2 & 1 & 0 \\ 6 & 3 & 4 \end{pmatrix}$$

3. Determine the value of:

$$\begin{vmatrix} 3 & -2 \\ 7 & 4 \end{vmatrix}$$

4. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$, show that $A \times B \neq B \times A$

4.2.7 Tools, Equipment, Supplies and Materials

- Scientific calculator
- 15 cm ruler
- Pen, pencil
- Ruled and graph paper
- Mark Pens
- Cards

4.2.8 References

Aitken, A.C (2017) Determinant and matrices Read Book Ltd.

Panju, M(2011) Iterative methods for computing eigen values and eigen vectors ar.xiv 11051185

Searle. S.R., & Khun, A.I. (2017) matrix algebra useful for statistics. John wiley & sons.

4.2.9 Answers to Self-Assessment

1. Add the matrices

$$\begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix} + \begin{pmatrix} -3 & 5 \\ 7 & 6 \end{pmatrix} = \begin{matrix} 4+(-3) & -2+5 \\ -1+7 & 3+6 \end{matrix} = \begin{pmatrix} 1 & -3 \\ 6 & 9 \end{pmatrix}$$

2. Subtract

$$\begin{pmatrix} 3 & 1 & -4 \\ 4 & 3 & 1 \\ 1 & 4 & -3 \end{pmatrix} - \begin{pmatrix} 2 & 7 & -5 \\ -2 & 1 & 0 \\ 6 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 3-2 & 1-7 & -4-(-5) \\ 4-(-2) & 3-1 & 1-0 \\ 1-6 & 4-3 & -3-4 \end{pmatrix} = \begin{pmatrix} 1 & -6 & 1 \\ 6 & 2 & 1 \\ -5 & 1 & -7 \end{pmatrix}$$

3. Determine the value of:

$$\begin{vmatrix} 3 & -2 \\ 7 & 4 \end{vmatrix} = (3 \times 4) - (-2 \times 7) = 26$$

4. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$, show that $A \times B \neq B \times A$

$$A \times B = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} [(2 \times 2) + (3 \times 0)] & [(2 \times 3) + (3 \times 1)] \\ [(1 \times 2) + (0 \times 0)] & [(1 \times 3) + (0 \times 1)] \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 9 \\ 2 & 3 \end{pmatrix}$$

$$B \times A = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} [(2 \times 2) + (3 \times 1)] & [(2 \times 3) + (3 \times 0)] \\ [(0 \times 2) + (1 \times 1)] & [(0 \times 3) + (1 \times 0)] \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 6 \\ 1 & 0 \end{pmatrix}$$

Therefore $A \times B \neq B \times A$

5 Learning Outcome 6: Apply Vectors in work

5.2.2 Introduction to the learning outcome

This learning outcome covers; derivation of vectors in two and three dimensions and performing various operations on vectors which includes obtaining vector position and performing resolution of vectors.

5.2.3 Performance Standard

1. Meaning of vector
2. Representations of vectors
3. Operations of vectors
4. Addition
5. Subtraction
6. Scalar and vector products
7. Determination of angles

5.2.4 Information Sheet

5.2.4.1 Definition of terms

Scalar: These are quantities with magnitude but no direction.

Vector: These are quantities with both magnitude and direction.

Position vector: This is a vector that originates form the origin to a given point e.g. velocity and acceleration.

Meaning of vector

A vector is a quantity having both magnitude and direction such as force, velocity etc. A vector is also defined as a tensor of order one. A scalar has only magnitude and is completely characterized by one number.

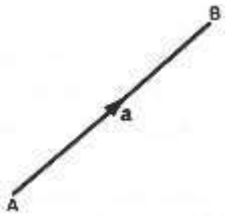
It is a tensor of order zero and temperature is an example of a scalar. A second order tensor, in a three-dimensional space, is represented by nine numbers (components).

Representations of vectors

The vector quantity can be represented graphically by a line, drawn so that:

i. The length of the line denotes the magnitude of the quantity according to the stated vector scale
ii. The direction of the line denotes the direction in which the vector quantity acts. The sense of the direction is indicated by an arrowhead.

The vector quantity AB is referred to as $\overline{AB}$ or a.



The magnitude of the vector quantity is written $|\overline{AB}|$ or $|a|$ or simply AB or a.

Operations of vectors

To add or to subtract vectors, we add or subtract the respective corresponding components.

I.e. let $\vec{P}$ = (a, b) and $\vec{Q}$ = (c, d)

Then $\vec{p}$ + $\vec{q}$ = $(a + c, b + d)$

Let $\vec{r}$ = (r_1, r_2) and $\vec{S}$ = (s_1, s_2)

=$(r_1 - s_1, r_2 - s_2)$

Example: Find

$(\vec{P} + \vec{Q})$ B) $(\vec{P} - \vec{Q})$ Given $\vec{P} = (4, 5)$ And $\vec{Q}$ = $(6, -1)$

Solution

$\vec{P}$ + $\vec{Q}$ = $(p_1 + q_1, p_2 + q_2)$

= $(4 + 6, 5 + -1)$

= $(10, 4)$

ii) $\vec{P}$ - $\vec{Q}$ = $(p_1 - q_1, p_2 - q_2)$

= $(4 - 6,\ 5 - (-1))$

$= (-2, 6)$

OR

Rewrite the difference $\vec{P}$ - $\vec{Q}$ as the sum $\vec{P}$ +$\left(\overrightarrow{-Q}\right)$. To determine $\overrightarrow{-Q}$, we do a scalar multiplication of -1 times $\overrightarrow{-Q}$

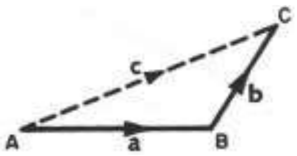
$\overrightarrow{-q}$ = -1 $(6, -1)$

$= (-6,\ 1)$

Hence $\vec{P}$ + $\vec{Q}$ $= (4 - 6, \quad 5 + 1) \quad = (-2 \quad, \ 6)$

Addition

The sum of two vectors, $\overline{AB}$ and $\overline{BC}$ is defined as the single equivalent or resultant vector$\overline{AC}$.



$\overline{AB} + \overline{BC} = \overline{AC}$

$a + b = c$

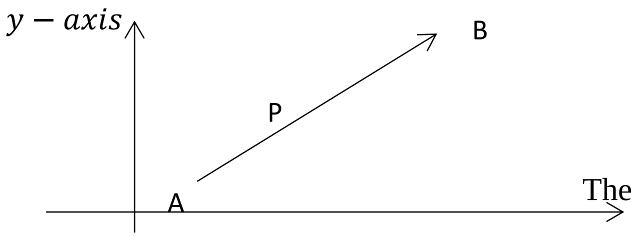
Example:

A force of 40N is acting in the direction due east represented by p and a force of 30N is acting in the direction due south represented as q, find the magnitude (r) of the vector of the two forces.

$r^2 = p^2 + q^2 = \sqrt{(p^2 + q^2)} = \sqrt{(40^2 + 30^2)}$=50

Scalar and vector products

In the concept of two-dimensions, we can use the Cartesian planes to draw our vectors



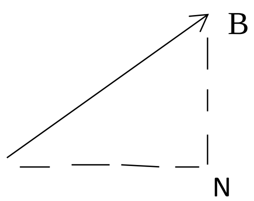
The vector can be identified $\vec{P}$

Or (A, B)

1 2 3 4 5 $x - axis$

Vector P is a 2-dimensional vector drawn with 3 units on the x-axis and 3 units on the y-axis.

The magnitude of vector P is simply the length of vector P which we can calculate using the basic Pythagoras theorem i.e. we can denote the vector components as $NB = 3$ and AN = 3

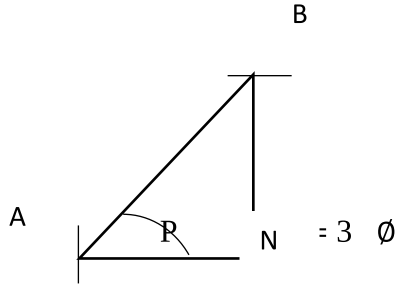


A

Magnitude of P (denoted as /p/)

$$|P| = \sqrt{3^2 + 3^2} = 4.24\ units$$

Direction of a vector normally we use degrees or radians from the horizontal in an anticlockwise direction to describe direction of a vector



Where AB – Hypotenuse side

BN - Adjacent side

NB – Opposite side to angle Ø

$P_X = 3$

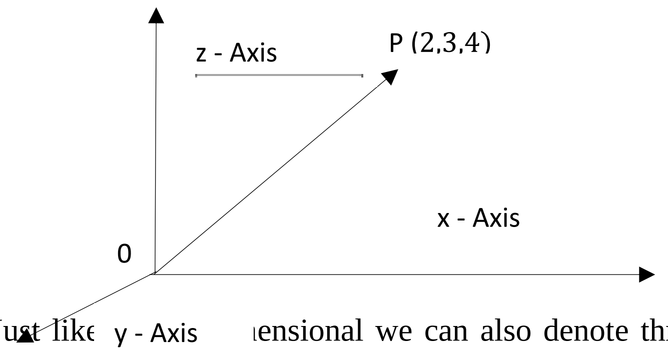
We first form a right-angled triangle by joining the ends of the vector then apply simple trigonometry.

So, we have: Tanθ =$\frac{3}{3}$ ($\frac{opposite}{adjacent}$) = 1

$$tan^{-1} = 45^0 = 0.785\ radians$$

Hence, we can describe our vector as having magnitude 4.24 units and direction 45^0 from the right horizontal axis. In three dimensional spaces, we construct three mutually perpendicular axes (commonly the x, y, z axis).

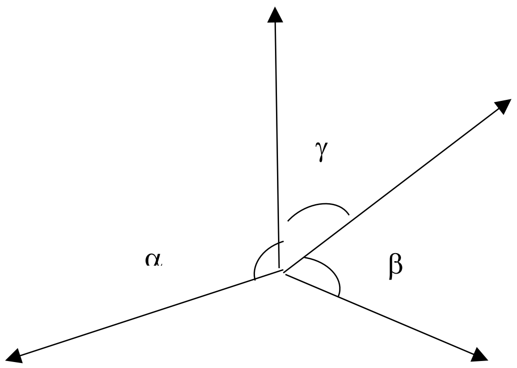
Example: Sketch point (2,3,4)



Just like ensional we can also denote three dimensional vectors using the standard unit vectors i, j, k, i=$(1, 0, 0)$ j=$(0, 1, 0)$, k=$(0, 0, 1)$ an then express the vector as sum of scalar multiples of these units vectors i.e. P=(P_1 P_2 P_3)= P_{1i} + P_{2j}+ P_{3k}

For vector OP_1

$|OP|$ = $\sqrt{2^2} + 3^2 + 4^2$ = 5.385units



δ-Angle between P and x-axis

Use scalar products to get the direction cosines.

I.e. $p_1 = \quad P.I \quad = 1 \times 1 \times cos\ \delta$

$p_2 \ = \ P.j \quad = 1 \times 1 \times cos\ \beta$

$p_3 \ = \ P.k \quad = 1 \times 1 \times cos\ \gamma$

So, we can write a unit vector P as:

$$P = \quad Cos\ \delta i \ + \ Cos\ \beta\ j \ + \ Cos\ \gamma\ k$$

Determination of angles

The direction of a vector in three directions is determined in angles which the vector makes with the three axes of reference.

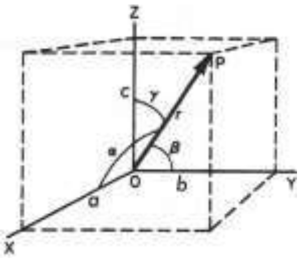


Figure 5: Directional Cosines

Let $\overline{OP} = r = ai + bj + ck$

Then; $\frac{a}{r} = cos\alpha \ \therefore a = rcos\alpha$

$\frac{b}{r} = cos\beta \ \therefore b = rcos\beta$

$\frac{c}{r} = cos\gamma \ \therefore c = rcos\gamma$

Also $a^2 + b^2 + c^2 = r^2$

$r^2cos^2\alpha + r^2cos^2\beta + r^2cos^2\gamma = r^2$

$cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

Scalar (Dot) product of vectors

Product of two non-zero vectors $\vec{a}$ and $\vec{b}$ denoted by $\vec{a}.\vec{b}$ = $|\vec{a}||\vec{b}|$ Cosθ

Where θ is the angle between $\vec{a}$ and $\vec{b}$.

When either $\vec{a}$ = 0 or $\vec{b}$ = 0 then θ is not defined and in this case, $\vec{a}$. $\vec{b}$ = 0

If both $\vec{a}$ and $\vec{b}$ are non-zero and yet $\vec{a}$. $\vec{b}$ = 0, then this implies that $\vec{a}$ and $\vec{b}$ are perpendicular to each other. The angle between two non-zero vectors is given by;

Cos θ = $\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

Or simply θ= cos -1 $\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

If the θ has not been given,

$\vec{a}$. $\vec{b}$ = $(a_1b_1, +a_2b_2)$ = K (a constant).

Example

Compute the product for each of the following.

$\vec{p}$ = 4i -3j $\vec{q}$ = 2i + j

Solution

$\vec{p}$. $\vec{q}$ = (4×2) + (-3×1)

= 8-3 = 5

$\vec{p}$ = $(8, 0, 3)$ $\vec{q}$ = $(3, 2, 7)$

Solution

$\vec{p}$. $\vec{q}$ = (8×3) + (0×2) + (3×7)

= 6 + 0 + 21

= 27

Determine the angle between $\vec{p}$ = $(3, -4, -1)$ and $(0, 5, 2)$

Solution

First, we need to calculate $\vec{p}$. $\vec{q}$ And then $|\vec{p}|$ and $|\vec{q}|$

$\vec{p}$. $\vec{q}$ = (3×0) + (-4×5) + (-1×2)

= 0+ -20-2

$= -22$

$|\vec{p}| = \sqrt{3^2 + (-4^2) + (-1^2)}$ $= \sqrt{26}$

$|\vec{q}| = \sqrt{0^2} + 5^2 + 2^2$ $= \sqrt{29}$

Cos $\theta = \frac{\vec{P}.\vec{Q}}{|\vec{P}||\vec{Q}|} = \frac{-22}{\sqrt{26}\sqrt{29}}$ =-0.8011927 =

$Cos^{-1}(-0.8011927)$ = 143.24° (2.5 radians)

Let $\vec{P} = (p_1, p_2, p_3)$ and $\vec{Q} = (q_1, q_2, q_3)$

Then $\vec{P} \times \vec{Q} = (p_2q_3, p_3q_3 - p_3q_2)$i - $(p_1q_3 - p_3q_1)$j - $(p_1q_2 - p_2q_1)$k

Position of vectors

Definition: A position vector is a vector that starts from the origin i.e. its origin is $(0, 0, 0)$

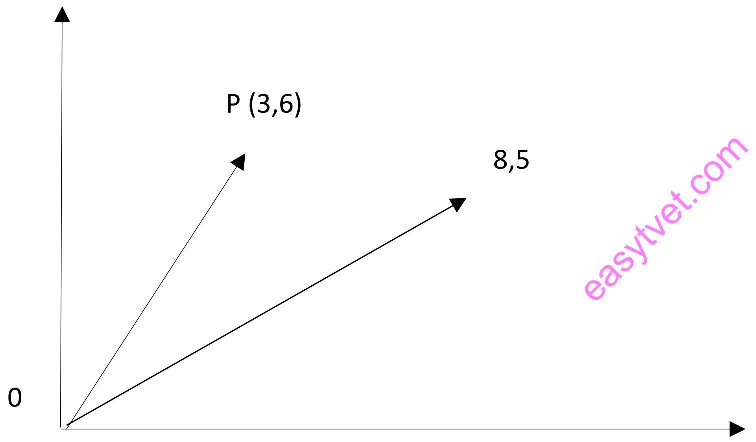


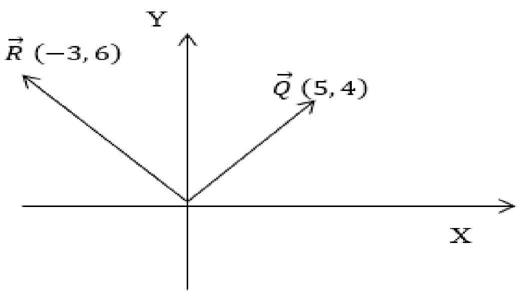
Figure 6: Position Vectors

The position vector of $\vec{P}$ is $\binom{3}{6}$ and position vector of $\vec{Q}$ $\binom{8}{5}$

Example 1

The position vector of $\vec{Q}$ is $\binom{5}{4}$ and the position vector and the position vector of $\vec{R}$ $\binom{-3}{6}$ is

Find the vector $\overrightarrow{QR}$



Solution

$\overrightarrow{QR} = \overrightarrow{QO} + \overrightarrow{OR}$

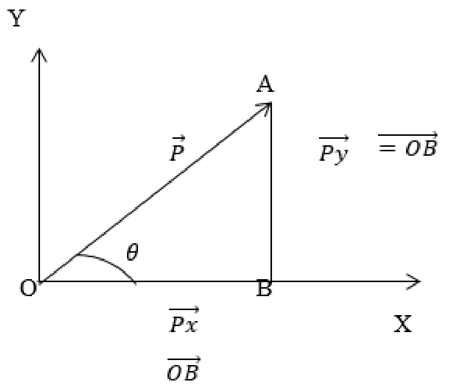
NB: $\overrightarrow{QO}$ = - $\overrightarrow{OQ}$ (opposite direction)

$-\binom{5}{4} + \binom{-3}{6} \quad = \binom{-8}{2}$

Generally, $\overrightarrow{AB}$= -a + b or simply b-a since from A to B, you must first move from A to the origin hence (-a) then from the origin to B.

Performance of vector resolution

Definition: A vector resolution is the breaking down of one vector into two or more smaller vectors. In e1this regard, we only discuss the rectangular components of a vector (X-component and Y component). Thus,



The rectangular components of $\vec{P}$ are $\vec{P}_X$ and $\vec{P}_y$

Cosθ = $\frac{OB}{OA}$

OB = $\vec{P}$ Cosθ

Recall Cosine = $\frac{Adjacent}{Hypotenuse}$

But OB = P_x

Hence $\boxed{P_x = \vec{P}\ \text{Cos}\theta}$ -The magnitude of horizontal component

Sinθ= $\frac{AB}{OB}\left[Sin\theta = \frac{Opposite}{Hypotenuse}\right]$

AB= OA Sinθ (but AB= P_y)

Hence $P_y = \vec{P}sin\theta$ is the magnitude of vertical component.

5.2.5 Learning Activities

1. Activity 1

Labeling Components

One of the first steps to understanding vectors is knowing what the different components are. Have your students work with partners for the first part of this activity. Each pair should create a graph that represents a vector. Their graphs should carefully represent the difference between a vector and a line, particularly because the graphs can have some similarities.

Then, have all students hang their graphs on a classroom wall, numbering each graph. Ask students to walk along the wall with their notebooks and notate the horizontal and vertical components of each of their classmates' vectors.

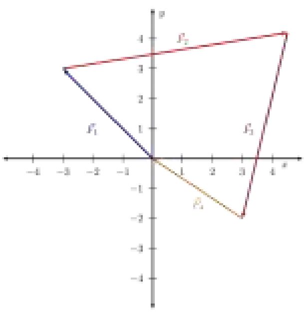
Once you have discussed their answers, ask each partnership to take their own graph and use colors and arrows to show how the horizontal and vertical components of their own vectors are determined.

2. Activity 2

To understand the principles of adding vectors by the graphical method and of adding vectors by component addition.

Two or more vectors can be added together to determine a vector sum or resultant. Two methods of adding vectors are the graphical or head-to-tail method and the trigonometric or component addition method. For each problem, vectors A, B and C are shown.

i. Sketch the head-to-tail addition of F_1, F_2, F_3 and F_4 on the empty grid; label each vector.
ii. Draw and label the resultant (R).
iii. Record the magnitude and direction of each component; use a + to indicate East or North; use a – to indicate a West or South.
iv. Sum the components to determine the components of the resultant.
v. Use the Pythagorean theorem and SOH CAH TOA to determine the magnitude and direction of R

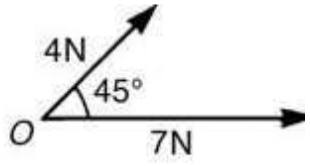


Magnitude of R =

Direction of R =

5.2.6 Self-Assessment

1. Calculate the resultant force of the two forces:

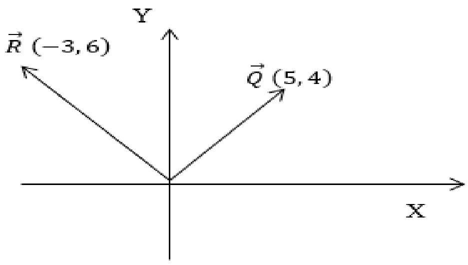


i. The magnitude of the resultant of vector addition

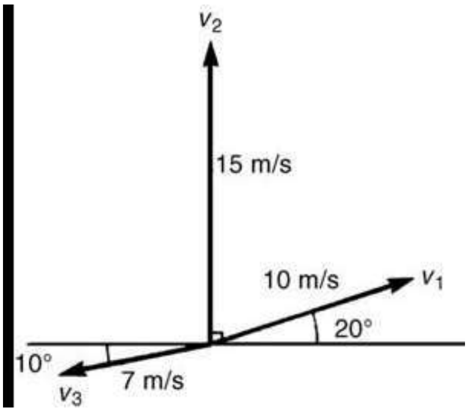
ii. The direction of the resultant of vector addition

1 The position vector of $\vec{Q}$ is $\binom{5}{4}$ and the position vector and the position vector of $\vec{R}\ is\ \binom{-3}{6}$

Find the vector $\overrightarrow{QR}$



3. Use a graphical method to determine the magnitude and direction of the resultant of the three velocities



5.2.7 Tools, Equipment, Supplies and Materials

- Calculator
- Ruler
- Black/white board
- Chalk/ board markers (Assorted)

5.2.8 References

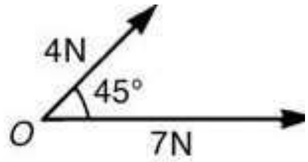
Pramod S. Joag. (2016). An introduction to vectors, vector operators and vector analysis. Delhi: Cambridge University Press.

Ray M. Bowen & C.C. Wang (2008). Introduction to vectors and tensors: Mineola: Dover Publications.

Richard E. Johnson (2010). Vector Algebra Meerut: Krishna Prakashan Media (P) Ltd.

5.2.9 Answers to Self-Assessment

1. Calculate the resultant force of the two forces:



Horizontal component of force,

$$H = 7COS0° + 4COS45°$$

$= 7 + 2.828 = 9.828$N

Vertical component of force,

$$V = 7sin0° + 4sin45° = 0 + 2.828 = 2.828$$

The magnitude of the resultant of vector addition

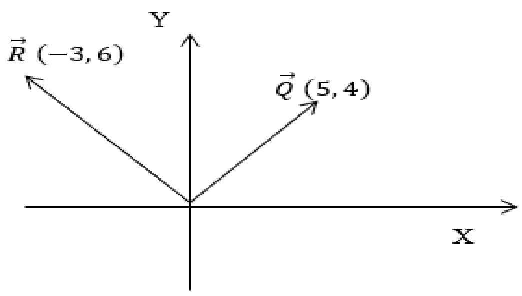
$= \sqrt{H^2 + V^2} = \sqrt{9.828^2 + 2.828^2}$=10.23N

The direction of the resultant of vector addition

$$= tan^{-1}\left(\frac{2.828}{9.828}\right) = 16.05^{\circ}$$

2. The position vector of $\vec{Q}$ is $\binom{5}{4}$ and the position vector and the position vector of $\vec{R}$ $\binom{-3}{6}$ is

Find the vector $\overrightarrow{QR}$



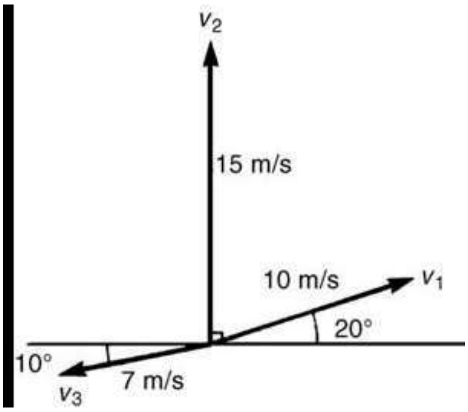
Solution

$\overrightarrow{QR}$ = $\overrightarrow{QO}$ + $\overrightarrow{OR}$

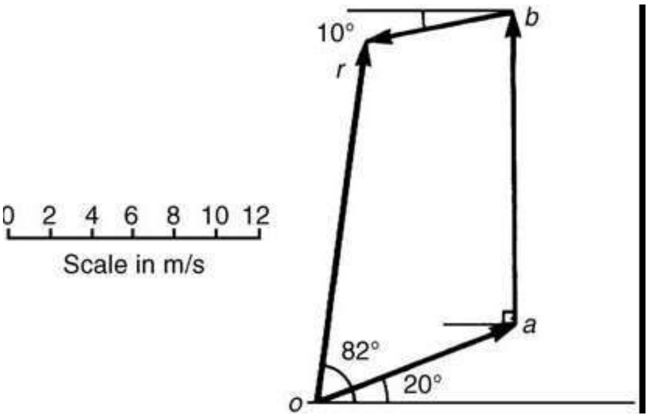
NB: $\overrightarrow{QO}$ = - $\overrightarrow{OQ}$ (opposite direction)

-$\binom{5}{4}$ + $\binom{-3}{6}$ = $\binom{-8}{2}$

3. Use a graphical method to determine the magnitude and direction of the resultant of the three velocities



$$v_1 + v_2 + v_3 = 17.5\,{}^{m}/_{s}\ at\ 82°\ to\ the\ horizontal$$



6 Learning Outcome 7: Collect, Organize and Interpret Statistical Data

6.2.2 Introduction to the learning outcome

This learning outcome covers obtainment of mean, median, mode and standard deviation from given data, performance of calculations based on laws of probability, performance of calculation involving probability distributions, mathematical expectation sampling distributions and application of sampling distribution methods in data analysis.

6.2.3 Performance Standard

1. Classification of data
a) Grouped data
b) Ungrouped data
2. Data collection
a) Tabulation of data
b) Class intervals
c) Class boundaries

d) Frequency tables

e) Cumulative frequency

3. Sampling

a) Importance of sampling

b) Errors in sampling

c) Types of sampling and their limitations

4. Diagrammatic and graphical presentation of data e.g.

a) Histograms

b) Frequency polygons

c) Bar charts

d) Pie charts

e) Cumulative frequency curves

5. Measures of central tendency

a) Measures

b) Properties

6. Calculation and interpretation of mean, mode and median, variance and standard deviation

6.2.4 Information Sheet

6.2.4.1 Definition of Terms

Mode: This is the number which is the most repeated in a series.

Standard deviation: This is the amount of variation of a set of numbers. It is the square root variance.

Variance: This is the mean of the squared differences of the number from the mean.

Classification of data

A variable may be of two kinds:

Discrete – a variable that can be counted, or for which there is a fixed set of values.

Continuous – a variable that can be measured on a continuous scale, the result depending on the precision of the measuring instrument or the accuracy of the observer.

Ungrouped data can be presented diagrammatically in several ways and these include:

(a) Pictograms, in which pictorial symbols are used to represent quantities

(b) Horizontal bar charts, having data represented by equally spaced horizontal rectangles

(c) Vertical bar charts, in which data are represented by equally spaced vertical rectangles

Trends in ungrouped data over equal periods of time can be presented diagrammatically by a percentage component bar chart.

Grouped data

When the number of members in a set is small, say ten or less, the data can be represented diagrammatically without further analysis, by means of pictograms, bar charts, percentage components bar charts or pie diagrams.

For sets having more than ten members, those members having similar values are grouped together in classes to form a frequency distribution. To assist in accurately counting members in the various classes, a tally diagram is used.

A frequency distribution is merely a table showing classes and their corresponding frequencies.

The new set of values obtained by forming a frequency distribution is called grouped data.

Data collection

We can use this example to define a number of terms, consider the class labelled 7.1 – 7.3

The class values stated in the table are the lower and upper limits of the class and their difference gives the class width.

The class boundaries are 0.05 below the lower class limit and 0.05 above the upper class limit.

The lower class boundary is 7.1 - 0.05 = 7.05

The upper class boundary is 7.3 + 0.05 = 7.35

The class interval is the difference between the upper and lower class boundaries.

Class interval = upper class boundary – lower class boundary

= 7.35 – 7.05 = 0.30

Where the classes are regular, the class interval can also be found by subtracting any lower class limit from the lower class limit of the following class.

The central value (mid value) of the class is the average of the upper and lower class boundaries.

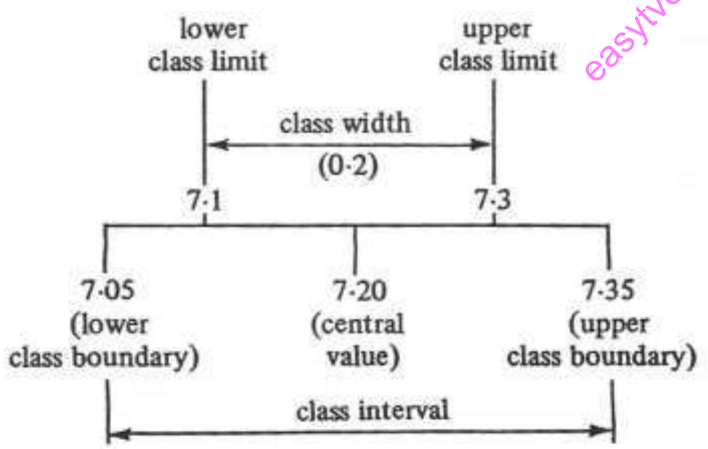


Figure 7: Class Boundaries

Sampling

Performance of calculations involving use of standard normal table, sampling distribution, t-distribution and estimation

Standard normal table (known Z table)

$$Z = \frac{x - \mu}{\alpha}$$

$$X = raw\ score$$

$$\mu = mean$$

$$\alpha \ = standard\ deviation$$

A Z found on the both tables should be used on the table provided to known the percentile and therefore compare. Note; if it is a negative then a table for negative values is used.

Sampling distribution

It is also known as probability distribution and the standard deviation of this topic is known as standard error. Sampling distribution mean is equal to the mean of the population.

$\mu_x = \mu$

Therefore, standard error is:

$$\sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{n-1}}$$

$$SE = \frac{\sigma}{\sqrt{n}}$$

SE Standard error

∂ =Standard deviation

μ=Size of population

n= size of sample

If $fpc = 1$ from factor$\sqrt{N-n)/N-1)}$, therefore standard arror formula can be approximated by

$$\surd(\mathrm{x} = \alpha)/\surd n$$

Distribution

It is a type of distribution similar with the normal distribution curve but with a bit shorter and fatter tail. Therefore distribution is used because small size is small.

$$t = \frac{x - \mu}{(\frac{s}{\sqrt{n}})}$$

Where,

$\bar{x}$ is the sample mean

μ is the population mean

s is the standard deviation

n is the size of the sample given

Estimation

This is the process of identifying a value by approximating due to a certain purpose. It can be done by rounding off to the nearest whole number.

Application of sampling distribution methods in data analysis

There are three main types;

- Normal distribution; commonly used in investing, finance, science and engineering. It fully based on its mean and standard deviation.
- Binomial distribution: It is discrete, as opposed to continuous, since 1 or 0 / yes or no is a valid response.
- Chi-squared distribution
- Poisson distribution

DIAGRAMMATIC AND GRAPHICAL PRESENTATION OF DATA

Histograms

Frequency histogram

A histogram is a graphical representation of a frequency distribution, in which vertical rectangular blocks are drawn so that:

i. The centre of the base indicates the central value of the class
ii. The area of the rectangle represents the class frequency

If the class intervals are regular, the frequency is then denoted by the height of the rectangle.

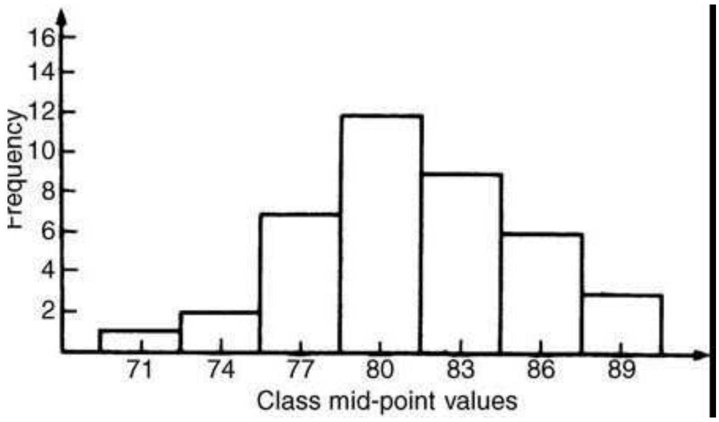


Figure 8: Histogram

Frequency Polygon

Another method of presenting grouped data diagrammatically is by using a **frequency polygon**, which is the graph produced by plotting frequency against class mid-point values and joining the coordinates with straight lines.

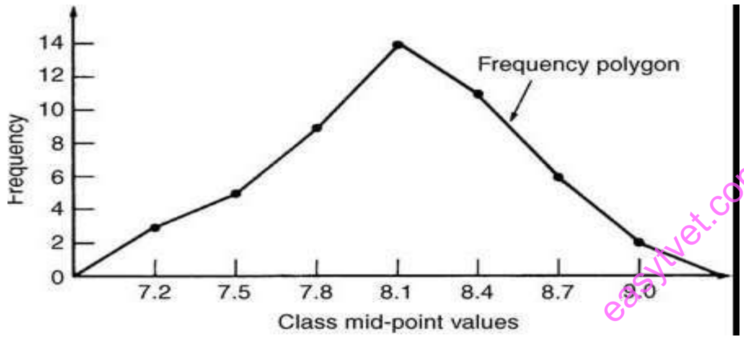


Figure 9: Frequency Polygon

A cumulative frequency distribution is a table showing the cumulative frequency for each value of upper class boundary. The cumulative frequency for a particular value of upper class boundary is obtained by adding the frequency of the class to the sum of the previous frequencies.

The curve obtained by joining the co-ordinates of cumulative frequency (vertically) against upper class boundary (horizontally) is called an **ogive** or a **cumulative frequency distribution curve**

MEASURES OF CENTRAL TENDENCY

A single value, which is representative of a set of values, may be used to give an indication of the general size of the members in a set, the word '**average**' often being used to indicate the single value. The statistical term used for 'average' is the arithmetic mean or just the **mean**.

Other measures of central tendency may be used and these include the **median** and the **modal** values.

Obtainment of mean, median, mode and standard deviation from given data

A **set** is a group of data and an individual value within the set is called a **member** of the set. Some members selected at random from a population are called a **sample**.

Mean: It is also known as average therefore it is addition of the number divided by the number of the numbers.

$$Mean = \frac{\sum of\ numbers}{no\ of\ numbers}$$

Median: This is the number in the middle after being arranged from the lowest to the highest number; if they are two, find the mean.

Mode: This is the number which is the most repeated in a series.

Variance: This is the mean of the squared differences of the number from the mean.

$$Variance = \sum(x_{1-}\mu)^2/n$$

Standard deviation: This is the amount of variation of a set of numbers known as square root variance.

Example:

Determine the mean, median and mode for the set:

{2, 3, 7, 5, 5, 13, 1, 7, 4, 8, 3, 4, 3}

Thus, **mean value**,

$\bar{x} = \frac{2+3+7+5+5+13+1+7+4+8+3+4+3}{13} = \frac{65}{13} = 5$

Median

To obtain the median value the set is ranked, that is, placed in ascending order of magnitude, and since the set contains an odd number of members the value of the middle member is the median value.

Ranking the set gives: [1,2,3,3,3,4,4,5,5,7,7,8,13]

{1, 2, 3, 3, 3, 4, 4, 5, 5, 7, 7, 8, 13}

The middle term is the seventh member, i.e. 4, thus the **median value is 4**.

Mode

The **modal value** is the value of the most commonly occurring member and is **3**, which occurs three times, all other members on occurring once or twice.

6.2.5 Learning Activities

1. As the teacher ask your students to collect 25 bottle tops and start making a histogram of their dates they forward the bottle tops. Each student has collected 25 bottle tops over the past few days. The students come to place the pennies in groups above a number line on the floor, according to the years in which the pennies were minted.

i. Construct a histogram of the dates of return within a week.
ii. Ask the class to estimate the mean and standard deviation of the distribution

6.2.6 Self-Assessment

1. The data given below refer to the gain of each of a batch of 40 transistors, expressed correct to the nearest whole number.

Form a frequency distribution for these data having seven classes.

81 83 87 74 76 89 82 84

86 76 77 71 86 85 87 88

84 81 80 81 73 89 82 79

81 79 78 80 85 77 84 78

83 79 80 83 82 79 80 77

a) Calculate the range

b) Calculate the range

c) Construct the frequency distribution

d) Construct the histogram

3. The frequency distribution for the value of resistance in ohms of 48resistors is as shown.

Determine the mean value of resistance.

20.5–20.9: 3, 21.0–21.4: 10, 21.5–21.9: 11,

22.0–22.4: 13, 22.5–22.9: 9, 23.0–23.4: 2

4. Determine the standard deviation from the mean of the set of numbers: (5, 6, 8, 4, 10, 3) correct to 4 significant figures.

6.2.7 Tools, Equipment, Supplies and Materials

- Scientific Calculators
- Rulers, pencils, erasers
- Charts with presentations of data
- Graph books
- Computers with an internet connection

6.2.8 References

K. A. Stroud, (2001). Engineering Mathematics, 5th Ed. Industrial Press Inc, New York.

John Bird, (2003). Engineering Mathematics, Fourth Edition. Elsevier Science, Oxford, UK

6.2.9 Answers to Self-Assessment

1. The data given below refer to the gain of each of a batch of 40 transistors, expressed correct to the nearest whole number.

Form a frequency distribution for these data having seven classes.

81 83 87 74 76 89 82 84

86 76 77 71 86 85 87 88

84 81 80 81 73 89 82 79

81 79 78 80 85 77 84 78

83 79 80 83 82 79 80 77

a) Calculate the range

The value obtained by taking the value of the smallest member from that of the largest member. Inspection of the set of data shows that, range=89−71=18

b) Construct a tally diagram

This is obtained by listing the classes in the left-hand column, and then inspecting each of the 40 members of the set in turn and allocating them to the appropriate classes by putting '1s' in the appropriate rows. Every fifth '1' allocated to the particular row is shown as an oblique line crossing the four previous '1s', to help with final counting.

Class	Tally
70–72	1
73–75	11
76–78	~~1111~~ 11
79–81	~~1111~~ ~~1111~~ 11
82–84	~~1111~~ 1111
85–87	~~1111~~ 1
88–90	111

Figure 10: Tally Diagram

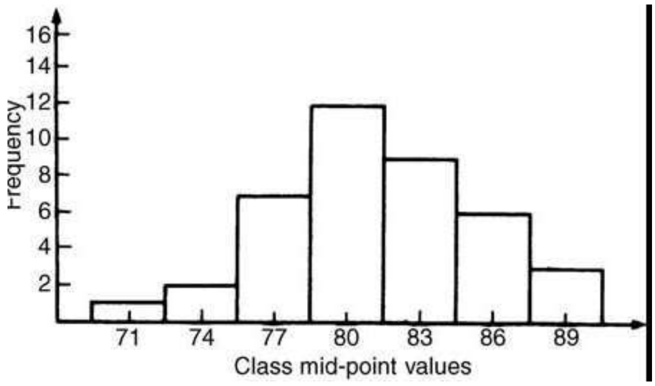
c) Construct the frequency distribution

Class	Class mid-point	Frequency
70 – 72	71	1
73 – 75	74	2
76 – 78	77	7
79 – 81	80	12
82 – 84	83	9
85 – 87	86	6
88 - 90	89	3

Figure 11: Frequency Distribution

d) Construct the histogram

The width of the rectangles correspond to the upper class boundary values minus the lower class boundary values and the heights of the rectangles correspond to the class frequencies. The easiest way to draw a histogram is to mark the class mid-point values on the horizontal scale and draw the rectangles symmetrically about the appropriate class mid-point values and touching one another.



5. The frequency distribution for the value of resistance in ohms of 48resistors is as shown. Determine the mean value of resistance.

20.5–20.9: 3, 21.0–21.4: 10, 21.5–21.9: 11,

22.0–22.4: 13, 22.5–22.9: 9, 23.0–23.4: 2

The class mid-point/frequency values are:

20.7 - 3, 21.2 - 10, 21.7 - 11, 22.2 - 13, 22.7 - 9 and 23.2 - 2

For grouped data, the mean value is given by:

$$\bar{x} = \frac{\sum(fx)}{\sum f} = \frac{(20.7 \times 3) + (21.2 \times 10) + (21.7 \times 11) + (22.2 \times 13) + (22.7 \times 9) + (23.2 \times 2)}{48}$$

$$= 21.9$$

6. Determine the standard deviation from the mean of the set of numbers: (5, 6, 8, 4, 10, 3) correct to 4 significant figures.

The arithmetic mean $\bar{x} = \frac{\sum x}{n} = \frac{5+6+8+4+10+3}{6} = 6$

Standard deviation $\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$

The $(x-\bar{x})^2$ values are $(5-6)^2$,$(6-6)^2$, $(8-6)^2$, $(4-6)^2$, $(10-6)^2$, $(3-6)^2$

The sum of the $(x-\bar{x})^2$ values, i.e.

$$\sum(x-\bar{x})^2 = 1+0+4+4+16+9 = 34$$

$$= \frac{\sum(x-\bar{x})^2}{n} = \frac{34}{6} = 5.6$$

Since, there are 6 members in the set.

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{5.6} = 2.38$$

7 Learning Outcome 8: Apply Concepts of probability for Work

7.2.2 Introduction to the learning outcome

This learning outcome covers: Probability, Types of probability events, Laws of probability, Counting techniques, Permutation, Combination, Tree diagrams and Venn diagrams.

7.2.3 Performance Standard

1. Probability
2. Types of probability events

- Dependent
- Independent
- Mutually exclusive

3. Laws of probability
4. Counting techniques
5. Permutation
6. Combination
7. Tree diagrams
8. Venn diagrams

7.2.4 Information Sheet

7.2.4.1 Definition of terms

Probability

Probability of something happening is the likelihood or chance of it happening. Values of probability lie between 0 and 1, where 0 represents an absolute impossibility and 1 represents an absolute certainty. The probability of an event happening usually lies somewhere between these two extreme values and is expressed either as a proper or decimal fraction.

If p is the probability of an event happening and q is the probability of the same event not happening, then the total probability is p + q and is equal to unity, since it is an absolute certainty that the event either does or does not occur, i.e. $\boldsymbol{p + q = 1}$

The expectation, E, of an event happening is defined in general terms as the product of the probability p of an event happening and the number of attempts made, n, i.e. $E = pn$.

TYPES OF PROBABILITY EVENTS

Dependent event

A dependent event is one in which the probability of an event happening affects the probability of another ever happening.

Independent event

An independent event is one in which the probability of an event happening does not affect the probability of another event happening.

Conditional probability

Conditional probability is concerned with the probability of say event B occurring, given that event A has already taken place. If A and B are independent events, then the fact that event A has already occurred will not affect the probability of event B. If A and B are dependent events, then event A having occurred will affect the probability of event B.

Mutually exclusive event

Two events are known as mutually exclusive, when the occurrence of one of them excludes the occurrence of the other. I.e. in tossing of a coin the outcomes are either head or tail.

Laws of probability

The addition law of probability

The addition law of probability is recognized by the word **'or'** joining the probabilities. If P_A is the probability of event A happening and P_B is the probability of event B happening, the probability of **event *A* or event *B*** happening is given by $P_A + P_B$(provided events A and B are **mutually exclusive**, i.e. A and B are events which cannot occur together).

Similarly, the probability of events ***A* or *B* or *C* or . . . *N*** happening is given by

$$\boldsymbol{P_A + P_B + P_C \dots \dots \dots \dots P_N}$$

The multiplication law of probability

The multiplication law of probability is recognized by the word **'and'** joining the probabilities. If P_A is the probability of event A happening and P_B is the probability of event B happening, the probability of **event *A* and event *B*** happening is given by$P_A \times P_B$.

Similarly, the probability of events ***A* and *B* and *C* and . . . *N*** happening is given by:

$$P_A \times P_B \times P_C \dots \dots \dots \dots . P_N$$

Not Mutually Exclusive Events

Consider the case where two events A and B are not mutually exclusive. The probability of that event that either A or B or both occur is given as

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Permutations

If n different objects are available, they can be arranged in different orders of selection. Each different ordered arrangement is called a **permutation**.

For example, permutations of the three letters X, Y and Z taken together are:

XYZ, XZY, YXZ, YZX, ZXY and ZYX

This can be expressed as ${}^{3}P_{3}$ =6, the upper 3 denoting the number of items from which the arrangements are made, and the lower 3 indicating the number of items used in each arrangement.

In general, ${}^{n}P_{r} = n(n-1)(n-2) \ldots \ldots \ldots \ldots (n-r+1)$ or $= \frac{n!}{(n-r)!}$

Combinations

Using three letters A, B, C we now make selections without regard to the order of the letters in each group i.e. AB is now the same as BA, etc. Each group is called a combination and ${}^{n}c_{r}, where$ n is the total number of items and r is the number in each selection, gives the number of possible combinations. Note that AB and BA are different permutations, but are not different combinations.

Tree Diagram

A **probability tree diagram** shows all the possible events. The first event is represented by a dot. From the dot, branches are drawn to represent all possible outcomes of the event. The probability of each outcome is written on its branch.

Venn Diagram

A set is a collection of things. A Venn diagram uses overlapping circles or other shapes to illustrate the logical relationships between two or more sets of items. Often, they serve to graphically organize things, highlighting how the items are similar and different.

Each friend is an "element" (or "member") of the set. It is normal to use **lowercase letters** for them.

Now let's say that alex, casey, drew and hunter play **Soccer**:

Soccer = {alex, casey, drew, hunter}

(It says the Set "Soccer" is made up of the elements alex, casey, drew and hunter.)

And casey, drew and jade play **Tennis**:

Tennis = {casey, drew, jade}

We can put their names in two separate circles:

You can now list your friends that play **Soccer OR Tennis**.

This is called a "Union" of sets and has the special symbol ∪:

Soccer ∪ Tennis = {alex, casey, drew, hunter, jade}

Not everyone is in that set ... only your friends that play Soccer or Tennis (or both).

In other words we combine the elements of the two sets.

"Intersection" is when you must be in BOTH sets.

In our case that means **they play both Soccer AND Tennis** ... which is casey and drew.

The special symbol for Intersection is an upside down "U" like this: ∩

And this is how we write it:

$$\text{Soccer} \cap \text{Tennis} = \{\text{casey, drew}\}$$

Difference

You can also "subtract" one set from another.

For example, taking Soccer and subtracting Tennis means people that **play Soccer but NOT Tennis** ... which is alex and hunter.

And this is how we write it:

$$\text{Soccer} - \text{Tennis} = \{\text{alex, hunter}\}$$

7.2.5 Learning Activities

1. In groups of 2 have the students play the Rock, Paper and Scissors game. Play the first round:

i. What is the probability that your friend will throw a rock?
ii. What is the probability that your friend will not throw a paper?

Now get a paper and pen and play 20 times and record your data.

i. How many times was rock thrown by the partner?
ii. What was the probability of it being thrown?
iii. How many times was paper thrown by the partner?
iv. What was the probability?
v. How many times was scissors thrown by the partner?
vi. What was the probability?

7.2.6 Self-Assessment

1. Determine the probabilities of selecting at random (a) a man, and (b) a woman from a crowd containing 20 men and 33 women.
2. Find the expectation of obtaining a 4 upwards with 3 throws of a fair dice.

3. The probability of a component failing in one year due to excessive temperature is $\frac{1}{20}$, due to excessive vibration is $\frac{1}{25}$ and due to excessive humidity is $\frac{1}{50}$. Determine the probabilities that during a one-year period component: (a) fails due to excessive temperature and excessive vibration, (b) fails due to excessive vibration or excessive humidity, and (c) will not fail because of both excessive temperature and excessive humidity.

7.2.7 Tools, Equipment, Supplies and Materials

- Scientific Calculators
- Rulers, pencils, erasers
- Charts with presentations of data
- Graph books
- Computers with an internet connection

7.2.8 References

K. A. Stroud, (2001). Engineering Mathematics, 5th Ed. Industrial Press Inc, New York.

John Bird, (2003). Engineering Mathematics, Fourth Edition. Elsevier Science, Oxford, UK

7.2.9 Answers to Self-Assessment

1. Determine the probabilities of selecting at random (a) a man, and (b) a woman from a crowd containing 20 men and 33 women.

Solution

a. The probability of selecting at random a man, p, is given by the ratio $= \frac{number\ of\ men}{number\ of\ crowd}$

$$P = \frac{20}{20+33} = \frac{20}{53} = 0.3774$$

b. The probability of selecting at random women, q, is given by the ratio$= \frac{number\ of\ women}{number\ of\ crowd}$

$$Q = \frac{33}{20+33} = \frac{33}{53} = 0.6226$$

2. Find the expectation of obtaining 4 upwards with 3 throws of a fair dice.
3. The probability of a component failing in one year due to excessive temperature is $\frac{1}{20}$, due to excessive vibration is $\frac{1}{25}$ and due to excessive humidity is $\frac{1}{50}$. Determine the probabilities that during a one-year period component: (a) fails due to excessive temperature and excessive vibration, (b) fails due to excessive vibration or excessive humidity, and (c) will not fail because of both excessive temperature and excessive humidity.

Solution

Let PA be the probability of failure due to excessive temperature, then

$$PA = \frac{1}{20}\ and\ \overline{PA} = \frac{19}{20}$$

$\overline{PA}$ The probability of not failing.

Let PB be the probability of failure due to excessive vibration, then

$$PB = \frac{1}{25} \; and \; \overline{PB} = \frac{24}{25}$$

$$\overline{PB}$$

Let PB be the probability of failure due to excessive humidity, then

$$PC = \frac{1}{50} \; and \; \overline{PC} = \frac{49}{50}$$

a. The probability of a component failing due to excessive temperature **and** excessive vibration is given by:

$$PA \times PB = \frac{1}{20} \times \frac{1}{25} = \frac{1}{500} = 0.002$$

b. The probability of a component failing due to excessive vibration **or** excessive humidity is:

$$PB + PC = \frac{1}{25} + \frac{1}{50} = \frac{3}{50} = 0.06$$

c. The probability that a component will not fail due to excessive temperature **and** will not fail due to excess humidity is:

$$\overline{PA} \times \overline{PC} = \frac{19}{20} \times \frac{49}{50} = \frac{931}{1000} = 0.931$$

8 Learning Outcome 9: Perform Commercial Calculation

8.2.2 Introduction to the learning outcome

The learning outcome covers: Product pricing, Average sales determination, Stock turnover, Calculation of incomes, Profit and Loss calculations, Salaries,

8.2.3 Performance Standard

1. Product pricing
2. Average sales determination
3. Stock turnover
4. Calculation of incomes
5. Profit and loss calculations
6. Salaries

- Gross
- Net

7. Wages

- Time rate
- Flat rate
- Overtime
- Piece rate
- Commission
- Percentage
- Bonus

8. Exchange rates calculation

- Devaluation
- Revaluation

8.2.4 Information Sheet

8.2.4.1 Definition of terms

Product pricing

Price is the money that customers must pay for a product or service. In other words, price is an offer to sell for a certain amount of currency.

Pricing is the art of translating into quantitative terms the value of a product to customers at a point of time.

There are three components to the overall pricing strategy:

- Choice of a ***Pricing Principle****: Cost-Plus, Competitive, Value-Based*
- Choice of a ***Price Positioning****: Market Skimming, Neutral, Penetration*
- Choice of a ***Pricing Structure****: Unit Pricing, Tiered Pricing, Bundled Pricing, Subscriptions etc.*

Average sales determination

The selling price formula is:

Selling Price = Cost Price + Profit Margin

Cost price is the price a retailer paid for the product. And the profit margin is a percentage of the cost price.

- **Cost Price**: The price a retailer paid for the product
- **Profit Margin**: A percentage of the cost price.

How to Calculate Selling Price per Unit

1. Determine the total cost of all units purchased.
2. Divide the total cost by the number of units purchased to get the cost price.
3. Use the selling price formula to calculate the final price: Selling Price = Cost Price + Profit Margin

Stock turnover

Inventory turnover is a ratio showing how many times a company has sold and replaced inventory during a given period. A company can then divide the days in the period by the inventory turnover formula to calculate the days it takes to sell the inventory on hand.

Inventory Turnover = Sales / Average Inventory

Where:

Average Inventory = (Beginning Inventory + Ending Inventory) / 2

Companies calculate inventory turnover by:

- Calculating the average inventory, which is done by dividing the sum of beginning inventory and ending inventory by two.
- Dividing sales by average inventory.

Profit and loss calculations

Cost price (C.P.): This is the price at which an article is purchased.

Selling price (S.P.): This is the price at which an article is sold.

Profit or Gain: If the selling price is more than the cost price, the difference between them is the profit incurred.

Formula: Profit or Gain = S.P. – C.P.

Loss: If the selling price is less than the cost price, the difference between them is the loss incurred.

Formula: Loss = Cost price (C.P.) – Selling Price (S.P.)

Profit or Loss is always calculated on the cost price.

Marked price: This is the price marked as the selling price on an article, also known as the listed price.

Discount or Rebate: This is the reduction in price offered on the marked or listed price.

Below is the list of some basic formulas used in solving questions on profit and loss:

- Gain % = (Gain / CP) * 100
- Loss % = (Loss / CP) * 100
- SP = [(100 + Gain%) / 100] * CP
- SP = [(100 – Loss %) / 100]*CP

Salaries

A salary is a form of payment from an employer to an employee, which may be specified in an employment contract. It is contrasted with piece wages, where each job, hour, or other unit is paid separately, rather than on a periodic basis.

Gross Pay

Gross income for an individual—also known as gross pay when it's on a paycheck—is the individual's total pay from his or her employer before taxes or other deductions. This includes income from all sources and is not limited to income received in cash; it also includes property or services received. Gross annual income is the amount of money a person earns in one year before taxes and includes income from all sources.

Net Pay

Net pay refers to the amount an employee takes home, not the amount it costs to employ them.

Wages

- Time rate
- Flat rate
- Overtime
- Piece rate
- Commission
- Percentage
- Bonus

Exchange rates calculation

The exchange rate is defined as the rate on the basis of which two countries involved in trade exchange marketable items or commodities. It is basically the cost of exchanging one currency for another currency.

Exchange rates fluctuate constantly throughout the week as currencies are actively traded. This pushes the price up and down, similar to other assets such as gold or stocks. Therefore, the exchange rate can be calculated as per the below-mentioned relationship: –

Exchange Rate = Money in Foreign Currency / Money in Domestic Currency

Additionally, it can also be determined as per the below-mentioned relationship: –

Exchange Rate = Money in After Exchange / Money before Exchange

Devaluation

Devaluation is the deliberate downward adjustment of the value of a country's money relative to another currency, group of currencies, or currency standard. Countries that have a fixed exchange rate or semi-fixed exchange rate use this monetary policy tool. It is often confused with depreciation and is the opposite of revaluation, which refers to the readjustment of a currency's exchange rate.

Given two exchange rates in terms of a Base Currency and a Quote Currency we can calculate appreciation and depreciation between them using the percentage change calculation. Letting V_1 be the starting rate and V_2 the final rate.

The percentage change of the Quote Currency relative to the Base Currency is

$$\%change = \frac{(V_1 - V_2)}{V_2} \times 100$$

The percentage change of the Base Currency relative to the Quote Currency is

$$\%change = \frac{(V_2 - V_1)}{V_1} \times 100$$

A positive change is appreciation and a negative change is depreciation.

- Revaluation

A revaluation is a calculated upward adjustment to a country's official exchange rate relative to a chosen baseline. The baseline can include wage rates, the price of gold, or a foreign currency.

Revaluation is the opposite of devaluation, which is a downward adjustment of a country's official exchange rate.

8.2.5 Learning Activities

Activity 1

The activity below is to enhance knowledge on taxes.

Materials:

- computer/internet
- W-2 form (pre-filled)
- 1040 tax form

Guiding Questions:

- What types of taxes do people in your community pay?
- What is the tax rate for each type of tax?
- How much of your income will go to taxes?

Procedures:

- Divide students into pairs or small groups.
- Have students determine their annual income if they work 40 hours a week 52 weeks a year at the current minimum wage rate.
- Have students fill out a 1040 tax form using a prepared W-2.
- Have students determine their income tax rate for federal and state taxes.
- Have students research what other taxes apply to their personal situation and figure out the amount.
- Have students create a slide presentation that explains the difference between their annual salary and the actual amount they are able to count as part of their budget after paying taxes.

Activity 2

In groups of 4, have the students buy 20 biro pens at wholesale price of KShs. 10 per pen, they will spend a total cost of KShs. 2.50 per pen to advertise and on labor. If they made a total of profit of KShs. 120 from sale of the 20pens Using the template below let them fill the results.

Cost of the pen?

Profit per pen?

Selling price?

8.2.6 Self-Assessment

1. The market oriented pricing, there are three pricing ways, outline them?
2. Hot Pie's Bakery Supply needs to calculate the selling price for its product line of bread machines. The business purchased 20 bread machines for $3,000.

a. Calculate the cost price

b. The cost price for each bread machine is $150, and the business hopes to earn a 40% profit margin. Calculate the selling price?

3. In a transaction, the profit percentage is 80% of the cost. If the cost further increases by 20% but the selling price remains the same, how much is the decrease in profit percentage?
4. A man bought two bicycles for Rs. 2500 each. If he sells one at a profit of 5%, then how much should he sell the other so that he makes a profit of 20% on the whole?
5. **A trader wants to make an investment in the exchange-traded funds traded in US markets. However, the trader lives in India and 1 INR corresponds to 0.014 USD. The trader has INR 10,000 to invest in the exchange-traded funds traded in the offshore market.**

   Help the trader determine the value of INR investment in terms of US currency.

8.2.7 Tools, Equipment, Supplies and Materials

- Scientific Calculators
- Rulers, pencils, erasers
- Charts with presentations of data
- Graph books
- Computers with an internet connection

8.2.8 References

K. A. Stroud, (2001). Engineering Mathematics, 5th Ed. Industrial Press Inc, New York.

John Bird, (2003). Engineering Mathematics, Fourth Edition. Elsevier Science, Oxford, UK

8.2.9 Answers to Self-Assessment

1. In market oriented pricing, there are three pricing ways, outline them.

- **Price above market**: Consciously pricing your product above the competition to brand yourself as having a higher-quality or better-performing item
- **Copy market**: Selling your item at the same price as your competition to maximize profit while staying competitive
- **Price below market**: Using data as a benchmark and consciously pricing a product below competitors, to lure customers into your store over theirs

2. Hot Pie's Bakery Supply needs to calculate the selling price for its product line of bread machines. The business purchased 20 bread machines for $3,000.

a. Calculate the cost price

Total cost of units purchased: $3,000

Number of units purchased: 20

Cost price: $= \frac{\$3,000}{20} = \$150$

b. The cost price for each bread machine is $150, and the business hopes to earn a 40% profit margin. Calculate the selling price?

Selling Price = $150 + (40% x $150)

Selling Price = $150 + (0.4 x $150)

Selling Price = $150 + $60

Selling Price = $210

3. In a transaction, the profit percentage is 80% of the cost. If the cost further increases by 20% but the selling price remains the same, how much is the decrease in profit percentage?

Solution:

Let us assume CP = Rs. 100.

Then Profit = Rs. 80 and selling price = Rs. 180.

The cost increases by 20% → New CP = Rs. 120, SP = Rs. 180.

Profit % = 60/120 * 100 = 50%.

Therefore, Profit decreases by 30%.

4. A man bought two bicycles for Rs. 2500 each. If he sells one at a profit of 5%, then how much should he sell the other so that he makes a profit of 20% on the whole?

Solution:

Before we start, it's important to note here that it is not 15% to be added to 5% to make it a total of 20%.

Let the other profit percent be x.

Then, our equation looks like this.

105/100 * 2500 + [(100+x)/100] * 2500 = 120/100 * 5000 → x= 35.

Hence, if he makes a profit of 35% on the second, it comes to a total of 20% profit on the whole.

5. **A trader wants to make an investment in the exchange-traded funds traded in US markets. However, the trader lives in India and 1 INR corresponds to 0.014 USD. The trader has INR 10,000 to invest in the exchange-traded funds traded in the offshore market.**

   Help the trader determine the value of INR investment in terms of US currency.

   Determine the value of exchange in terms of US dollars.

$$Money\ in\ after\ exchange = Exchange\ rate \times Money\ before\ exchange$$

   The value of exchange in terms of US dollars = 0.014×10000

   Money in After Exchange = $140

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