

# TVET CURRICULUM DEVELOPMENT, ASSESSMENT AND CERTIFICATION COUNCIL (TVET CDACC) 

| Qualification Code | $:$ | 071606T4MCT |
| :--- | :---: | :--- |
| Qualification | $:$ | Mechatronics Technician Level 6 |
| Unit Code | $:$ | ENG/OS/MC/CC/07/6 |
| Unit of Competency : | Apply Fluid Mechanics Principles |  |

## WRITTEN ASSESSMENT ASSESSOR'S GUIDE

## INSTRUCTIONS TO ASSESSOR

1. Marks for each question are indicated in the brackets.
2. Answers provided are model answers.

## SECTION A: (40 MARKS)

1. State two criteria of classifying reciprocating pumps
(2 Marks)
i. According to the water being in contact with one side or both sides of the piston
ii. According to the number of cylinders provided
(Award 1 mark for each correct response)
2. List four types of fluids classified according to the presence of viscosity. (4 Marks)
i. Ideal fluid.
ii. Real fluid.
iii. Newtonian fluid.
iv. Non-Newtonian fluid.
v. Ideal plastic fluid.
(Award 1 mark for each correct response, any 4)
3. List three applications of dimensional homogeneity
i. To convert units of one system into the units of other system.
ii. To check the correctness of an equation.
iii. To establish a relation among various physical quantities.
(Award 1 mark for each correct response)
4. Define Newtonian fluid and give two examples of Newtonian fluids

A Newtonian fluid is defined as one with constant viscosity, with zero shear rate at zero shear stress, that is, the shear rate is directly proportional to the shear stress.

Examples of Newtonian fluids
i. water
ii. oil
iii. gasoline
iv. alcohol
v. glycerin
(Award 2 marks for definition, and one mark for each correct example, any 2)
5. Outline three types of casings adopted in centrifugal pumps
(3 Marks)
i. Volute casing
ii. Vortex casing
iii. Casing with guide plates
(Award 1 mark for each correct response)
6. Define the following terms used in centrifugal pumps
(4 Marks)
i. Suction head- it is the vertical height of the centre line of the centrifugal pump above the water surface in the tank or pump from which water is to be lifted.
ii. Delivery head- it is the vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered.
iii. Static head- it is the sum of suction head and delivery head.
iv. Manometric head- it is the head against which a centrifugal pump has to work.
(Award 1 mark for each correct definition)
7. Define dimensional analysis
(2 Marks)
It is a mathematical technique used in research work for design and for conducting model tests. It deals with the dimensions of the physical quantities involved in the phenomenon.
(Award 2 marks for the correct definition)
8. What is the importance of model analysis?

It is used for predicting the performance of the hydraulic structures or hydraulic machines before actually constructing or manufacturing. Models of the structures or machines are made and tests are performed on them to obtain the desired information.
(Award 2 mark for the correct response)
9. List four forces acting on a moving fluid through a pipe
i. Viscous force
ii. Pressure force
iii. Inertia force
iv. Gravity force
v. Pressure force
(Award 1 mark for each correct response, any 4)
10. State two types of energy loses experienced by fluids moving through pipes and their causes
(4 Marks)
i. Major losses- caused by friction
ii. Minor losses- sudden expansion or contraction of pipe, bend in pipes, pipe fittings and an obstruction in pipe.
(Award 1 mark for identification and 1 mark for the cause)
11. Name two types of centrifugal pumps and one application for each
i. Radial Centrifugal Pumps- cooling water for thermal and nuclear power plants, handling sea water, fresh water supply, boiler feed, mine dewatering
ii. Axial Centrifugal Pumps- circulate fluids in power plants, sewage digesters, irrigation and evaporators
(Award 1 mark for identification and 1 mark for the cause)
12. Compare the flow of fluids in series and parallel pipes
i. In a series system, the flow rate through the entire system remains constant while in parallel system head loss is the same in each pipe.
ii. In a series system, the total head loss in this case is equal to the sum of the head losses in individual pipes while in parallel system the total flow rate is the sum of the flow rates in individual pipes.
(Award 2 marks for each correct comparison)

## SECTION B: (60 MARKS)

13. 

a. A single acting reciprocating pump running at $50 \mathrm{r} . \mathrm{p} . \mathrm{m}$., delivers $0.01 \mathrm{~m}^{3} / \mathrm{s}$ of water. The diameter of the piston is 200 mm and stroke length 400 mm . determine:
i. The theoretical discharge of the pump,
ii. Coefficient of the pump, and
iii. Slip and percentage slip of the pump

## Solution. Given :

Speed of the pump,

$$
N=50 \text { r.p.m. }
$$

Actual discharge,

$$
Q_{a c t}=.01 \mathrm{~m}^{3} / \mathrm{s}
$$

Dia. of piston,

$$
D=200 \mathrm{~mm}=.20 \mathrm{~m}
$$

$\therefore$ Area,

$$
A=\frac{\pi}{4}(.2)^{2}=.031416 \mathrm{~m}^{2}
$$

Stroke,

$$
L=400 \mathrm{~mm}=0.40 \mathrm{~m} .
$$

(i) Theoretical discharge for single-acting reciprocating pump is given by equation (20.1) as

$$
Q_{t h}=\frac{A \times L \times N}{60}=\frac{.031416 \times .40 \times 50}{60}=\mathbf{0 . 0 1 0 4 7} \mathrm{m}^{3} / \mathrm{s} . \text { Ans. }
$$

(ii) Co-efficient of discharge is given by

$$
C_{d}=\frac{Q_{a c l}}{Q_{t h}}=\frac{0.01}{.01047}=\mathbf{0 . 9 5 5} . \mathrm{Ans}
$$

(iii) Using equation (20.8), we get

$$
\begin{aligned}
\text { Slip } & =Q_{t h}-Q_{a c t}=.01047-.01=0.00047 \mathrm{~m}^{3} / \mathrm{s} . \text { Ans. } \\
& =\frac{\left(Q_{t h}-Q_{a c t}\right)}{Q_{t h}} \times 100=\frac{(.01047-.01)}{.01047} \times 100 \\
& =\frac{.00047}{.01047} \times 100=4.489 \% . \text { Ans. }
\end{aligned}
$$

And percentage slip
b. Briefly describe any two efficiencies of a centrifugal pump
i. Manometric efficiency- it is the ratio of the manometric head imparted by the impeller to the water.
ii. Mechanical efficiency-it is the ratio of the power available at the impeller to the power at the shaft of the centrifugal pump.
iii. Overall efficiency- it is the ratio of power output of the pump to the power input of the pump.

## (Award 1 mark for identification and 1 mark for brief description)

c. The double-acting reciprocating pump, running at $40 \mathrm{r} . \mathrm{p} . \mathrm{m}$., is discharging 1.0 m of water per minute. The pump has a stroke of 400 mm . The diameter of the piston is 200 mm . The delivery of suction head are 20 m and 5 m respectively. Find the slip of the pump required to drive the pump. (6 Marks)

Speed of pump,

$$
N=40 \text { r.p.m. }
$$

Actual discharge,

$$
Q_{a c t}=1.0 \mathrm{~m}^{3} / \mathrm{min}=\frac{1.0}{60} \mathrm{~m}^{3} / \mathrm{s}=0.01666 \mathrm{~m}^{3} / \mathrm{s}
$$

Stroke,

$$
\begin{aligned}
L & =400 \mathrm{~mm}
\end{aligned}=0.40 \mathrm{~m}, ~(200 \mathrm{~mm}=0.20 \mathrm{~m} .
$$

Diameter of piston,
$\therefore$ Area,

$$
A=\frac{\pi}{4} D^{2}=\frac{\pi}{4}(.2)^{2}=0.031416 \mathrm{~m}^{2}
$$

Suction head,

$$
\begin{aligned}
& h_{s}=5 \mathrm{~m} \\
& h_{d}=20 \mathrm{~m} .
\end{aligned}
$$

Delivery head,
Theoretical discharge for double-acting pump is given by equation (20.5) as,

$$
Q_{t h}=\frac{2 A L N}{60}=\frac{2 \times .031416 \times 0.4 \times 40}{60}=.01675 \mathrm{~m}^{3} / \mathrm{s} .
$$

Using equation (20.8), Slip $=Q_{l t}-Q_{a c t}=.01675-.01666=.00009 \mathbf{~ m}^{3} / \mathbf{s}$. Ans.
Power required to drive the double-acting pump is given by equation (20.7) as,

$$
\begin{aligned}
P & =\frac{2 \times \rho g \times A L N \times\left(h_{s}+h_{d}\right)}{60,000}=\frac{2 \times 1000 \times 9.81 \times .031416 \times .4 \times 40 \times(5+20)}{60,000} \\
& =4.109 \mathrm{~kW} . \text { Ans. }
\end{aligned}
$$

(Award 6 marks for the correct workings, correct steps, correct units and the correct answer)
14.
a. An oil of viscosity $0.1 \mathrm{Ns} / \mathrm{m}^{2}$ and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and of length 300 m . the rate of flow of fluid through the pipe is 3.5 litre/s. find the pressure drop in a length of 300 m and also shear stress at the pipe wall. (10 marks)

Solution. Given : Viscosity, $\mu=0.1 \mathrm{Ns} / \mathrm{m}^{2}$
$\begin{array}{ll}\text { Relative density } & =0.9 \\ \therefore \quad \rho_{0} \text { or density of oil } & =0.9 \times 1000=900 \mathrm{~kg} / \mathrm{m}^{3} \quad\left(\because \quad \text { Density of water }=1000 \mathrm{~kg} / \mathrm{m}^{3}\right)\end{array}$

$$
\begin{aligned}
& D=50 \mathrm{~mm}=.05 \mathrm{~m} \\
& L=300 \mathrm{~m} \\
& Q=3.5 \text { litres } / \mathrm{s}=\frac{35}{1000}=.0035 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

Find (i) Pressure drop, $p_{1}-p_{2}$
(ii) Shear stress at pipe wall, $\tau_{0}$
(i) Pressure drop $\left(\mathbf{p}_{1}-\mathbf{p}_{2}\right)=\frac{32 \mu \bar{u} L}{D^{2}}$, where $\bar{u}=\frac{Q}{\text { Area }}=\frac{.0035}{\frac{\pi}{4} D^{2}}=\frac{.0035}{\frac{\pi}{4}(.05)^{2}}=1.782 \mathrm{~m} / \mathrm{s}$

The Reynolds number ( $R_{e}$ ) is given by, $R_{e}=\frac{\rho V D}{\mu}$
where $\rho=900 \mathrm{~kg} / \mathrm{m}^{3}, V=$ average velocity $=\bar{u}=1.782 \mathrm{~m} / \mathrm{s}$

$$
\therefore \quad R_{e}=900 \times \frac{1.782 \times .05}{0.1}=801.9
$$

As Reynolds number is less than 2000 , the flow is viscous or laminar

$$
\begin{aligned}
\therefore \quad p_{1}-p_{2} & =\frac{32 \times 0.1 \times 1.782 \times 3000}{(.05)^{2}} \\
& =684288 \mathrm{~N} / \mathrm{m}^{2}=68428 \times 10^{-4} \mathrm{~N} / \mathrm{cm}^{2}=68.43 \mathrm{~N} / \mathrm{cm}^{2} . \text { Ans. }
\end{aligned}
$$

(ii) Shear Stress at the pipe wall ( $\tau_{0}$ )

The shear stress at any radius $r$ is given by the equation (9.1)

$$
\text { i.e., } \quad \tau=-\frac{\partial p}{\partial x} \frac{r}{2}
$$

$\therefore$ Shear stress at pipe wall, where $r=R$ is given by

$$
\begin{aligned}
& \tau_{0}=\frac{-\partial p}{\partial x} \frac{R}{2} \\
& \text { Now } \\
& \text { and } \\
& \text { Now } \\
& \text { and } \\
& \frac{-\partial p}{\partial x}=\frac{-\left(p_{2}-p_{1}\right)}{x_{2}-x_{1}}=\frac{p_{1}-p_{2}}{x_{2}-x_{1}}=\frac{p_{1}-p_{2}}{L} \\
& =\frac{684288}{300} \frac{\mathrm{~N} / \mathrm{m}^{2}}{\mathrm{~m}}=2280.96 \mathrm{~N} / \mathrm{m}^{3} \\
& R=\frac{D}{2}=\frac{.05}{2}=.025 \mathrm{~m} \\
& \tau_{0}=2280.96 \times \frac{.025}{2} \frac{\mathrm{~N}}{\mathrm{~m}^{2}}=\mathbf{2 8 . 5 1 2} \mathrm{N} / \mathrm{m}^{2} \text {. Ans. }
\end{aligned}
$$

## (Award 10 marks for the correct workings, correct steps, correct units and the correct answer)

b. Two tanks A and B have 70 m difference in water levels, and are connected by a pipe 0.25 m diameter and 6 km long with 0.002 friction coefficient. The pipe is tapped at its mid-point to leak out $0.04 \mathrm{~m} 3 / \mathrm{s}$ flow rate. Minor losses are ignored. Determine the discharge leaving tank A and the discharge entering tank B ? (10 marks)

$$
\begin{aligned}
& \text { Solution } \\
& \mathbf{h}_{\mathrm{f}}=\mathrm{h}_{\mathrm{f}}+\mathrm{h}_{\mathrm{f}} \\
& 70=h_{\mathrm{f} 1}+\mathrm{h}_{\mathrm{R}} \\
& 70=\mathrm{k}_{1} \mathrm{Q}_{1}{ }^{2}+\mathrm{k}_{2} \mathrm{Q}_{2}{ }^{2} \\
& \mathrm{k}_{1}=\mathrm{k}_{2}=\frac{32 \mathrm{fL}}{\pi^{2} \mathrm{~g} \mathrm{~d}^{5}}=\frac{32^{*} 0.002 * 3000}{\pi^{2 * 9.81 * 0.25}}=2032.7 \\
& \therefore 70=\mathrm{k}_{1} \mathrm{Q}_{1}{ }^{2}+\mathrm{k}_{1} \mathrm{Q}_{2}{ }^{2} \\
& \mathrm{Q}_{1}=\mathrm{Q}_{2}+\mathrm{Q}_{3}=\mathrm{Q}_{2}+0.04 \\
& \therefore 70=\mathrm{k}_{1}\left(\mathrm{Q}_{2}+0.04\right)^{2}+\mathrm{k}_{1} \mathrm{Q}_{2}{ }^{2} \\
& =\mathrm{k}_{1}\left(\mathrm{Q}_{2}{ }^{2}+0.08 \mathrm{Q}_{2}+0.0016\right)+\mathrm{k}_{1} \mathrm{Q}_{2}{ }^{2} \\
& =\mathrm{k}_{1} \mathrm{Q}_{2}{ }^{2}+0.08 \mathrm{k}_{1} \mathrm{Q}_{2}+0.0016 \mathrm{k}_{1}+\mathrm{k}_{1} \mathrm{Q}_{2}{ }^{2} \\
& =2 \mathrm{k}_{1} \mathrm{Q}_{2}{ }^{2}+0.08 \mathrm{k}_{1} \mathrm{Q}_{2}+0.0016 \mathrm{k}_{1} \\
& =4065.4 \mathrm{Q}_{2}{ }^{2}+162.6 \mathrm{Q}_{2}+3.25 \\
& 0.0172=\mathrm{Q}_{2}{ }^{2}+0.04 \mathrm{Q}_{2}+0.0008 \\
& \mathrm{Q}_{2}{ }^{2}+0.04 \mathrm{Q}_{2}-0.0164=0 \\
& \mathrm{Q} 2=\frac{-0.04 \pm \sqrt{(-0.04)^{2}-4(1)(-0.0164)}}{2(1)} \\
& \therefore \mathrm{Q}_{2}=0.11 \mathrm{~m}^{3} / \mathrm{s} \quad \& \quad \mathrm{Q}_{1}=0.15 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

(Award 10 marks for the correct working, steps with the correct answer)
15.
a. The internal and external diameter of an impeller of a centrifugal pump which is running at $1000 \mathrm{r} . \mathrm{p} . \mathrm{m}$., are 200 mm and 400 mm respectively. The discharge through pump is $0.04 \mathrm{~m}^{3} / \mathrm{s}$ and velocity of flow is constant and equal to $2.0 \mathrm{~m} / \mathrm{s}$. the diameters of the suction and delivery pipes are 150 mm and 100 mm respectively and suction and delivery heads are 6 m (abs.) and 30 m (abs.) of water respectively. If the outlet vane angle is $45^{\circ}$ and power required to drive the pump is 16.186 kW , determine:
i. Vane angle of the impeller at inlet
ii. Overall efficiency of the pump
iii. Manometric efficiency of the pump
(14 marks)

Solution. Given :
Speed,

$$
\begin{aligned}
N & =1000 \mathrm{r} . \mathrm{p} . \mathrm{m} . \\
D_{1} & =200 \mathrm{~mm}=0.2 \mathrm{~m} \\
D_{2} & =400 \mathrm{~mm}=0.4 \mathrm{~m} \\
Q & =0.04 \mathrm{~m}^{3} / \mathrm{s} \\
V_{f_{1}} & =V_{f_{2}}=2.0 \mathrm{~m} / \mathrm{s} \\
D_{s} & =150 \mathrm{~mm}=0.15 \mathrm{~m} \\
D_{d} & =100 \mathrm{~mm}=0.10 \mathrm{~m} \\
h_{s} & =6 \mathrm{~m} \text { (abs.) } \\
h_{d} & =30 \mathrm{~m} \text { (abs.) } \\
\phi & =45^{\circ}
\end{aligned}
$$

Internal dia.,
External dia.,
Discharge,
Velocity of flow,
Dia. of suction pipe,
Dia. of delivery pipe,
Suction head,
Delivery head,
Outlet vane angle,
Power required to drive the pump, $P=16.186 / \mathrm{kW}$
(i) Vane angle of the impeller at inlet ( $\theta$ ).


Fig. 19.9

From inlet velocity, we have $\tan \theta=\frac{V_{f_{1}}}{u_{1}}=\frac{2.0}{u_{1}}$, where $u_{1}=\frac{\pi D_{1} N}{60}=\frac{\pi \times 0.2 \times 1000}{60}=10.47 \mathrm{~m} / \mathrm{s}$ $\therefore \quad \tan \theta=\frac{2.0}{10.47}=0.191$ or $\theta=\tan ^{-1} .191=10^{\circ} \mathbf{4 8}^{\prime}$. Ans.
(ii) Overall efficiency of the pump $\left(\eta_{0}\right)$.

Using equation (19.10), we have $\eta_{o}=\frac{\left(\frac{W H_{m}}{1000}\right)}{\text { S.P. }}$
where S.P. $=$ Power required to drive the pump and equal to $P$ here.

$$
\begin{align*}
\eta_{o} & =\frac{\left(\frac{\rho \times g \times Q \times H_{m}}{1000}\right)}{P}=\frac{\rho g \times Q \times H_{m}}{1000 \times P} \\
& =\frac{1000 \times 9.81 \times .04 \times H_{m}}{1000 \times 16.186}=0.02424 H_{m} \tag{i}
\end{align*}
$$

Now $H_{m}$ is given by equation (19.6) as

$$
\begin{equation*}
H_{m}=\left(\frac{p_{o}}{\rho g}+\frac{V_{o}^{2}}{2 g}+Z_{o}\right)-\left(\frac{p_{i}}{\rho g}+\frac{V_{i}^{2}}{2 g}+Z_{i}\right) \tag{ii}
\end{equation*}
$$

where $\quad \frac{p_{o}}{\rho g}=$ Pressure head at outlet of pump $=h_{d}=30 \mathrm{~m}$

$$
\frac{V_{o}^{2}}{2 g}=\text { Velocity head at outlet of pump }=\frac{V_{d}^{2}}{2 g}
$$

$$
\frac{p_{i}}{\rho g}=\text { Pressure head at inlet of pump }=h_{s}=6 \mathrm{~m}
$$

$$
\frac{V_{i}^{2}}{2 g}=\text { Velocity head at inlet of pump }=\frac{V_{s}^{2}}{2 g}
$$

$Z_{o}$ and $Z_{i}=$ Vertical height at outlet and inlet of the pump from datum line.
If $\quad Z_{o}=Z_{i}$ then equation $(i i)$ becomes as

$$
\begin{equation*}
H_{m}=\left(30+\frac{V_{d}^{2}}{2 g}\right)-\left(6+\frac{V_{s}^{2}}{2 g}\right) \tag{iii}
\end{equation*}
$$

Now

$$
V_{d}=\frac{\text { Discharge }}{\text { Area of delivery pipe }}=\frac{0.04}{\frac{\pi}{4}\left(D_{d}\right)^{2}}=\frac{.04}{\frac{\pi}{4} \times .1^{2}}=5.09 \mathrm{~m} / \mathrm{s}
$$

And

$$
V_{s}=\frac{.04}{\text { Area of suction pipe }}=\frac{.04}{\frac{\pi}{4} D_{s}^{2}}=\frac{.04}{\frac{\pi}{4} \times .15^{2}}=2.26 \mathrm{~m} / \mathrm{s}
$$

Substituting these values in equation (iii), we get

$$
\begin{aligned}
H_{m} & =\left(30+\frac{5.09^{2}}{2 \times 9.81}\right)-\left(6+\frac{2.26^{2}}{2 \times 9.81}\right) \\
& =(30+1.32)-(6+.26)=31.32-6.26=25.06 \mathrm{~m}
\end{aligned}
$$

Substituting the value of ' $H_{m}$ ' in equation (i), we get

$$
\eta_{o}=.02424 \times 25.06=0.6074=\mathbf{6 0 . 7 4} \% . \text { Ans. }
$$

(iii) Manometric efficiency of the pump ( $\eta_{\text {man }}$ ).

Tangential velocity at outlet is given by

$$
u_{2}=\frac{\pi D_{2} \times N}{60}=\frac{\pi \times 0.4 \times 1000}{60}=20.94 \mathrm{~m} / \mathrm{s}
$$

From outlet velocity triangle, we have

$$
\begin{array}{rlrl}
\tan \phi & =\frac{V_{f_{2}}}{u_{2}-V_{w_{2}}}=\frac{2.0}{20.94-V_{w_{2}}} \\
\therefore \quad & 20.94-V_{w_{2}} & =\frac{2.0}{\tan \phi}=\frac{2.0}{\tan 45}=2.0 \\
\therefore \quad & V_{w_{2}} & =20.94-2.0=18.94 .
\end{array}
$$

Using equation (19.8), $\quad \eta_{\text {man }}=\frac{g H_{m}}{V_{w_{2}} u_{2}}=\frac{9.81 \times 25.06}{18.94 \times 20.94}=0.6198=61.98 \%$. Ans.
b. Find the diameter of a pipe of length 2000 m when the rate of flow of water through the pipe is 200 litres/s and the head lost due to friction is 4 m . take the value of $\mathrm{C}=50$ in Chezy's formula
(6 marks)

$$
\begin{array}{lrl}
\text { Length of pipe, } & L & =2000 \mathrm{~m} \\
\text { Discharge, } & Q & =200 \text { litre/s }=0.2 \mathrm{~m}^{3} / \mathrm{s} \\
\text { Head lost due to friction, } & h_{f} & =4 \mathrm{~m} \\
\text { Value of Chezy's constant, } C & =50 \\
\text { Let the diameter of pipe }=d
\end{array}
$$

Hydraulic mean depth, $\quad m=\frac{d}{4}$
Loss of head per unit length, $i=\frac{h_{f}}{L}=\frac{4}{2000}=.002$
Chezy's formula is given by equation (11.4) as $V=C \sqrt{m i}$
Substituting the values of $V, m, i$ and $C$, we get

$$
\frac{0.2 \times 4}{\pi d^{2}}=50 \sqrt{\frac{d}{4} \times .002} \text { or } \sqrt{\frac{d}{4} \times .002}=\frac{0.2 \times 4}{\pi d^{2} \times 50}=\frac{.00509}{d^{2}}
$$

Squaring both sides, $\frac{d}{4} \times .002=\frac{.00509^{2}}{d^{4}}=\frac{.0000259}{d^{4}}$ or $d^{5}=\frac{4 \times .0000259}{.002}=0.0518$

$$
\therefore \quad d=\sqrt[5]{0.0518}=(.0518)^{1 / 5}=0.553 \mathrm{~m}=\mathbf{5 5 3} \mathbf{~ m m} . \text { Ans. }
$$

(Award 6 marks for the correct working, steps with the correct answer)
16. In the figure below, when a sudden contraction is introduced in a horizontal pipe line from 50 cm to 25 cm , the pressure changes from $10,500 \mathrm{~kg} / \mathrm{m}^{2}\left(103005 \mathrm{n} / \mathrm{m}^{2}\right)$ to 6900 $\mathrm{kg} / \mathrm{m}^{2}\left(67689 \mathrm{n} / \mathrm{m}^{2}\right)$. Calculate the rate of flow. Assume the co-efficient of contraction of jet to be 0.65 . Following this if there is a sudden enlargement from 25 cm to 50 cm and if the pressure at the 25 cm section is $6900 \mathrm{~kg} / \mathrm{m}^{2}\left(67689 \mathrm{n} / \mathrm{m}^{2}\right)$ what is the pressure at the 50 cm enlarged section? (20 marks)


Dia. of large pipe, $\quad D_{1}=50 \mathrm{~cm}=0.5 \mathrm{~m}$
Area,
$A_{1}=\frac{\pi}{4}(.5)^{2}=0.1963 \mathrm{~m}^{2}$
Dia. of smaller pipe, $\quad D_{2}=25 \mathrm{~cm}=0.25 \mathrm{~m}$
$\therefore$ Area,
$A_{2}=\frac{\pi}{4}(.25)^{2}=0.04908 \mathrm{~m}^{2}$
Pressure in large pipe, $\quad p_{1}=10500 \mathrm{~kg} / \mathrm{m}^{2}$ or $103005 \mathrm{~N} / \mathrm{m}^{2}$
Pressure in smaller pipe, $\quad p_{2}=6900 \mathrm{~kg} / \mathrm{m}^{2}$ or $67689 \mathrm{~N} / \mathrm{m}^{2}$
Co-efficient of contraction, $C_{c}=0.65$

Head lost due to contraction is given by equation (11.6),

$$
\begin{equation*}
h_{c}=\frac{V_{2}^{2}}{2 g}\left(\frac{1}{C_{c}}-1.0\right)^{2}=\frac{V_{2}^{2}}{2 g}\left(\frac{1}{0.65}-1\right)^{2}=0.2899 \frac{V_{2}^{2}}{2 g} \tag{i}
\end{equation*}
$$

From continuity equation, we have

$$
\begin{align*}
A_{1} V_{1} & =A_{2} V_{2} \text { or } V_{1}=\frac{A_{2} V_{2}}{A_{1}}=\frac{\frac{\pi}{4} D_{2}^{2} \times V_{2}}{\frac{\pi}{4} D_{1}^{2}} \\
& =\left(\frac{D_{2}}{D_{1}}\right)^{2} \times V_{2}=\left(\frac{0.50}{0.25}\right)^{2} \times V_{2}=\frac{V_{2}}{4} \tag{ii}
\end{align*}
$$

Applying Bernoulli's equation at sections 1-1 and 2-2,

But

$$
\begin{aligned}
\frac{p_{1}}{\rho g}+\frac{V_{1}^{2}}{2 g}+Z_{1} & =\frac{p_{2}}{\rho g}+\frac{V_{2}^{2}}{2 g}+Z_{2}+h_{c} \\
Z_{1} & =Z_{2} \text { (as pipe is horizontal) }
\end{aligned}
$$

$$
\therefore \quad \frac{p_{1}}{\rho g}+\frac{V_{1}^{2}}{2 g}=\frac{p_{2}}{\rho g}+\frac{V_{2}^{2}}{2 g}+h_{c}
$$

Substituting the values of $p_{1}, p_{2}, h_{c}$ and $V_{1}$, we get

$$
\frac{103005}{1000 \times 9.81}+\frac{\left(V_{2} / 4\right)^{2}}{2 g}=\frac{67689}{1000 \times 9.81}+\frac{V_{2}^{2}}{2 g}+.2899 \frac{V_{2}^{2}}{2 g}
$$

or

$$
10.5+\frac{V_{2}^{2}}{16 \times 2 g}=6.9+1.2899 \frac{V_{2}^{2}}{2 g}
$$

or

$$
10.5-6.9=1.2899 \frac{V_{2}^{2}}{2 g}-\frac{1}{16} \times \frac{V_{2}^{2}}{2 g}=1.2274 \frac{V_{2}^{2}}{2 g}
$$

or

$$
3.6=1.2274 \times \frac{V_{2}^{2}}{2 g}
$$

$$
\therefore \quad V_{2}=\sqrt{\frac{3.6 \times 2 \times 9.81}{1.2274}}=7.586 \mathrm{~m} / \mathrm{s}
$$

(i) Rate of flow of water, $Q=A_{2} V_{2}=0.04908 \times 7.586$

$$
=0.3723 \mathrm{~m}^{3} / \mathrm{s} \text { or } 372.3 \mathrm{lit} / \mathrm{s} . \text { Ans. }
$$

(ii) Applying Bernoulli's equation to sections 3-3 and 4-4,

$$
\frac{p_{3}}{\rho g}+\frac{V_{3}^{2}}{2 g}+Z_{3}=\frac{p_{4}}{\rho g}+\frac{V_{4}^{2}}{2 g}+Z_{4}+\text { head loss due to sudden enlargement }\left(h_{e}\right)
$$

But

$$
\begin{aligned}
& p_{3}=6900 \mathrm{~kg} / \mathrm{m}^{2}, \text { or } 67689 \mathrm{~N} / \mathrm{m}^{2} \\
& V_{3}=V_{2}=7.586 \mathrm{~m} / \mathrm{s} \\
& V_{4}=V_{1}=\frac{V_{2}}{4}=\frac{7.586}{4}=1.8965 \\
& Z_{3}=Z_{4}
\end{aligned}
$$

And head loss due to sudden enlargement is given by equation (11.5) as

$$
h_{e}=\frac{\left(V_{3}-V_{4}\right)^{2}}{2 g}=\frac{(7.586-1.8965)^{2}}{2 \times 9.81}=1.65 \mathrm{~m}
$$

Substituting these values in Bernoulli's equation, we get
or

$$
\frac{67689}{1000 \times 9.81}+\frac{7.586^{2}}{2 \times 9.81}=\frac{p_{4}}{1000 \times 9.81}+\frac{1.8965^{2}}{2 \times 9.81}+1.65
$$

$$
6.9+2.933=\frac{p_{4}}{1000 \times 9.81}+0.183+1.65
$$

$$
\therefore \quad \frac{p_{4}}{1000 \times 9.81}=6.9+2.933-0.183-1.65=9.833-1.833=8.00
$$

$$
\therefore \quad p_{4}=8 \times 1000 \times 9.81=78480 \mathrm{~N} / \mathrm{m}^{2} . \text { Ans. }
$$

(Award 20 marks for the correct working, steps with the correct answer)

