TVET CURRICULUM DEVELOPMENT, ASSESSMENT AND CERTIFICATION COUNCIL

(TVET CDACC)

Qualification Code	**:**	071606T4MCT
Qualification	**:**	Mechatronics Technician Level 6
Unit Code	**:**	ENG/OS/MC/CC/02/6/A
Unit of Competency	**:**	Apply Engineering Mathematics

ASSESSOR'S GUIDE

INSTRUCTIONS TO ASSESSOR

1. Marks for each question are indicated in the brackets.
2. Answers provided are model answers.

SECTION A (40 MARKS)

(These only serves as a guide to expected responses)

1. Determine the mean of the following {2,3,7,5,5,13,1,7,4,8,3,4,3}. (2 Marks)

   Solution

   Mean, $M = \frac{fx}{n} = \frac{65}{13}$=5 **award 1mark for correct formula**

   Award 1mark correct answer

2. If p = 15i -10j + 30k, find the modulus of p (2 Marks)

   Solution

   Modulus /P/=$\sqrt{15^2+10^2+30^2}=\sqrt{1225}$=35 units **award 1mark formula**

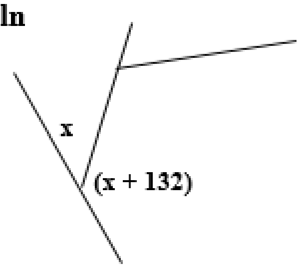
   Award 1mark for correct answer

3. The interior angle of a n-sided regular polygon exceeds its exterior angle by 132^o. Calculate the number of sides. (3 Marks)

   Solution

Let the size of exterior be x, then interior becomes (x + 132)

sln



But Interior + exterior $= 180^0$

x + (x + 132) = 180^0
2x = 180-132 = 48
And x = 24^0

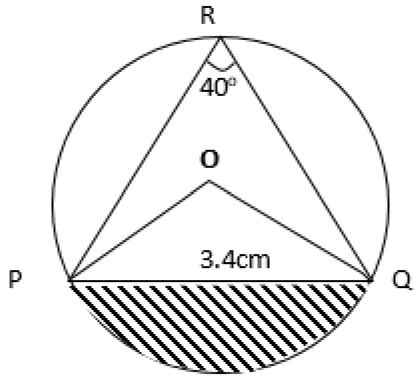
But:

$Number\ of\ sides\ n = \frac{360^0}{Size\ of\ exterior(\theta)}$

$Number\ of\ sides\ n = \frac{360^0}{24^0} = \mathbf{15\ sides}$

Award 1mark formula, Award 1mark simplification and 1mark correct answer.

4. The diagram below shows a circle with a chord PQ = 3.4cm and angle PRQ = 40^o.
Calculate the area of the shaded segment. (5 Marks)

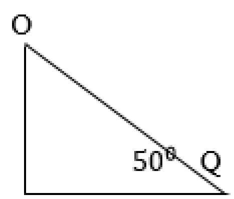


Solution

By Angle properties of a circle, Angle formed at the Centre is twice the angle at the Circumference i.e 2x40 = 80^0

Base Angle < OPQ = < PQO = 50^0
Dropping a perpendicular bisector from point O to divide line PQ into two equal parts

Award 2mark for proper description
1marks for formulas
1mark for proper substitution
1marks for correct answer



The radius OQ can be calculated by SOHCAHTOA

Cos 50 $=\frac{Ad}{Hy} = \frac{1.7}{OQ}, \rightarrow OQ = \frac{1.7}{Cos\ 50} = 2.644cm$

Hence radius r = 2.644
Area of Shaded= area of sector - Area of Triangle OPQ

1
c
s

$= \frac{22}{7} x\ 2.644^2 - \frac{1}{2} \times 2.644x2.644Sin80$
= 22.30 – 3.442
$= \mathbf{18.86} cm^2$

5. Given that A = $\begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix}$ and

B = $\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$ and that C = AB, evaluate C^{-1} (3 Marks)

Solution

C= AB

$= \begin{pmatrix} 2 & 1 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} 2+2 & -4+-1 \\ 0+-4 & 0+2 \end{pmatrix}$

$C = \begin{pmatrix} 4 & -5 \\ -4 & 2 \end{pmatrix}$

Hence Invers, $C^{-1} = \frac{1}{(4x2)-(-5x-4)}\begin{pmatrix} 2 & 5 \\ 4 & 4 \end{pmatrix}$

$= \frac{1}{-12}\begin{pmatrix} 2 & 5 \\ 4 & 4 \end{pmatrix}$

Or

$\begin{pmatrix} -0.1667 & -0.4167 \\ -0.3333 & -0.3333 \end{pmatrix}$

1mark for proper substitution, 2marks for correct answer

6. The surface area of a sphere is 201.1cm2.find the diameter of the sphere. (4 Marks)

Solution

$$S.A = 4\pi r^2$$
$$201.1 = 4\pi r^2$$

r =√201.1/4

$\boldsymbol{r = 7.0904}$**cm**

Diameter=2r=14.19cm

Award 1 mark for correct formula, 2marks for calculating radius and 1 mark for correct diameter answer.

7. A mother is now $2\frac{1}{2}$ times as old as her daughter Mary; four years ago, the ratio of their ages was 3:1. What is the present age of the mother? (3 Marks)

Solution

Let the present age of Mary be x

Mother,,, ,,,, ,,, ,,, ,,, ,,2.5x

Now fill the columns of the tables as indicated,

	Ages in 4years ago	*Present ages*
Mary	*(x-4)*	*x*
mother	*(2.5x – 4)*	*2x*

Now according to the statements, we can equate the ages of Mary and that of the Mother as shown.

***Note**: Ratio is a fraction*

$$\frac{Mother}{Mary} = \frac{2.5x - 4}{x - 4} = \frac{3}{1}$$

And cross multiplying;

$3(x-4) = 2.5x - 4$

$3x - 2.5x = -4 + 12$

$0.5x = 8$

x=16 years now

*Mother, Now = 2.5 x 16 = **40 years***

Award 1mark formula, 2marks for correct simplification and answer.

8. Given that: - r = 5i – 2j and m = -2i + 6j – k are the position vectors for R and M respectively. Find the length of vector RM (2 Marks)

Solution

$$\mathbf{RM} = \mathbf{M}\text{-}\mathbf{R} = \begin{pmatrix}-2\\6\\-1\end{pmatrix} - \begin{pmatrix}5\\-2\\0\end{pmatrix} = \begin{pmatrix}-7\\8\\-1\end{pmatrix}$$

$$Magnitude = \sqrt{(-7)^2 + (8)^2 + (-1)^2}$$

$$= \sqrt{49 + 64 + 1}$$

$$\sqrt{114} = \mathbf{10.67}$$

Award 1mark formula, 1mark for correct answer

9. Use Binomial theorem to expand $(\mathbf{2+3x})^{\frac{1}{2}}$ **as far as the term in** x^3, and state the range of values of x which the expansion is valid. (4 Marks)

Solution

NOTE: *Only Upto the 4th Term*

$(2+3x)^{½} = \left[2\left(1+\frac{3x}{2}\right)\right]^{½}$

$= \left[2^{1/2}\left(1+\frac{3x}{2}\right)^{½}\right]$ **Let p =** $\frac{3x}{2}$

$2^{\frac{1}{2}}\left[1+\text{np}+\frac{n(n-1)(p)^2}{2!}+\frac{n(n-1)(n-2)(p)^3}{3!}\right]$

$2^{\frac{1}{2}}\left[1+\frac{1}{2}\left(\frac{3x}{2}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{3x}{2}\right)^2}{2}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{3x}{2}\right)^3}{6}\right]$

Hence , 1.4142 +1.06x + $0.3977x^2$ + $0.2983\,x^3$

and Valid when $-\frac{2}{3} \le x \le \frac{2}{3}$

Award 1mark formula, 1mark for proper substitution and simplification, 1mark for correct answer

10. Solve $\frac{dy}{dx}$ from the first principle, given that $y = \frac{1}{x+4}$ (5 Marks)

Solution

$$f^{-1}x = \frac{limit}{h-0}\left(\frac{f(x+h)-f(y)}{h}\right)$$

$$\frac{\delta y}{\delta x} = \frac{limit}{h-0}\left(\frac{\left(\frac{1}{x+h+4}\right)-\left(\frac{1}{x+4}\right)}{h}\right)$$

$$\frac{\delta y}{\delta x} = \frac{limit}{h-0}\left(\frac{\frac{(x+4-(x+h+4)}{(x+h+4)(x+4)}}{h}\right)$$

$$= \frac{limit}{h-0}\left(\frac{\frac{-h}{(x)^2+8x+(x+4)h+16}}{h}\right)$$

$$= \frac{-h}{(x)^2+8x+(x+4)h+16} \text{ x } \frac{1}{h}$$

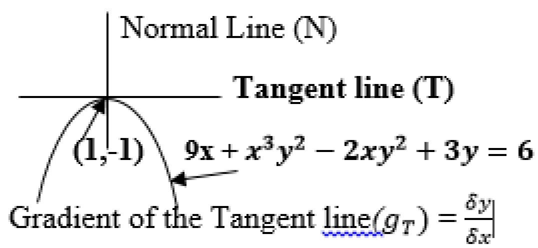
$$= \frac{limit}{h-0}\left(\frac{-h}{(x)^2+8x+(x+4)h+16}\right. = \frac{-1}{x^2+8x+16}$$

Award 1 marks formula, 2mark for proper substitution and simplification, 2marks for correct answer

11. Use implicit differentiation of $\mathbf{9x} + \boldsymbol{x^3y^2 - 2xy^2 + 3y = 6}$, to determine the equation of the tangent at point (1,1) (4 Marks)

Solution

Generally the Gradient of the curve at appoint is equal to the gradient of the Tangent touching the curve at that same point



Award 1 marks formula ,2mark for proper substitution and calculations

$$\frac{dy}{dx}9x + \frac{dy}{dx}\boldsymbol{x^3y^2} - \frac{dy}{dx}\boldsymbol{2xy^2} + \frac{dy}{dx}3y = \frac{dy}{dx}6$$
$$= 9 + (\boldsymbol{x^3}\tfrac{dy}{dx}\boldsymbol{y^2} + \boldsymbol{y^2}\tfrac{dy}{dx}\boldsymbol{x^3}) - (2x\tfrac{dy}{dx}\boldsymbol{y^2} + \boldsymbol{y^2}$$
$$\tfrac{dy}{dx}\boldsymbol{2x}) + \tfrac{dy}{dx}3y = 0$$

Note : *Use* ***Product Rule*** *To differentiate the part* $\boldsymbol{x^3y^2}$ ***and*** $\boldsymbol{2xy^2}$ *and differentiate it implicitly*

$$9 + 2x^3\tfrac{dy}{dx}y + 3x^2y^2 - (4xy\tfrac{dy}{dx} + 2y^2) + 3\tfrac{dy}{dx} = 0,$$

$$9 + 2x^3\tfrac{dy}{dx}y + 3x^2y^2 - 4xy\tfrac{dy}{dx} - 2y^2 + 3\tfrac{dy}{dx} = 0,$$

$$\tfrac{dy}{dx}(2x^3y - 4xy + 3) = 2y^2 - 3x^2y^2 - 9$$

And making $\frac{dy}{dx}$ the Subject,

$$\frac{dy}{dx} = \frac{2y^2 - 3x^2y^2 - 9}{(2x^3y - 4xy + 3)}$$

And at (1,-1)

Gradient, $\frac{dy}{dx} = \frac{2y^2 - 3x^2y^2 - 9}{(2x^3y - 4xy + 3)}$

$$= \frac{2(-1)^2 - 3(1)^2(-1)^2 - 9}{(2(1)^3(-1) - 4(1)(-1) + 3)} = -2$$

Gradient of tangent = dy/dx

$$\frac{-2}{1} = \frac{y - (-1)}{x - 1}$$

y+1 = -2(x-1)

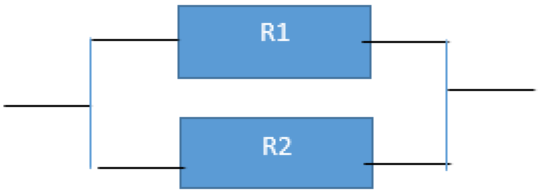
y = 2x +1

1marks for correct answer

12. Two resistors when connected in Parallel, give a total resistance of 9.6 ohms. When connected in series, the total resistance was 40 Ohms. If one of the resistors is R Ohms, Show that ; $\boldsymbol{R^2 - 40R + 384 = 0}$

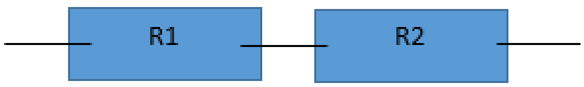
(3 Marks)

Solution



1mark for proper parallel substitution

In parallel $\frac{R1R2}{R1+R2} = \mathbf{9.6}$ eqtn 1



1mark for proper series substitution

In series: **R1 + R2 = 40** **eqtn 2**

So , R1= 40 –R2

$$\frac{(40 - \mathrm{R2})\mathrm{R2}}{(40 - \mathrm{R2}) + \mathrm{R2}} = 9.6$$

$$\frac{40\mathrm{R2} - \mathrm{R2}^2}{40 - \mathrm{R2} + \mathrm{R2}} = \frac{9.6}{1}$$

$= \frac{40\mathrm{R2} - \mathrm{R2}^2}{40} = \frac{9.6}{1}$

And $40\mathrm{R2} - \mathrm{R2}^2 = 384$

$= 40\mathrm{R2} - \mathrm{R2}^2 - 384 = 0$

1 mark for correct answer

So that $\boldsymbol{R2^2 - 40R2 + 384 = 0}$

SECTION B (60 MARKS)

13. study the matrices set below and use Cramer's theorem to find;

$4x + 8y + z = -6$
$2x - 3y + 2z = 0$
$x + 7y - 3z = -8$

a) Its determinant (4 Marks)

Solution

Here the determinant of the coefficients is:

$$|A| = \begin{vmatrix} -4 & 2 & -9 \\ 3 & 4 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= -4(8 + 3) - 2(6 - 1) - 9(-9 - 4)$$

$$= -44 - 10 + 117$$

$$|A| = 63$$

Award 2 marks correct formula, 2 marks for correct answer

b) Its inverse (7 Marks)

Solution

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$A^{-1} = 1/63 \begin{matrix} 11 & 5 & 13 \\ 23 & 5 & 10 \\ -38 & 23 & 22 \end{matrix}$$

Award 2 mark for correct inverse formula, 3marks for correct substitution and simplifications, 2 marks for correct answer.

c) Solutions x,y and z of the matrix (9 Marks)

Solution

for $|A_x|$, replacing the first column of $|A|$ with the corresponding constants 2, 5 and 8, we have

$$|A_x| = \begin{vmatrix} 2 & 2 & -9 \\ 5 & 4 & 1 \\ 8 & -3 & 2 \end{vmatrix}$$

$$= 2(11) - 2(2) - 9(-47) = 22 - 4 + 423$$

$$|A_x| = 441$$

Similarly,

$$|A_y| = \begin{vmatrix} -4 & 2 & -9 \\ 3 & 5 & 1 \\ 1 & 8 & 2 \end{vmatrix}$$

$$= -4(2) - 2(5) - 9(19)$$

$$= -8 - 10 - 171$$

$$|A_y| = -189$$

and

$$|A_z| = \begin{vmatrix} -4 & 2 & 2 \\ 3 & 4 & 5 \\ 1 & -3 & 8 \end{vmatrix}$$

$$= -4(47) - 2(19) + 2(-13)$$

$$= -188 - 38 - 26$$

$$|A_z| = -252$$

Hence $x = \frac{|A_x|}{|A|} = \frac{441}{63} = 7$

$$y = \frac{|A_y|}{|A|} = \frac{-189}{63} = -3$$

$$z = \frac{|A_z|}{|A|} = \frac{-252}{63} = -4$$

So the solution set of the system is $\{(7, -3, -4)\}$

Award 2mark for each step of /A/ and 1mark for each solution. NB; accept any other valid formula

14. The following table gives the frequency distribution of the number of orders received each day during the past 50 days at the office of a mail-ordercompany. Calculate;

No. of order	f
10 – 12	4
13 – 15	12
16 – 18	20
19 – 21	14
Total	n = 50

i) Variance (12 Marks)

Solution:

No. of order	f	x	fx	fx^2
10 – 12	4	11	44	484
13 – 15	12	14	168	2352
16 – 18	20	17	340	5780
19 – 21	14	20	280	5600
Total	n = 50		832	14216

Variance, $$s^2 = \frac{\sum fx^2 - \frac{\left(\sum fx\right)^2}{n}}{n-1}$$

$$= \frac{14216 - \frac{(832)^2}{50}}{50-1}$$

$$= 7.5820$$

Award 7marks for correct table, 2marks for variance formula, 2marks for correct substitution and simplification,1marks for correct answer.

ii) Standard deviation (8 Marks)

Standard Deviation, $s = \sqrt{s^2} = \sqrt{7.5820} = 2.75$

Award 3marks for correct formula, 5marks answer. NB;(accept any other formula applied by the trainee)

15. Given that $A = 3i + 4k$ and $B = 4i + 3k$,evaluate;

I. A.B (6 Marks)

Solution

$\vec{A} \cdot \vec{B}$

$= A_1 \cdot B_1 + A_2 \cdot B_2$

$= 3 \cdot 4 + 4 \cdot 3$

$= 12 + 12$

$= 24$

Award 2 mark for correct formula, 2marks for correct substitutions, 2 marks for correct answer.

II. /A/ and /B/ (5 Marks)

Solutions

$$|\vec{A}|$$

$$= \sqrt{A_1^2 + A_2^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$|\vec{B}|$$

$$= \sqrt{B_1^2 + B_2^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

Award 1/2mark for correct formula (each), 1marks for correct substitutions (each), 1marks for correct answer (each)

III. The angle between A and B (9 Marks)

$$\cos(\theta) = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|}$$

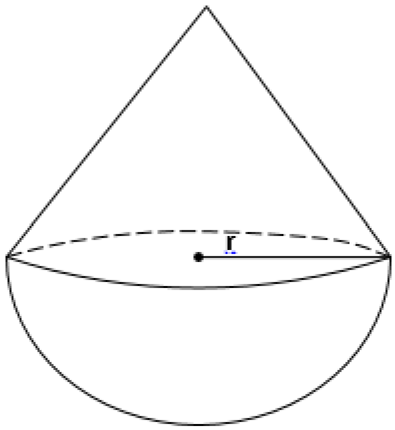
$$\cos(\theta) = \frac{24}{5 \cdot 5} = 0.96$$

$$\therefore \theta = \cos^{-1}(0.96) = 0.2838 \text{ rad}$$

$$\therefore \theta = 16.2602^{\circ}$$

Award 3mark for correct formula, 2marks for correct substitutions, 2marks for correct answer in rads and 2marks for correct answer in degrees.

16. The diagram below shows a metal solid consisting of a cone mounted on hemisphere. The height of the cone is $1\frac{1}{2}$ times its radius;



a) The radius of the cone (5 Marks)

Solution

This becomes simple, the radius $= r$

and height of the solid to be $h = 1\frac{1}{2}r$

so,

Total. Volume = V. Hemispher + V.Cone

$31.5\pi\ cm^3 = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$31.5\pi\ cm^3 = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

And substituting for h, we have:

$31.5\pi\ cm^3 = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 (1\frac{1}{2}r)$

$31.5\pi\ cm^3 = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^3 (\frac{3}{2})$

$31.5\pi\ cm^3 = \pi r^3 \left[\frac{2}{3} + \frac{3}{6}\right]$

$31.5\pi\ cm^3 = \frac{7}{6}\pi r^3$

$r^3 = \frac{31.5 \times 6}{7} = 27$

$\boldsymbol{r = \sqrt[3]{27} = 3\ cm}$

Award 1 mark for correct formula, 2marks for correct substitutions, 2marks for correct answer.

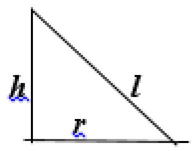
b) The surface area of the solid (7 Marks)

Solution

Total.S.A = S.A of hemisphere + S.A of the curved part of Cone

$= 2\pi r^2 + \pi r l$

$= 2x\ \frac{22}{7} x\ 3^2 + \frac{22}{7} x\ 3\ x\ l$



but: $h = 1\frac{1}{2}r = \frac{3}{2}x(3) = 4.5$ *and,*

Slant height $l = \sqrt{r^2 + h^2}$

$l = \sqrt{3^2 + (4.5)^2} = 5.408\ cm$

Therefore Volume become:

$= 2x\ \frac{22}{7} x\ 3^2 + \frac{22}{7} x\ 3\ x\ 5.408$

$= 56.57 + 50.99$

$= \mathbf{107.6\ Cm^2}$

Award 2marks for correct formula, 2marks for correct substitutions, 3marks for correct answer.

c) How much water will rise if the solid is immersed totally in a cylindrical container which Contains some water, given the radius of the cylinder is 4cm. (5 Marks)

Solution

In this case, we first, find the base area of the cylinder,

Base Area of a cylinder $= \pi r^2$

$= \frac{22}{7} x\ 4^2 = \mathbf{50.29 Cm^2}$

Now assuming that there was water inside the cylinder and above solid

$\boldsymbol{V = 31.5\pi\ cm^3}$

Is immersed, then 31.5π would be increased volume:

Therefore

Volume = Base Area of Cylinder** x **height

$31.5\pi = 50.29\ h$

$h = \frac{31.5\pi}{50.29} = \frac{31.5 \ x \ 3.142}{50.29} = \mathbf{1.968 \ cm}$

Award 1mark for correct formula, 3marks for correct substitution and simplification, 1marks for correct answer.

d) The density, in kg/m^3 of the solid given that the mass of the solid is 144 Kg.

(3 Marks)

Solution

Density $= \frac{Mass}{Volume} = \frac{144000}{31.5\pi} = \frac{14000}{31.5 \ x \ 3.142}$

$= 141.5 \ g/cm^3$

In kg/m^3 :

= 141.5 x 1000 = 141500 kg/m^3

Award 1mark for correct formula, 1mark for correct substitutions, 1mark for correct answer.

END