CHAPTER 1: BASIC MATHEMATICS

Unit of learning code CON/CU/PL/CC/01/5/A

Related Unit of Competency in Occupational Standard Apply Basic Mathematics

1.1 Introduction to the unit of learning

This unit describes the competencies required in applying basic: algebra, trigonometry statistics, indices and logarithms and ratio. It also involves performing geometrical calculations, business calculations, carrying out basic mensuration and plotting simple graphs.

1.2 Summary of Learning Outcomes

1. Apply basic algebra
2. Apply basic trigonometry
3. Perform geometrical calculations
4. Carry out basic mensuration
5. Apply basic statistics
6. Plot simple graphs
7. Apply Indices and Logarithms
8. Perform business calculations
9. Apply Ratios

1.2.1 Learning Outcome 1: Apply Basic Algebra

1.2.1.1. Introduction to the learning outcome

This unit describes the competencies required in applying basic mathematics on algebra.

1.2.1.2. Performance Standard

1. Calculations involving Indices are performed based on the concept
2. Linear equations are represented based on the concept
3. Scientific calculator is used in solving mathematical problems in line with manufacturer's manual
4. Simultaneous equations are performed based on mathematical rules
5. Simple algebraic equations are formed based on the concept
6. Simple algebraic equations are solved based on the concept

1.2.1.3. Information Sheet

Laws of indices

1. Law of Multiplication $a^n x a^m = a^{n+m}$
2. $Law\ of\ Division\ a^n \div a^m = a^{n-m}$
3. $Law\ of\ Power\ to\ power\ (a^n)^m = a^{nxm}$

Example leaving answers on indices form

i. $\mathbf{2^2 x 2^4 = 2^{2+4} = 2^2}$

ii. $\mathbf{3^2 x 3^3 = 2^{2+3} = 2^5}$

iii. $\mathbf{5^4 \div 5^2 = 5^{4-2} = 5^2}$

iv. $\mathbf{\frac{2^3 x 2^4}{2^7 x 2^5} = \frac{2^{3+4}}{2^{7+5}} = \frac{2^7}{2^{12}} = 2^{7-12} = 2^{-5}}$

v. **Evaluate:** $\mathbf{\frac{(10^2)^3}{10^4 x 10^2} = \frac{10^6}{10^6} = 10^{6-6} = 10^0 = 1}$

vi. **Find the value of:** $\mathbf{\frac{2^3 x 3^5 x (7^2)^2}{7^4 x 2^4 x 3^3} = 2^{3-4} x 3^{5-2} x 7^{2x2-4}}$

$\mathbf{= 2^{-1} x 3^2 x 7^0}$

$\mathbf{= \frac{1}{2} x9x1}$

=4.5

Linear equations; are equations of the first order.

Examples:

Solve for x

- 2x – 3 = 0,

  2x=3

  x=3/2

  x=1.5

- 2y = 8

  y=8/2

  y=4

- m + 1 = 0,

m=-1

- x/2 = 3

x=3x2

x=6

- x + y = 2

x=2-y

- 3x – y + z = 3

3x-y=3-z

3x=3-z+y

$X=\frac{3-z+y}{3}$

Formulas

There are different forms to write linear equations. Some of them are:

Linear Equation	General Form	Example
Slope intercept form	$y = mx + c$	$y + 2x = 3$
Point–slope form	$y – y_1 = m (x – x_1)$	$y – 3 = 6(x – 2)$
General Form	$Ax + By + C = 0$	$2x + 3y – 6 = 0$
Intercept form	$x/x_0 + y/y_0 = 1$	$x/2 + y/3 = 1$

Example:

i. Solve (2x – 10)/2 = 3(x – 1)

Step 1: Clear the fraction

x – 5 = 3(x – 1)

Step 2: Simplify both sides equations

2x – 5 = 3x – 3

x = 3x + 2

x – 3x = 2

Step 3: Isolate x

-2x = 2

x = -1

ii. **Solve $x = 12(x + 2)$ Solution:**

$x = 12(x + 2)$

$x = 12x + 24$

Subtract 24 from each side

$x – 24 = 12x + 24 – 24$

$x – 24 = 12x$

Simplify

$11x = -24$

Isolate x, by dividing each side by 11

$11x / 11 = -24/11$

$x = -24/11$

iii. **Solve $x – y = 12$ and $2x + y = 22$**

Solution:

Name the equations

$x – y = 12$ ——— (1)

$2x + y = 22$ ——— (2)

Isolate Equation (1) for x,

$x = y + 12$

Substitute $y + 12$ for x in equation (2)

$2(y+12) + y = 22$

$3y + 24 = 22$

$3y = -2$

or $y = -2/3$

Substitute the value of y in $x = y + 12$

$x = y + 12$

$x = -2/3 + 12$

$x = 34/3$

Answer: $x = 34/3$ and $y = -2/3$

Simultaneous equations

They're several methods of solving simultaneous equations including

1) Elimination method
2) Substitution method
3) Graphical method
4) Matrix method

Let us discuss the first two;

1) Elimination method

Example1.

Solve the following pair of simultaneous linear equations:

Equation 1: $2x + 3y = 8$

Equation 2: $3x + 2y = 7$

Step 1: Multiply each equation by a suitable number so that the two equations have the same leading coefficient. An easy choice is to multiply Equation 1 by 3, the coefficient of *x* in Equation 2, and multiply Equation 2 by 2, the *x* coefficient in Equation 1:

3×Eqn 1….>3×(2x +3y)=8)…..

>6x +9y=24

2×Eqn 2…..>2×(3x+2y)=7)…..>6x+4y=14 Both equations now have the same leading coefficient

Step 2: Subtract the second equation from the first.

$-(6x + 9y = 24$

$-(6x + 4y = 14)$

$5y = 10$

Step 3: Solve this new equation for *y*.

$y = 10/5 = 2$

Step 4: Substitute $y = 2$ into either Equation 1 or Equation 2 above and solve for *x*. We'll use Equation 1.

$2x + 3(2) = 8$

$2x + 6 = 8$ Subtract 6 from both sides

$2x = 2$ Divide both sides by 2

$x = 1$

Solution: $x = 1$, $y = 2$ or (1,2).

Substitution Method

$$x + y = 24$$
$$2x - y = -6$$

In the substitution method, we manipulate one of the equations such that one variable is defined in terms of the other:

$$x + y = 24$$
$$\downarrow$$
$$y = 24 - x$$

Defining y in terms of x

Then, we take this new *definition* of one variable and *substitute* it for the same variable in the other equation.

In this case, we take the definition of y, which is 24 - x and substitute this for the y term found in the other equation:

$$y = 24 - x$$
$$\downarrow \quad \text{Substitute}$$
$$2x - y = -6$$
$$\downarrow$$
$$2x - (24 - x) = -6$$

$$2x-24+x=-6$$

$$3x=-6+24$$

$3x=18$

$X=6$

$Y=24-x$

$Y=24-6$

$Y=1$

1.2.1.4. Learning Activities

With guidance from the trainer, manipulate different algebraic expressions through addition, subtraction, multiplication and Simplification.

1.2.1.5. Self-Assessment

Solve the following linear equations:

1. $5y-11=3y+9$
2. $3x + 4 = 7 – 2x$
3. $9 – 2(y – 5) = y + 10$

Solve the simultaneous questions below

4. $5x + 3y = 41$

   $2x + 3y = 20$

5. $4x - 4y = 24$

   $x - 4y = 3$

1.2.1.6. Tools, Equipment, Supplies and Materials

Tools/Equipment:	**Materials:**
• Scientific Calculators • Rulers	• Charts with presentations of data • Graph books

• Pencils • Erasers • Computers with internet connection	• Text books

1.2.1.7. References

Ministry of Education (2003). *Secondary Mathematics Students' Book One (3rd ed)*. Nairobi: Kenya Literature Bureau

Ministry of Education (2003). *Secondary Mathematics Students' Book Two (3rd ed)*. Nairobi: Kenya Literature Bureau

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John Bird (2005). *Basic Engineering Mathematics, (4th Ed).* MA: Elsevier ltd

1.2.1.8. Model answers

1. 5y-11=3y+9

Solution:

Putting like times together

$5y - 3y = 9 + 11$

$2y = 20$

$\frac{\cancel{2}y}{\cancel{2}} = \frac{\cancel{20}10}{\cancel{2}} = 10$

$y = 10$

2. 3x + 4 = 7 – 2x

Solution:

Putting like terms together

$3x + 2x = 7 - 4$

$5x = 3$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{3}{5}$

$x = \frac{3}{5}$

3. 9 – 2(y – 5) = y + 10

Solution:

Opening the brackets: $9 - 2y + 10 = y + 10$

Putting like terms together: $9 + 10 - 10 = y + 2y$

Simplify: $9 = 3y$

$\frac{\cancel{9}3}{\cancel{3}} = \frac{\cancel{3}y}{\cancel{3}}$

$3 = y$

4. 5x + 3y = 41

2x + 3y = 20

Solution by subtraction method:

$$\begin{array}{c} 5x + 3y = 41 \\ \underline{2x + 3y = 20}\ - \\ 3x + 00 = 21 \\ 3x = 21 \end{array}$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{\cancel{21}7}{\cancel{3}}$$

$$x = 7$$

5. **4x − 4y = 24**

   x − 4y = 3

Solution: Adding the equations

$$\begin{array}{r} 4x - 4y = 24 - \\ x - 4y = 3 \\ \hline 3x - 0y = 21 \end{array}$$

$$3x = 21$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{\cancel{21}7}{\cancel{3}}$$

$$x = 7$$

1.2.2 Learning Outcome 2: Apply basic trigonometry

1.2.2.1. Introduction to the learning outcome

This unit describes the competencies required in applying basic trigonometry.

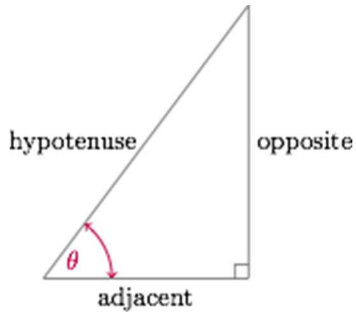
1.2.2.2. Performance Standard

1. Trigonometric ratios are derived based on trigonometric rules.
2. Calculations are performed based on trigonometric rules

1.2.2.3. Information Sheet

Trigonometric relationships

- There are six trigonometric ratios, sine, cosine, tangent, cosecant, secant and cotangent.
- These six trigonometric ratios are abbreviated as sin, cos, tan, csc, sec, cot.
- These are referred to as ratios since they can be expressed in terms of the sides of a right-angled triangle for a specific angle θ.



- Using the triangle above:
- $\sin\theta = \frac{opposite}{hypotenuse}$
- $\cos\theta = \frac{adjascent}{hypotenuse}$
- $\tan\theta = \frac{opposite}{adjascent}$

➢ $\frac{1}{sin\theta} = \csc\theta = \frac{hypotenus}{opposite}$

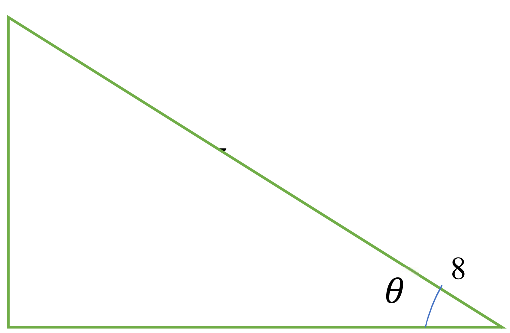
➢ $\frac{1}{cos\theta} = \sec\theta = \frac{hypotenuse}{adjascent}$

➢ $\frac{1}{tan\theta} = \cot\theta = \frac{adjascent}{opposite}$

Examples

1. Given the right triangle below, find

sin A, cos A, tan A, sec A, csc A and cot A.



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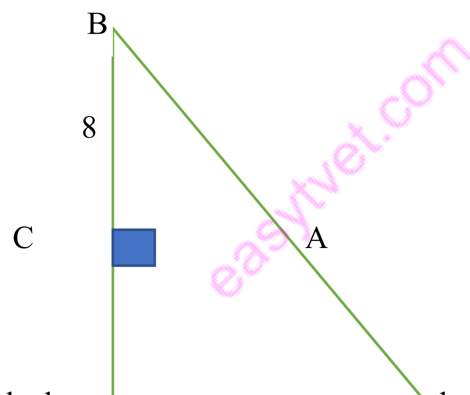
Given the lengths of the three sides of a right-angled triangle find the values of the trig functions, corresponding to the angle θ. (Round your answers to 2 decimal places)

2. Given the right triangle below, find
   sin A, cos A, tan A, sec A, csc A and cot A.



Solution ;

First we need to find the hypotenuse using Pythagora's theorem.

$(\text{hypotenuse})^2 = 8^2 + 6^2 = 100$

and hypotenuse = 10

We now use the definitions of the six trigonometric ratios given above to find sin A, cos A, tan A, sec A, csc A and cot A.

sin A = side opposite angle A / hypotenuse = 8 / 10 = 4 / 5

cos (A) = side adjacent to angle A / hypotenuse = 6 / 10 = 3 / 5

tan (A) = side opposite angle A / side adjacent to angle A

= 8 / 6 = 4 / 3

sec (A) = hypotenuse / side adjacent to angle A = 10 / 6

= 5 / 3

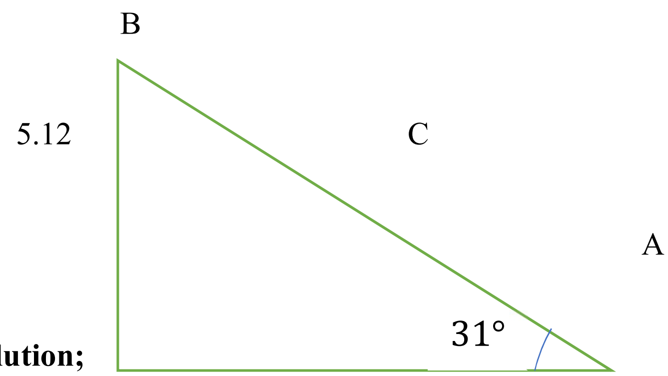
csc (A) = hypotenuse / side opposite to angle A

= 10 / 8 = 5 / 4

cot (A) = side adjacent to angle A / side opposite angle A

= 6 / 8 = 3 / 4

3. Find c in the figure below.



Solution;

We are given angle A and the side opposite to it with c the hypotenuse. The sine ratio gives a relationship between the angle, the side opposite to it and the hypotenuse as follows

sin A = opposite / hypotenuse

Angle A and opposite side are known, hence

sin 31 ^{o} = 5.12 / c

Solve for c

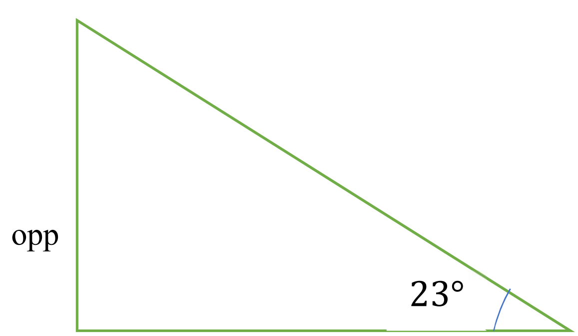
c = 5.12 / sin 31 ^{o}

and use a calculator to obtain

c (approximately) = 9.94

4. An electricity pylon stands on horizontal ground. At a point 80 m from the base of the pylon, the angle of elevation of the top of the pylon is 23◦. Calculate the height of the pylon to the nearest metre.

Solution;



80m

From the ratios we shall use tan

$$\tan 23° = \frac{opp}{80}$$

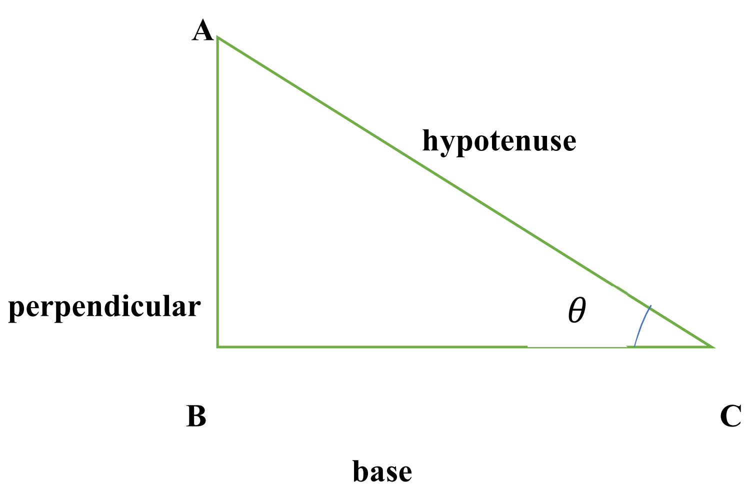
Opp=80 tan23°

=33.96m

=34m to the nearest metres

5. A surveyor measures the angle of elevation of the top of a perpendicular building as 19°. He moves 120 m nearer the building and finds the angle of elevation is now 47°. Determine the height of the building.

Trigonometric identities



In a right-angled triangle, by the Pythagorean theorem, we know,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

Therefore, in ΔABC, we have;

$AB^2 + BC^2 = AC^2$ ….. (1)

Dividing equation (1) by AC^2, we get

$$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

$(\text{Sin}\theta)^2 + (\text{Cos}\theta)^2 = 1^2$

$\text{Cos}^2\,\theta + \text{Sin}^2\,\theta = 1$ …...(2)

If θ = 0, then,

- $\text{Cos}^2\,0 + \text{Sin}^2\,0 = 1$
- $1^2 + 0^2 = 1$
- $1 + 0 = 1$
- $1 = 1$

And if we put θ = 90, then

- $\text{Cos}^2\,90 + \text{Sin}^2\,90 = 1$
- $0^2 + 1^2 = 1$
- $0 + 1 = 1$
- $1 = 1$

For all angles, $0° \leq \theta \leq 90°$, equation (2) is satisfied. Hence, equation (2) is a trigonometric identity.

Again, divide equation (1) by AB^2, we get

$$\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$$

$\text{Cot}^2\,\theta + 1 = \text{Cosec}^2\,\theta$

$(1)^2 + (\text{Cot})^2 = (\text{Csc}\ \theta)^2$

$1 + (\text{Cot}\theta)^2 = (\text{Csc}\ \theta)^2$

$\text{Cot}^2\,\theta + 1 = \text{Cosec}^2\,\theta$ …. (3)

If θ = 0, then equation (3) can be written as;

- $Cot^2 0 + 1 = Cosec^2 0$
- $\infty + 1 = \infty$
- $\infty = \infty$

Both the sides are equal.

And if θ = 90, then equation (3) can be written as;

- $Cot^2 90 + 1 = Cosec^2 90$
- $0^2 + 1 = 1^2$
- 1 = 1

Let's see what we get if we divide equation (1) by BC^2, we get,

$$\left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2$$

$(\text{Tan}\theta)^2 + (1)^2 = (\sec\theta)^2$

$(\text{Tan}\theta)^2 + (1)^2 = (\sec\theta)^2$

If θ = 0, then,

- $1 + \tan^2 0 = \sec^2 0$
- $1 + 0^2 = 1^2$
- 1 = 1

And if we put θ = 90, then

- $1 + \tan^2 90 = \sec^2 90$
- $1 + \infty = \infty$
- $\infty = \infty$

As you can see, the values of both sides are equal. Therefore, it proves that for all the values between 0^0 and 90^0, the equation (4) is satisfied. So, it is also a trigonometric identity.

Examples involving identities

1. $(1 - \sin A)/(1 + \sin A) = (\sec A - \tan A)^2$

Solution:

L.H.S = (1 - sin A)/(1 + sin A)

= $(1 - \sin A)^2/(1 - \sin A)(1 + \sin A)$,[Multiply both numerator and denominator by (1 - sin A)

= $(1 - \sin A)^2/(1 - \sin^2 A)$

= $(1 - \sin A)^2/(\cos^2 A)$, [Since $\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$]

= $\{(1 - \sin A)/\cos A\}^2$

= $(1/\cos A - \sin A/\cos A)^2$

= $(\sec A – \tan A)^2$ = R.H.S. *Proved.*

2. Prove that, $\sqrt{\{(\sec \theta – 1)/(\sec \theta + 1)\}} = \operatorname{cosec} \theta - \cot \theta$.

Solution:

L.H.S.= $\sqrt{\{(\sec \theta – 1)/(\sec \theta + 1)\}}$

= $\sqrt{[\{(\sec \theta - 1)(\sec \theta - 1)\}/\{(\sec \theta + 1)(\sec \theta - 1)\}]}$; [multiplying numerator and denominator by (sec θ - l) under radical sign]

= $\sqrt{\{(\sec \theta - 1)^2/(\sec^2 \theta - 1)\}}$

=$\sqrt{\{(\sec \theta - 1)^2/\tan^2 \theta\}}$; [since, $\sec^2 \theta = 1 + \tan^2 \theta \Rightarrow \sec^2 \theta - 1 = \tan^2 \theta$]

= $(\sec \theta – 1)/\tan \theta$

$= (\sec\theta/\tan\theta) - (1/\tan\theta)$

$= \{(1/\cos\theta)/(\sin\theta/\cos\theta)\} - \cot\theta$

$= \{(1/\cos\theta) \times (\cos\theta/\sin\theta)\} - \cot\theta$

$= (1/\sin\theta) - \cot\theta$

$= \operatorname{cosec}\theta - \cot\theta$ = R.H.S. *Proved.*

3. $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Solution:

L.H.S $= \tan^4\theta + \tan^2\theta$

$= \tan^2\theta\,(\tan^2\theta + 1)$

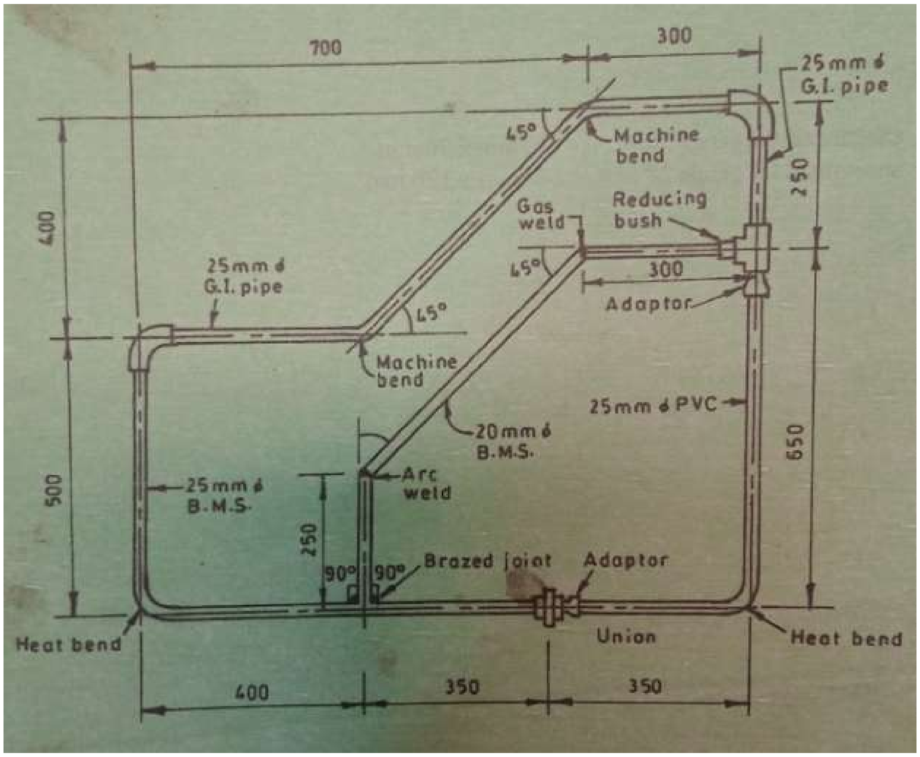
$= (\sec^2\theta - 1)(\tan^2\theta + 1)$ [since, $\tan^2\theta = \sec^2\theta - 1$]

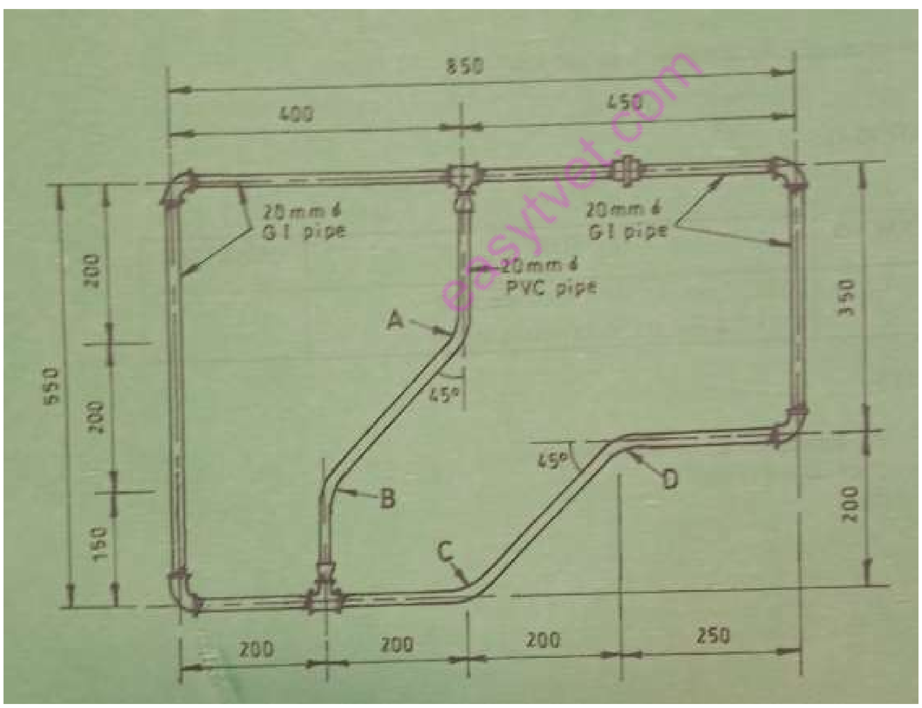
$= (\sec^2\theta - 1)\sec^2\theta$ [since, $\tan^2\theta + 1 = \sec^2\theta$]

$= \sec^4\theta - \sec^2\theta$ = R.H.S. *Proved.*

1.2.2.4. Learning Activities

1. Using knowledge in trigonometry, calculate the lengths of pipework required for the projects given
2. Fig 1

3. 

4. 

1.2.2.5. Self-Assessment

Prove the following identities

1. $\cos\theta/(1 - \tan\theta) + \sin\theta/(1 - \cot\theta) = \sin\theta + \cos\theta$

2. Show that, $1/(\csc A - \cot A) - 1/\sin A = 1/\sin A - 1/(\csc A + \cot A)$

3. $(\tan \theta + \sec \theta - 1)/(\tan \theta - \sec \theta + 1) = (1 + \sin \theta)/\cos \theta$

1.2.2.6. Tools, Equipment, Supplies and Materials

Tools/Equipment:	Materials:
• Scientific Calculators • Rulers • Pencils • Erasers • Computers with internet connection	• Charts with presentations of data • Graph books • Text books

1.2.2.7. References

Ministry of Education (2003). *Secondary Mathematics Students' Book One (3rd ed)*. Nairobi: Kenya Literature Bureau

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John Bird (2005). *Basic Engineering Mathematics, (4th Ed).* MA: Elsevier ltd

1.2.2.8. Model Answers

1. $\cos\theta/(1 - \tan\theta) + \sin\theta/(1 - \cot\theta) = \sin\theta + \cos\theta$

Solution:

L.H.S $= \cos\theta/(1 - \tan\theta) + \sin\theta/(1 - \cot\theta)$

$= \cos\theta/\{1 - (\sin\theta/\cos\theta)\} + \sin\theta/\{1 - (\cos\theta/\sin\theta)\}$

$= \cos\theta/\{(\cos\theta - \sin\theta)/\cos\theta\} + \sin\theta/\{(\sin\theta - \cos\theta/\sin\theta)\}$

$= \cos^2\theta/(\cos\theta - \sin\theta) + \sin^2\theta/(\cos\theta - \sin\theta)$

$= (\cos^2 \theta - \sin^2 \theta)/(\cos \theta - \sin \theta)$

$= [(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)]/(\cos \theta - \sin \theta)$

$= (\cos \theta + \sin \theta)$ = R.H.S. *Proved.*

2. Show that, 1/(csc A - cot A) - 1/sin A = 1/sin A - 1/(csc A + cot A)

Solution:

We have,

$1/(\csc A - \cot A) + 1/(\csc A + \cot A)$

$= (\csc A + \cot A + \csc A - \cot A)/(\csc^2 A - \cot^2 A)$

$= (2 \csc A)/1$; [since, $\csc^2 A = 1 + \cot^2 A \Rightarrow \csc^2 A - \cot^2 A = 1$]

$= 2/\sin A$; [since, $\csc A = 1/\sin A$]

Therefore,

$1/(\csc A - \cot A) + 1/(\csc A + \cot A) = 2/\sin A$

$\Rightarrow 1/(\csc A - \cot A) + 1/(\csc A + \cot A) = 1/\sin A + 1/\sin A$

Therefore, $1/(\csc A - \cot A) - 1/\sin A = 1/\sin A - 1/(\csc A + \cot A)$ *Proved.*

3. (tan θ + sec θ - 1)/(tan θ - sec θ + 1) = (1 + sin θ)/cos θ

Solution:

L.H.S = (tan θ + sec θ - 1)/(tan θ - sec θ + 1)

= [(tan θ + sec θ) - (sec^2 θ - tan^2 θ)]/(tan θ - sec θ + 1), [Since, sec^2 θ - tan^2 θ = 1]

= {(tan θ + sec θ) - (sec θ + tan θ) (sec θ - tan θ)}/(tan θ - sec θ + 1)

= {(tan θ + sec θ) (1 - sec θ + tan θ)}/(tan θ - sec θ + 1)

= {(tan θ + sec θ) (tan θ - sec θ + 1)}/(tan θ - sec θ + 1)

= tan θ + sec θ

= (sin θ/cos θ) + (1/cos θ)

= (sin θ + 1)/cos θ

= (1 + sin θ)/cos θ = R.H.S. *Proved.*

1.2.3 Learning Outcome 3:Perform Geometrical Calculations

1.2.3.1. Introduction to the learning outcome

This unit describes the competencies required in applying basic mathematics it involves performing geometrical calculations.

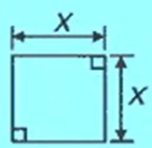
1.2.3.2. Performance Standard

1. Areas of regular figures are calculated based on the given formulae
2. Areas of irregular figures are calculated based on concept
3. Apply Pythagoras' theorem based on the concept

1.2.3.3. Information Sheet

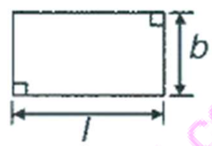
Area refers to the space that is occupies by a two-dimensional object. Or the space occupied by a flat object or a figure. There are different types of figures. Squares, Rectangles, Circles, Rhombus, Parallelogram, Trapezium, Sector & Segments. Each shape has its own applicable formula dependent on the shape of the edges and the outline.

(i) Square



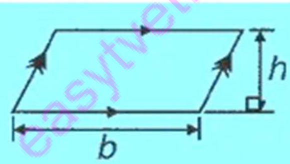
Area $= x^2$

(ii) Rectangle



Area $= l \times b$

(iii) Parallelogram

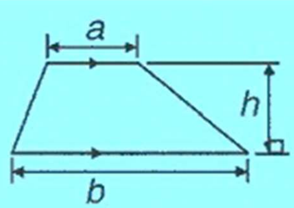


Area $= b \times h$

(iv) Triangle

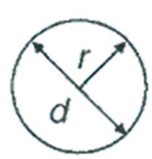
Area $= \frac{1}{2} \times b \times h$

(v) Trapezium



Area $= \frac{1}{2}(a + b)h$

(vi) Circle



Area $= \pi r^2$ or $\frac{\pi d^2}{4}$

Figure 1: Geometric Shapes

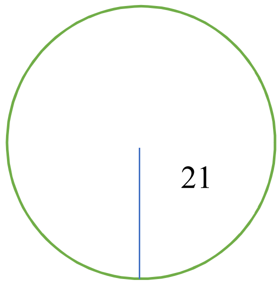
Table 3: Fomulae of Different Shapes

Shape Name	**Area**	**Perimeter (or Circumference)**
Circle	πr^2	$2\pi r$
Square	a^2	$4a$
Rectangle	$l \times b$	$2(l + b)$
Triangle	$\frac{1}{2} \times \text{height} \times \text{base}$	$a + b + c$
Parallelogram	$b \times h$	$2(l + b)$
Rhombus	$\frac{1}{2} \times d_1 \times d_2$	$4 \times \text{side}$
Trapezium	$\frac{1}{2} \times h\,(a + b)$	$a + b + c + d$

Shape Name	**TSA**	**LSA (or CSA)**	**Volume**
Cube	$6a^2$	$4a^2$	a^3
Cuboid	$2\,(lb + bh + hl)$	$2h\,(l + b)$	$l \times b \times h$
Cone	$\pi r\,(r + l)$	Πrl	$(\frac{1}{3}) \times \pi r^2 h$
Cylinder	$2\pi rh + 2\pi r^2$	$2\pi rh$	$\pi r^2 h$
Sphere	$4\pi r^2$	$4\pi r^2$	$(4/3) \times \pi r^3$
Hemisphere	$3\pi r^2$	$2\pi r^2$	$(\frac{2}{3}) \times \pi r^3$

Areas

1. A circle has a radius of 21 cm. Find its circumference and area. (Use $\pi = 22/7$)



Solution: We know,

Circumference of circle = $2\pi r$ = 2 x (22/7) x 21 = 2 x 22 x 3 = 132 cm

Area of circle = πr^2 = (22/7) x 21^2 = 22/7 x 21 x 21 = 22 x 3 x 21

Area of circle with radius, 21cm = 1386 cm^2

2.2.1.1 If one side of a square is 4 cm, then what will be its area and perimeter?

4cm

4cm

Solution: Given,

Length of side of square = 4 cm

Area = $side^2$ = 4^2 = 4 x 4 = 16 cm^2

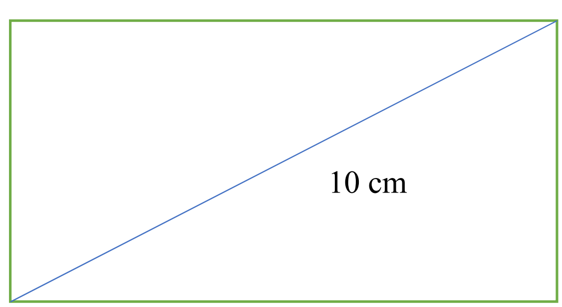
Perimeter of square = sum of all its sides

Since, all the sides of the square are equal, therefore;

Perimeter = 4+4+4+4 = 16 cm

2.2.1.1 Suppose a quadrilateral having a diagonal of length 10 cm, which divides the quadrilateral into two triangles and the heights of triangles with diagonals as the base, are 4 cm and 6 cm. Find the area of the quadrilateral.

4 cm



Solution: Given,

Diagonal, $d = 10$ cm

Height of one triangle, $h_1 = 4$cm

Height of another triangle, $h_2 = 6$cm

Area of quadrilateral = $\frac{1}{2} d(h_1+h_2) = \frac{1}{2} \times 10 \times (4+6) = 5 \times 10 = 50$ sq.cm.

4. A rhombus having diagonals of length 10 cm and 16 cm, respectively. Find its area.

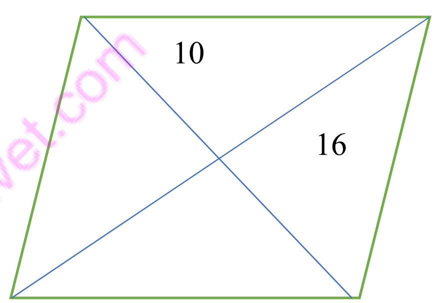
Solution: $d_1 = 10$ cm

$d_2 = 16$ cm

Area of rhombus = $\frac{1}{2} d_1 d_2$

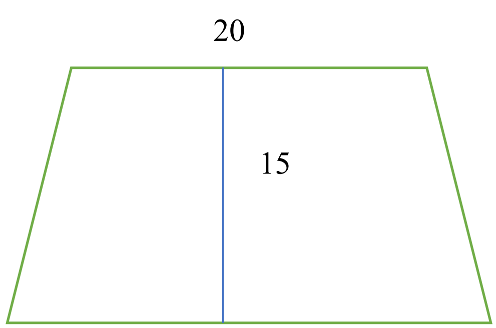
$A = \frac{1}{2} \times 10 \times 16$

$A = 80 \text{ cm}^2$



5. The area of a trapezium shaped field is 480 m², the distance between two parallel sides is 15 m and one of the parallel sides is 20 m. Find the other parallel side.

Solution: One of the parallel sides of the trapezium is $a = 20$ m, let another parallel side be b, height $h = 15$ m.



The given area of trapezium = 480 m^2

We know, by formula;

Area of a trapezium = ½ h (a+b)

480 = ½ (15) (20+b)

20 + b = (480×2)/15

b = 64 – 20 = 44 m

1.2.3.4. Learning Activities

With the trainers guidance, estimate the dimension of a given workshop and all the walk ways and find the area.

1.2.3.5. Self-Assessment

1. The height, length and width of a cuboidal box are 20 cm, 15 cm and 10 cm, respectively. Find its area.
2. If a cube has its side-length equal to 5cm, then its area is?
3. Find the height of a cylinder whose radius is 7 cm and the total surface area is 968 cm^2.
4. Find the height of a cuboid whose volume is 275 cm3 and base area is 25 cm^2.
5. A rectangular piece of paper 11 cm × 4 cm is folded without overlapping to make a cylinder of height 4 cm. Find the volume of the cylinder.

1.2.3.6. Tools, Equipment, Supplies and Materials

Tools/Equipment:	**Materials:**
• Scientific Calculators • Rulers • Pencils • Erasers • Computers with internet connection •	• Charts with presentations of data • Graph books • Text books •

1.2.3.7. References

Ministry of Education (2003). *Secondary Mathematics Students' Book One (3rd ed)*. Nairobi: Kenya Literature Bureau

Ministry of Education (2003). *Secondary Mathematics Students' Book Two (3rd ed)*. Nairobi: Kenya Literature Bureau

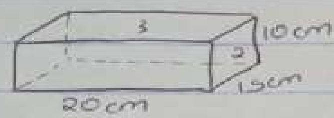
Ministry of Education (2003). *Secondary Mathematics Students' Book Three (3rd ed)*. Nairobi: Kenya Literature Bureau

John Bird (2007). *Engineering Mathematics (5th Ed).* MA: Elsevier ltd

John Bird (2005). *Basic Engineering Mathematics, (4th Ed).* MA: Elsevier ltd

1.2.3.8. Model Answers

1. The height, length and width of a cuboidal box are 20cm 15cm and 10cm. Find the area.



Cuboid = 6 faces (rectangular)

$A = L \times W$

Area 1 $= 20 \times 10 \times 2 = 400cm^2$

2 $= 15 \times 10 \times 2 = 300cm^2$

3 $= 20 \times 15 \times 2 = \underline{600cm^2}$

AREA $= 1300cm^2$

2. If a cube has its sidelength equal to 5cm, then its area is?

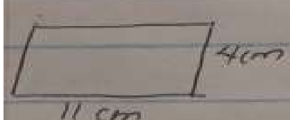
Solution

A cube is a composition of squares.

$A = L \times L$

$= 5 \times 5 \times 6\text{squares} = 150cm^2$

3. A rectangular piece of paper is folded without overlapping to make a cylinder of height 4cm. Find the volume.



length of paper is the circumference the base of the cylinder

circumference $(\pi D) = 11cm$ $\pi = 3.142$ or $22/7$

$\frac{3.142 \times D}{3.142} = \frac{11cm}{3.142} = 3.5cm$ (diametre)

V of a cylinder $= \pi r^2 h = \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 4$

$= 38.5cm^3$

1.2.4 Learning Outcome 4: Carry out basic mensuration

1.2.4.1. Introduction to the learning outcome

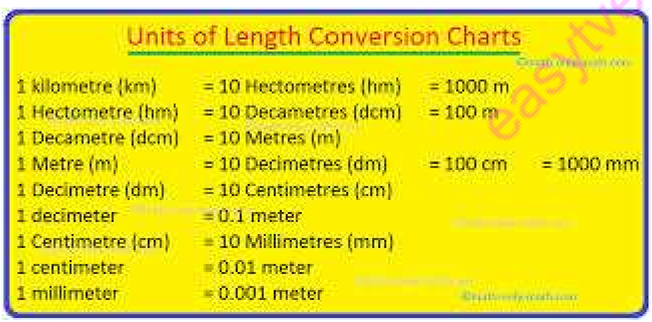
This unit describes the competencies required in applying basic mathematics on carrying out basic mensuration.

1.2.4.2. Performance Standard

1. Various ***units of measurements*** are identified based on the course requirements
2. Units are converted based on best practices
3. Perimeter and areas of regular ***figures*** are obtained based on known formulae
4. Area of irregular figures are obtained based on best practice
5. Volume and Surface area of solids are obtained based on given formulae.

1.2.4.3. Information Sheet

There are different units of measurement and subsequent conversion factors


Units of Length Conversion Charts

1 kilometre (km)	= 10 Hectometres (hm)	= 1000 m	
1 Hectometre (hm)	= 10 Decametres (dcm)	= 100 m	
1 Decametre (dcm)	= 10 Metres (m)		
1 Metre (m)	= 10 Decimetres (dm)	= 100 cm	= 1000 mm
1 Decimetre (dm)	= 10 Centimetres (cm)		
1 decimeter	= 0.1 meter		
1 Centimetre (cm)	= 10 Millimetres (mm)		
1 centimeter	= 0.01 meter		
1 millimeter	= 0.001 meter		

The different units of length and their equivalents are given here:

1 kilometer (km) = 10 Hectometers (hm) = 1000 m

1 Hectometer (hm) = 10 Decathletes (dcm) = 100 m

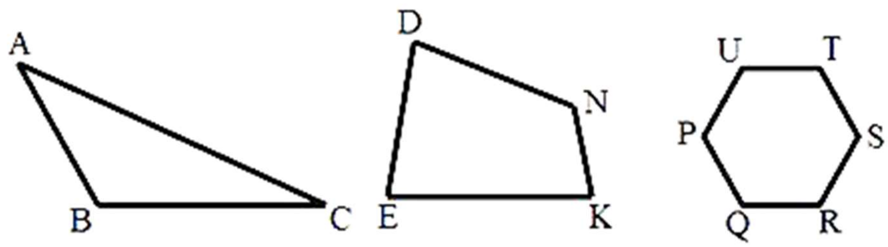
1 Demetre (dcm) = 10 Metres (m)

1 Meter (m) = 10 Decimeters (dm) = 100 cm = 1000 mm

1 Decimeter (dm) = 10 Centimetres (cm)

1 Centimeter (cm) = 10 Millimetres (mm)

The perimeter of a simple closed figure is the sum of the measures of line-segments which have surrounded the figure.



Perimeter of ΔABC = length (AB + BC + CA)

Perimeter of the quadrilateral DEKN = length (DE + EK + KN + ND)

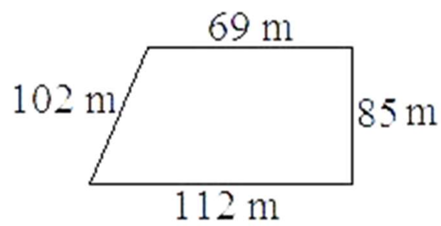
Perimeter of the hexagon PQRSTU = length (PQ + QR + RS + ST + TU + UP)

Calculation of perimeters will be applied in the determination of total length of pipework needed for a project.

Examples

1. Ken walks around a playground in his daily morning walk. How far does he walk every morning? The playground is of the shape of a quadrilateral having sides of length 112 m, 85 m, 69 m and 102 m.

   Solution:



Perimeter of the playground

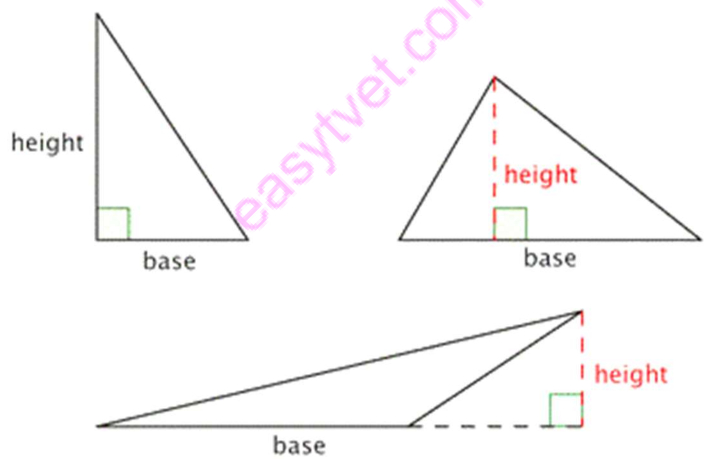
= 112 m + 85 m + 69 m + 102 m

= 368m

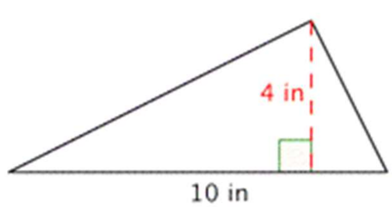
Ken walks 368 m around the playground, i.e. around the perimeter, every morning.

Area of Triangles and Trapezoids

$$A = \frac{1}{2}bh$$



2. **A triangle has a height of 4 inches and a base of 10 inches. Find the area.**



$A = \frac{1}{2}bh$ Start with the formula for the area of a triangle.

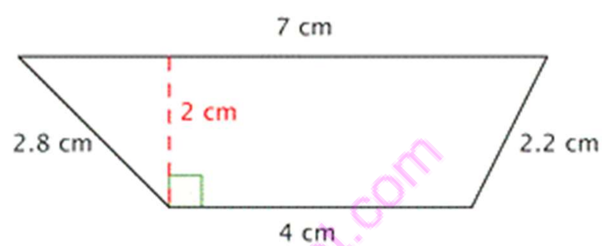
$A = \frac{1}{2} \cdot 10 \cdot 4$ Substitute 10 for the base and 4 for the height.

$A = \frac{1}{2} \cdot 40$

$A = 20$

A = 20 in^2

3. **Find the area of the trapezoid.**



Start with the formula for the area of a trapezoid

$A = \frac{(b_1 + b_2)}{2}h$

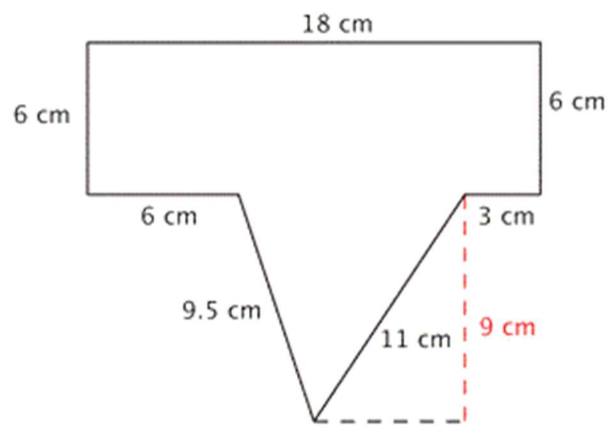
$A = \frac{(4+7)}{2} \cdot 2$

$A = \frac{11}{2} \cdot 2$

$A = 11$

The area of the trapezoid is 11 cm^2

4. **Find the area and perimeter of the polygon.**

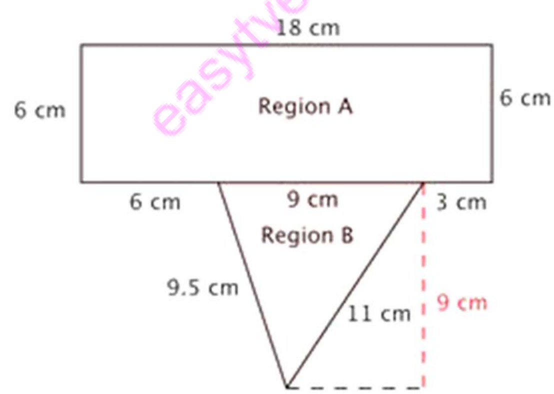


5. To find the perimeter, add together the lengths of the sides. Start at the top and work clockwise around the shape.

$P = 18 + 6 + 3 + 11 + 9.5 + 6 + 6$

$P = 59.5$ cm

6. To find the area, divide the polygon into two separate, simpler regions. The area of the entire polygon will equal the sum of the areas of the two regions.



Area of Polygon = (Area of A) + (Area of B)

Region A is a rectangle. To find the area, multiply the length (18) by the width (6).

$$\text{Area of Region A} = l \cdot w = 18 \cdot 6 = 108$$

Region B is a triangle

$$\text{Area of Region B} = \frac{1}{2} b \bullet h$$

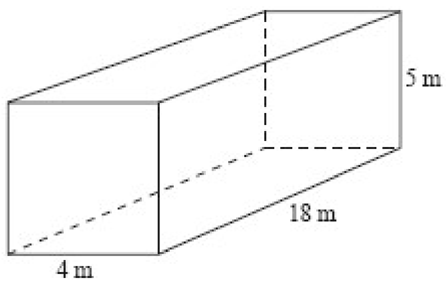
$$= \frac{1}{2} \bullet 9 \bullet 9$$

$$= \frac{1}{2} \bullet 81$$

$$= 40.5$$

$108 \text{ cm}^2 + 40.5 \text{ cm}^2 = 148.5 \text{ cm}^2$

7. Calculate the volume of the cuboid shown.



Volume = $4 \times 18 \times 5 = 360 \text{ m}^3$

(b)

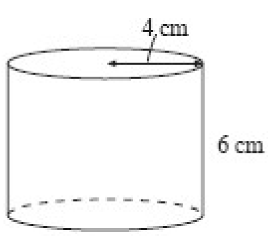
Calculate the surface area of the cuboid shown.

Surface area $= (2 \times 4 \times 18) + (2 \times 4 \times 5) + (2 \times 5 \times 18)$

$= 144 + 40 + 180$

$= 364 \text{ m}^2$

8. Calculate the volume and total surface area of the cylinder shown.



Volume $= \pi r^2 h = \pi \times 4^2 \times 6$

$= 96\pi$

$= 301.5928 \text{ cm}^3$

$= 302 \text{ cm}^3$

Area of curved surface = $2\pi rh = 2 \times \pi \times 4 \times 6$

= 48π

= $150.7964 cm^2$

Area of each end = $\pi r^2 = \pi \times 4^2$

= 16π

= $50.2654 cm^2$

Total surface area = $150.7964 + (2 \times 50.2654)$

= $251.327 cm^2$

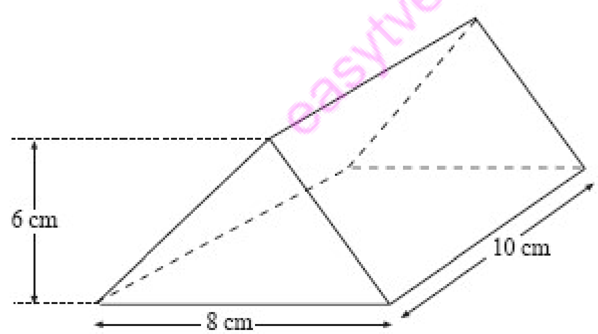
= $251\ cm^2$

Calculation of Areas will be applied during site clearance, site/ trench excavation and laying of drainage pipes.

1.2.4.4. Learning Activities

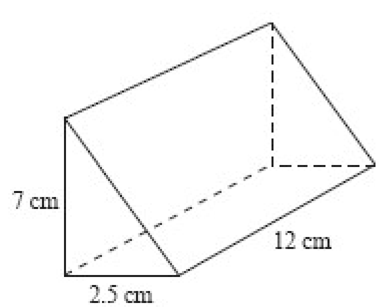
In groups of two or more discuss the following

1. Calculate the *volume* of this prism.

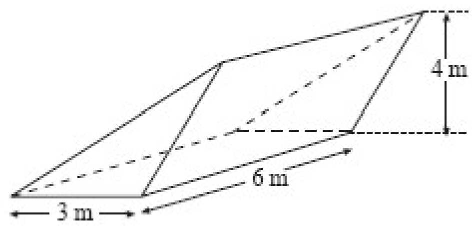


2. Calculate the *volume* of each of the following prisms:

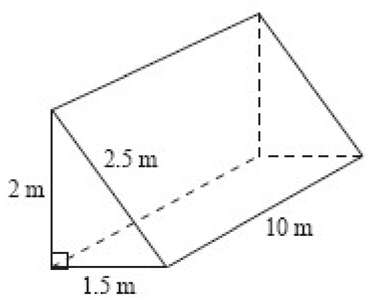
(a)



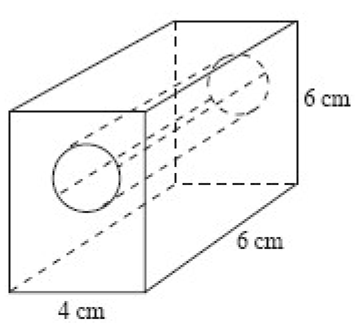
(b)



3.Calculate the volume and surface area of the following prism:

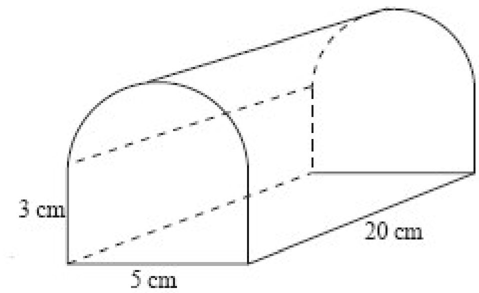


4. The diagram shows a wooden block that has had a hole drilled in it. The diameter of the hole is 2 cm. Calculate the volume of this solid, giving your answer correct to 2 decimal places.



5. The diagram shows a prism.

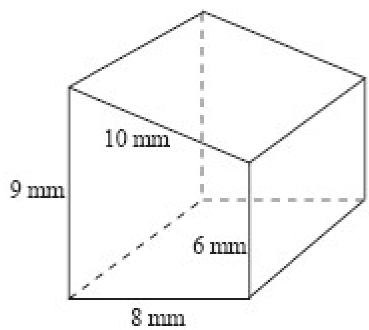
The cross-section of the prism consists of a rectangle and a semicircle.



(a) Calculate the *volume* of the prism. Give your answer to the nearest cm^3.

(b) Calculate the *total surface area* of the prism. Give your answer to the nearest cm^2.

6. The volume of the prism shown is 720 mm^3.



(a) Determine the *length* of the prism.

(b) Calculate the *surface area* of the prism.

7. A cylinder has a diameter of 12 cm and a curved surface area of 132π or 415 cm^2 (to 3 significant figures).

(a) Determine the *height* of the cylinder.

(b) Calculate the *volume* of the cylinder, giving your answer to the nearest cm^3.

1.2.4.5. Self-Assessment

1. Find the total surface area of the cylinder, whose radius is 5cm and height is 10cm?

2. What is the volume of a cylindrical shape water container that has a height of 7cm and diameter of 10cm?

3. Calculate the cost required to paint a container which is in the shape of a right circular cylinder having a base radius of 7 m and height 13 m. If the painting cost of the container is Ksh 2.5/m2. (Take $\pi = 22/7$)

4. Find the total surface area of a container in cylindrical shape whose diameter is 28 cm and height is 15 cm.

1.2.4.6. Tools, Equipment, Supplies and Materials

Tools/Equipment:	Materials:
• Scientific Calculators • Rulers • Pencils • Erasers • Computers with internet connection	• Charts with presentations of data • Graph books • Text books

1.2.4.7. References

Ministry of Education (2003). *Secondary Mathematics Students' Book One (3rd ed)*. Nairobi: Kenya Literature Bureau

Ministry of Education (2003). *Secondary Mathematics Students' Book Two (3rd ed)*. Nairobi: Kenya Literature Bureau

Ministry of Education (2003). *Secondary Mathematics Students' Book Three (3rd ed)*. Nairobi: Kenya Literature Bureau

John Bird (2007). *Engineering Mathematics (5th Ed).* MA: Elsevier ltd

John Bird (2005). *Basic Engineering Mathematics, (4th Ed).* MA: Elsevier ltd

1.2.4.8. Model Answers

1. Surface Area of a cylinder $= \pi r^2 + \pi r^2 + 2\pi r h$

Top Bottom Curved surface

$= 3.142 \times 5^2 + 3.142 \times 5^2 + 2 \times 3.142 \times 5 \times 10$

$78.55 + 78.55 + 314.2$

$= 471.3\text{ cm}^2$

2. Vol of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 5 \times 5 \times 7$

$= 550\text{cm}^3$

3. Calculate cost required to paint

SA of cylinder container $= \pi r^2 + 2\pi r h$

Because the top is hollow since it is a container

$= \frac{22}{7} \times 7 \times 7 + 2 \frac{22}{7} \times 7 \times 13$

$154 + 572 = 726\text{cm}^2$

Cost of painting $= 2.50$ sh per m^2

$? = 726$

$= 726 \times 2.50 =$ Sh 957.

4. SA of cylindrical container

$= \pi r^2 + 2\pi r h$

$= \frac{22}{7} \times 14 \times 14 + 2 \times \frac{22}{7} \times 14 \times 15$

$616 + 1320$

$= 1936\text{cm}^2$

1.2.5 Learning Outcome 5: Apply Basic Statistics

1.2.5.1 Introduction to the learning outcome

This unit describes the competencies required in applying basic mathematics on basic statistics.

1.2.5.2 Performance Standard

1. Grouped and ungrouped data is identified and interpreted based on given sample
2. Ungrouped data is organized based on the concept
3. Data is represented in frequency tables based on the concept
4. The median, mode and mean of grouped and ungrouped data is calculated based on the concept
5. Data is presented in a chart form based on the concept

1.2.5.3 Information Sheet

Statistics refers to the science of collecting and analyzing numerical data in large quantities for the purpose of proportioning into a representative sample.

There are different methods of data presentation: Frequency Tables, Pie-charts, Bar charts, Line graphs, Histograms, Frequency polygons e.t.c.

Data representation in frequency tables

A **frequency table** is constructed by arranging collected data values in ascending order of magnitude with their corresponding frequencies.

Example 1

The marks awarded for an assignment set for a Year 4 class of 20 students were as follows:

6	7	5	7	7	8	7	6	9	7
4	10	6	8	8	9	5	6	4	8

Present this information in a frequency table.

Solution

To construct a frequency table, we proceed as follows:

Step 1:

Construct a table with three columns. The first column shows what is being arranged in ascending order from 4 in the first column to 10 as shown below.

Mark	Tally	Frequency
4		
5		
6		
7		
8		
9		
10		

Step 2:

Go through the list of marks. The first mark in the list is 6, so put a tally mark against 6 in the second column. The second mark in the list is 7, so put a tally mark against 7 in the second column. The third mark in the list is 5, so put a tally mark against 5 in the third column as shown below.

Mark	Tally	Frequency
4		
5	|	
6	|	
7	|	
8		
9		
10		

We continue this process until all marks in the list are tallied.

Step 3:

Count the number of tally marks for each mark and write it in third column. The finished frequency table is as follows:

Mark	Tally	Frequency
4	II	2
5	II	2
6	IIII	4
7	~~IIII~~	5
8	IIII	4
9	II	2
10	I	1

For Grouped Data;

When the set of data values are spread out, it is difficult to set up a frequency table for every data value as there will be too many rows in the table. So we group the data into class intervals /groups to help us organize, interpret and analyze the data.

Example 1

The number of calls from motorists per day for roadside service was recorded for the month of December 2003. The results were as follows:

28	122	217	130	120	86	80	90	120	140
70	40	145	187	113	90	68	174	194	170
100	75	104	97	75	123	100	82	109	120
81									

Set up a frequency table for this set of data values.

Solution:

To construct a frequency table, we proceed as follows:

Smallest data value = 28

Highest data value = 217

$$\begin{aligned} \text{Difference} &= \text{Highest value} - \text{Smallest value} \\ &= 217 - 28 \\ &= 189 \end{aligned}$$

Let the width of the class interval be 40.

$$\therefore \text{ Number of class intervals} = \frac{189}{40} = 4.7 = 5 \qquad \{\text{Round up to the next integer}\}$$

There are at least 5 class intervals. This is reasonable for the given data.

Step 1: Construct a table with three columns, and then write the data groups or class intervals in the first column. The size of each group is 40. So, the groups will start at 0, 40, 80, 120, 160 and 200 to include all of the data. Note that in fact we need 6 groups (1 more than we first thought).

Class interval	Tally	Frequency
0 - 39		
40 - 79		
80 - 119		
120 - 159		
160 - 199		
200 - 239		

Step 2: Go through the list of data values. For the first data value in the list, 28, place a tally mark against the group 0-39 in the second column. For the second data value in the list, 122, place a tally mark against the group 120-159 in the second column. For the third data value in the list, 217, place a tally mark against the group 200-239 in the second column.

Class interval	Tally	Frequency
0 - 39	I	
40 - 79		
80 - 119		
120 - 159	I	
160 - 199		
200 - 239	I	

We continue this process until all of the data values in the set are tallied.

Step 3: Count the number of tally marks for each group and write it in the third column. The finished frequency table is as follows:

Class interval	Tally	Frequency
0 - 39	I	1
40 - 79	卌	5
80 - 119	卌 卌 II	12
120 - 159	卌 III	8
160 - 199	IIII	4
200 - 239	I	1
	Sum =	31

Example 2

1. The data given below refer to the gain of each of a batch of 40 transistors, expressed correct to the nearest whole number. Form a frequency distribution for these data having seven classes

   81 83 87 74 76 89 82 84 86 76 77 71 86 85 87 88
   84 81 80 81 73 89 82 79 81 79 78 80 85 77 84 78
   83 79 80 83 82 79 80 77

Class	**Tally**
70–72	1
73–75	11
76–78	1111 11
79–81	1111 1111 11
82–84	1111 1111
85–87	1111 1
88–90	111

Class	mid-point	Frequency
70–72	71	1
73–75	74	2
76–78	77	7

79–81	80	12
82–84	83	9
85–87	86	6
88–90	89	3

Now, using the above data, you can present the data using histograms, bar graphs and others.

Measures of Central Tendency

Median

The median is the middle score for a set of data that has been arranged in order of magnitude. (from the largest to smallest or vice versa). In order to calculate the median, suppose we have the data below:

65 55 89 56 35 14 56 55 87 45 92

We first need to rearrange that data into order of magnitude (smallest first):

14 35 45 55 55 **56** 56 65 87 89 92

Our median mark is the middle mark - in this case, 56 (highlighted in bold). It is the middle mark because there are 5 scores before it and 5 scores after it.

This works fine when you have an odd number of scores, but what happens when you have an even number of scores? What if you had only 10 scores? Well, you simply have to take the middle two scores and average the result. So, if we look at the example below:

65 55 89 56 35 14 56 55 87 45

We again rearrange that data into order of magnitude (smallest first):

14 35 45 55 **55** **56** 56 65 87 89

Only now we have to take the 5th and 6th score in our data set and average them to get a median of 55.5.

1.2.5.4 Learning Activities

Take a sample of 30 different persons and collect data on : their age, gender, favourite sports team, favorite food, favorite day of the week.

Represent this data in the form of Pie Charts, Frequency Tables and Pictograms.

1.2.5.5 Self-Assessment

1. The data below shows the mass of 40 students in a class. The measurement is to the nearest kg.

55	70	57	73	55	59	64	72
60	48	58	54	69	51	63	78
75	64	65	57	71	78	76	62
49	66	62	76	61	63	63	76
52	76	71	61	53	56	67	71

Construct a frequency table for the data using an appropriate scale.

2. Find the mean of the following data.

(a) 9, 7, 11, 13, 2, 4, 5, 5

(b) 16, 18, 19, 21, 23, 23, 27, 29, 29, 35

(c) 2.2, 10.2, 14.7, 5.9, 4.9, 11.1, 10.5

(d) $1\frac{1}{4}$, $2\frac{1}{2}$, $5\frac{1}{2}$, $3\frac{1}{4}$, $2^{1}/2$

1.2.5.6 Tools, Equipment, Supplies and Materials

Tools/Equipment:	**Materials:**
• Scientific Calculators • Rulers • Pencils • Erasers • Computers with internet connection	• Charts with presentations of data • Graph books • Text books

1.2.5.7 References

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1.2.5.8 Model Answers

1.**Step 1**: Find the range.

The range of a set of numbers is the difference between the least number and the greatest number in the set.

In this example, the greatest mass is 78 and the smallest mass is 48. The range of the masses is then 78 – 48 = 30. The scale of the frequency table must contain the range of masses.

Step 2: Find the intervals
the **intervals** separate the scale into equal parts.
We could choose intervals of 5. We then begin the scale with 45 and end with 79

Step 3: Draw the frequency table using the selected scale and intervals.

Mass (kg)	Frequency
45 – 49	2
50 – 54	4
55 – 59	7
60 – 64	10
65 – 69	4
70 – 74	6
75 – 79	7

2.

(a) 7
(b) 24
(c) 8.5
(d) 3

1.2.6 Learning Outcome 6: Plot simple graphs

1.2.6.1 Introduction to the learning outcome

This unit describes the competencies required in applying basic mathematics it involves plotting simple graphs.

1.2.6.2 Performance Standard

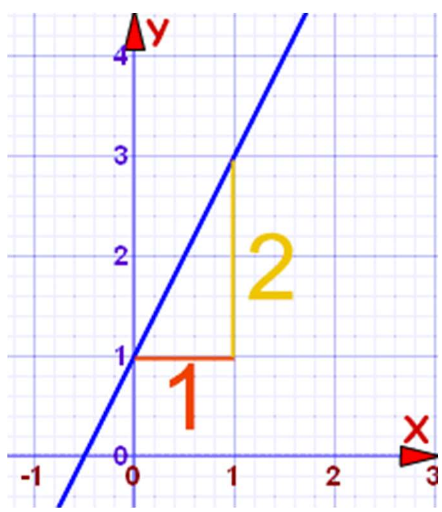
1. A ***graph*** is plotted for given set of data based on data
2. Information from a given graph is interpreted based on data

1.2.6.3 Information Sheet

We've different types of graphs that are drawn for data representation like the line graphs, bar graphs, histograms etc

We plot graphs of equations of different lines of the y=mx +c where m is the gradient of the line and c is the y-intercept.

Example 1



m = **21** = 2

b = 1 *(value of **y** when x=0)*

So: $y = 2x + 1$

With that equation you can now choose any value for x and find the matching value for y

For example, when x is 1:

$$y = 2\times\mathbf{1} + 1 = 3$$

Check for yourself that x=1 and y=3 is actually on the line.

Or we could choose another value for x, such as 7:

$$y = 2\times\mathbf{7} + 1 = 15$$

And so when x=7 you will have y=15

Positive or Negative Slope

Going from left-to-right, the cyclist has to **P**ush on a **P**ositive Slope:

Example 2

If $m = \frac{3}{1} = -3$

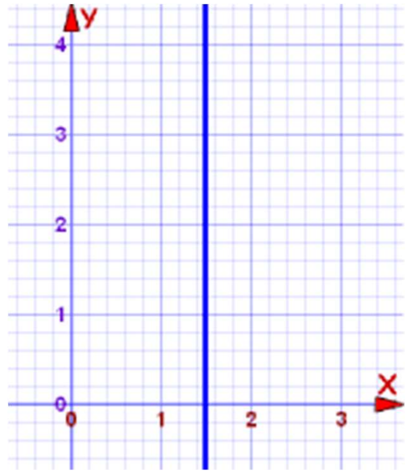
b = 0

This gives us $\mathbf{y = -3x + 0}$

We do not need the zero!

So: $y = -3x$

Example 3: Vertical Line



What is the equation for a vertical line?

The slope is **undefined** ... and where does it cross the Y-Axis?

In fact, this is a **special case**, and you use a different equation, not "**y**=...", but instead you use "**x**=...".

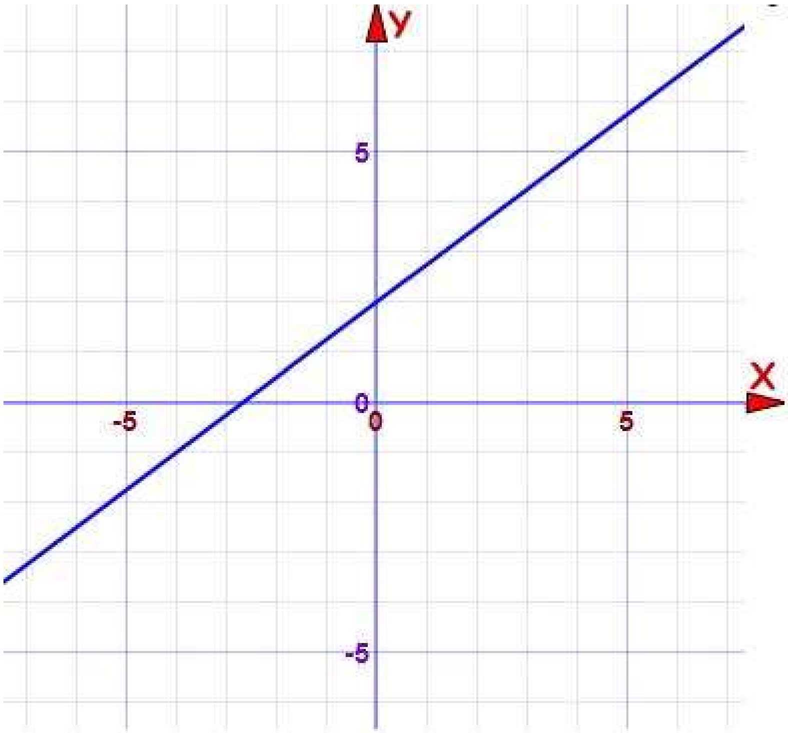
Like this:

x = 1.5

Every point on the line has **x** coordinate **1.5**,

that is why its equation is **x = 1.5**

What is the equation of the following line?



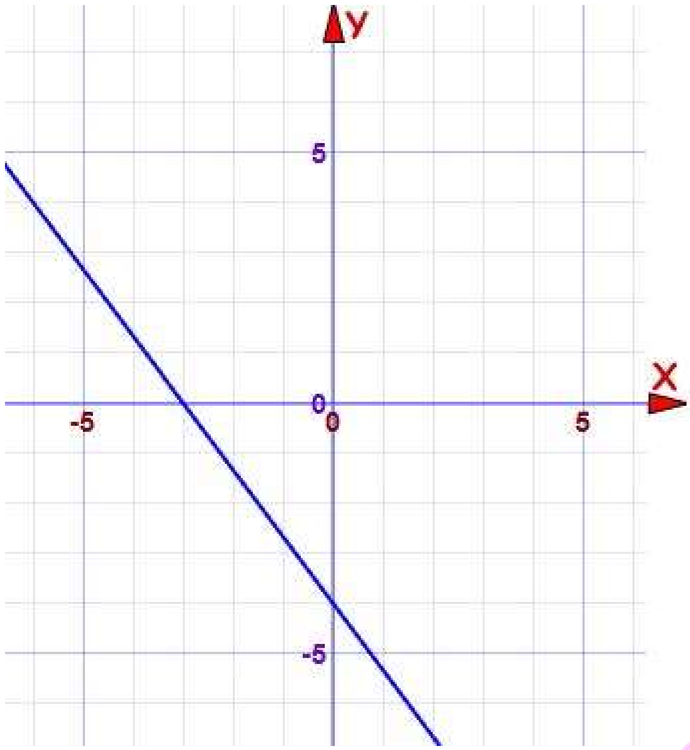
From the graph we can see m = rise/run = 3/4 = 0.75

and b is the y coordinate of the y-intercept, so b = 2

Substitute into y = mx + b

Therefore y = 0.75x + 2

What is the equation of the straight line shown in the diagram?



First find the Slope (also called the Gradient):

Slope = (change in y) / (change in x) = -4/3

Now find the y-intercept:

y-intercept = (0, -4)

So b = -4

The general equation of a straight line is y = mx + b

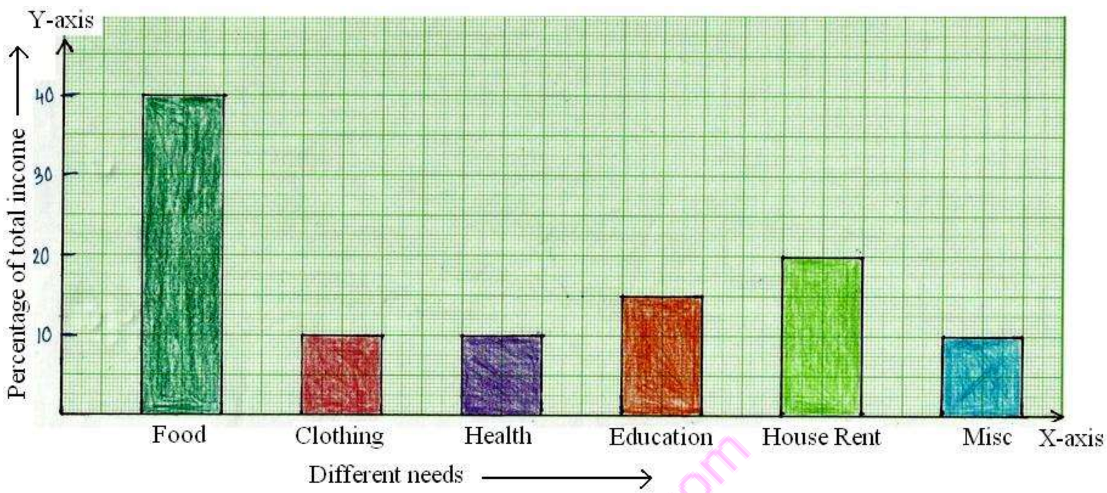
So the given line has equation

$y= -\frac{-4}{3}\,x -4$

The percentage of total income spent under various heads by a family is given below.

Different Heads	Food	Clothing	Health	Education	House Rent	Miscellaneous
% Age of Total Number	40%	10%	10%	15%	20%	5%

Represent the above data in the form of bar graph.



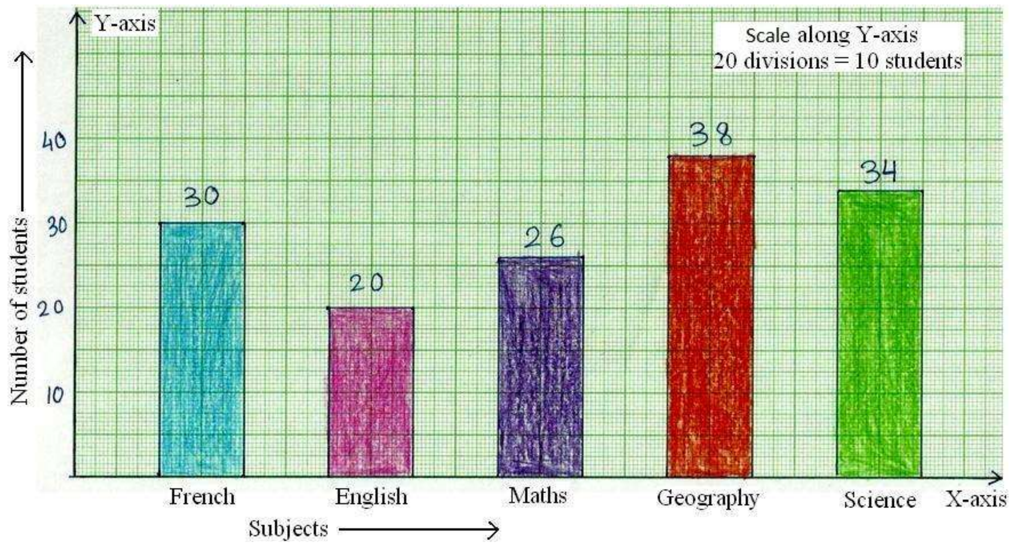
2. 150 students of class VI have popular school subjects as given below:

Subject	French	English	Maths	Geography	Science
Number of Students	30	20	26	38	34

Draw the column graph/bar graph representing the above data.

Solution:

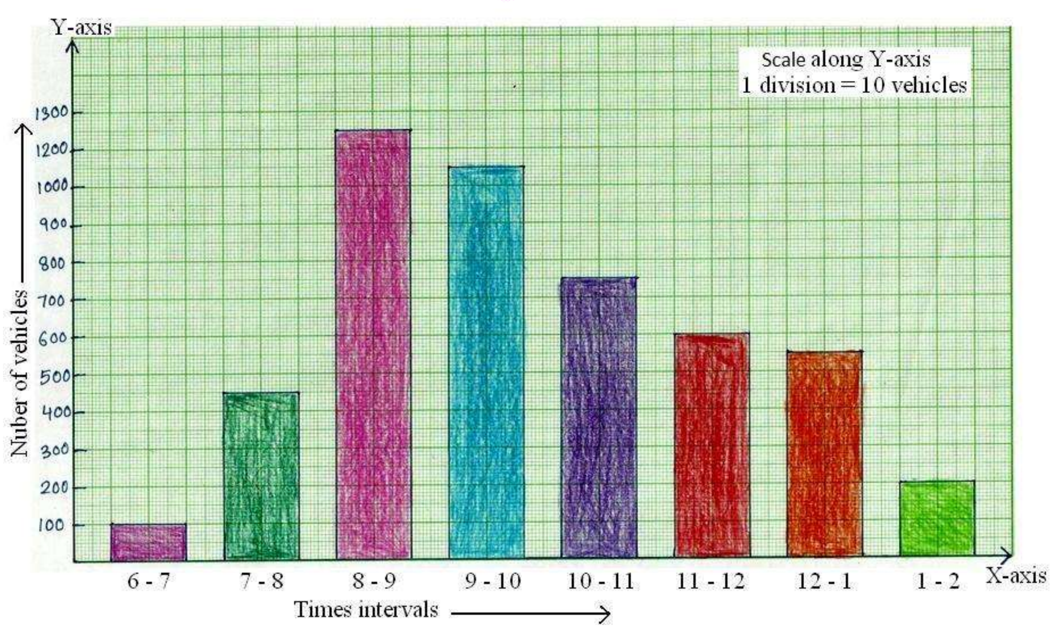
Take the subjects along **x-axis**, and the number of students along **y-axis**



Bar graph gives the information of favorite subjects of 150 students.

3. The vehicular traffic at a busy road crossing in a particular place was recorded on a particular day from 6am to 2 pm and the data was rounded off to the nearest tens.

Time in Hours	6 - 7	7 - 8	8 - 9	9 – 10	10 - 11	11 - 12	12 - 1	1 - 2
Number of Vehicles	100	450	1250	1050	750	600	550	200



Bar graph gives the information of number of vehicles passing through the crossing during different intervals of time.

1.2.6.4 Learning Activities

Graphs can also be used to find solutions of simultaneous equations. Plot the following pairs and give solutions of;

1. $5x + 3y = 41$

   $2x + 3y = 20$

2. $5x + y = 11$

   $3x - y = 9$

3. $x + 7y = 64$

   $x + 3y = 28$

4. $4x - 4y = 24$

   $x - 4y = 3$

1.2.6.5 Self-Assessment

1. ((a) Complete the table of values for $y = 2x + 5$
   (b) On the grid, draw the graph of $y = 2x + 5$ for the values from $x = -2$ to $x = 2$

2. Complete the table of values for $y = 2x – 3$

   (b) On the grid, draw the graph of $y = 2x – 3$

   (c) Use your graph to find
   (i) the value of y when $x = -1.5$
   (ii) the value of x when $y = 6$

1.2.6.6 Tools, Equipment, Supplies and Materials

Tools/Equipment:	Materials:
• Scientific Calculators • Rulers • Pencils • Erasers • Computers with internet connection	• Charts with presentations of data • Graph books • Text books

1.2.6.7 References

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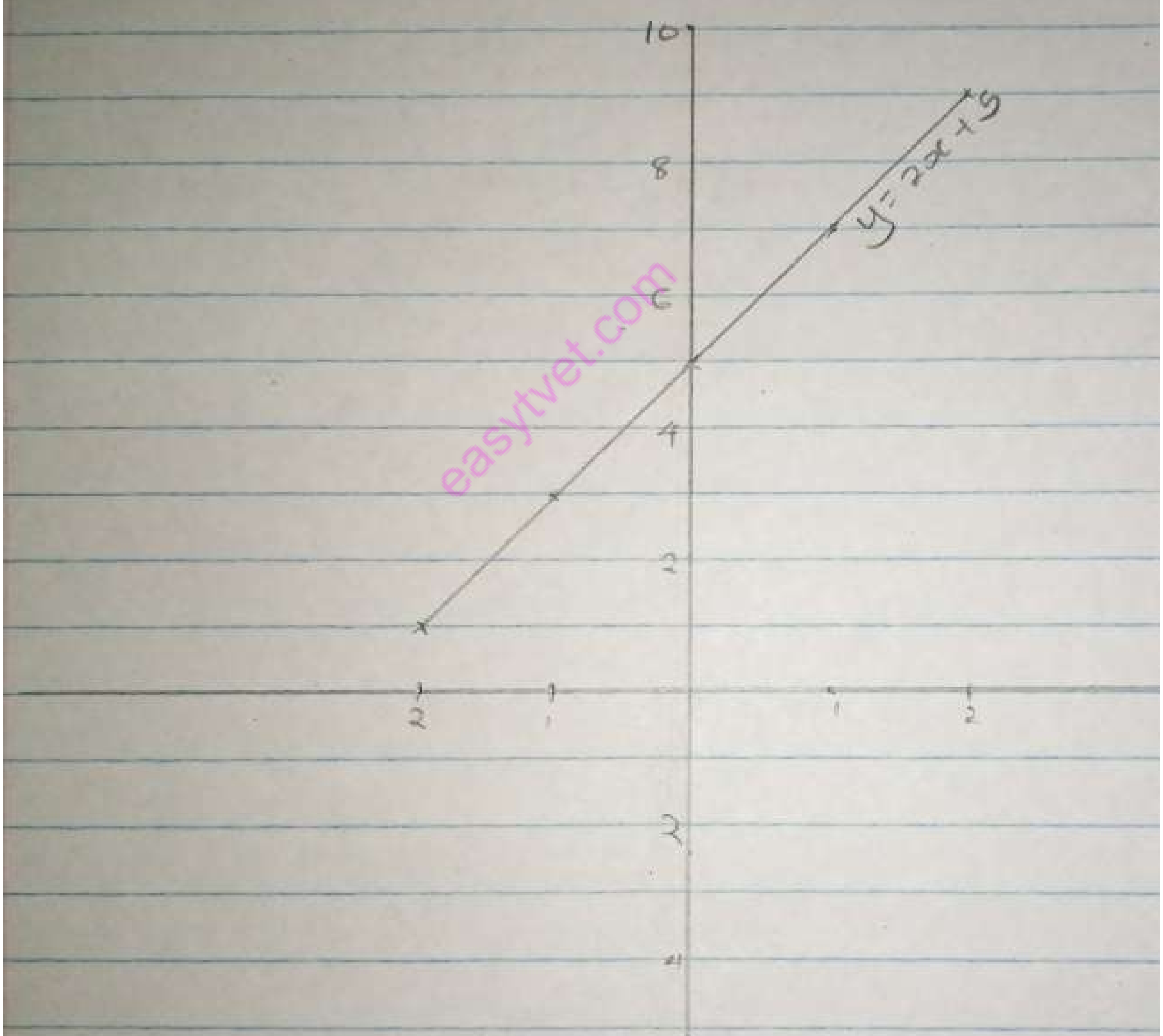
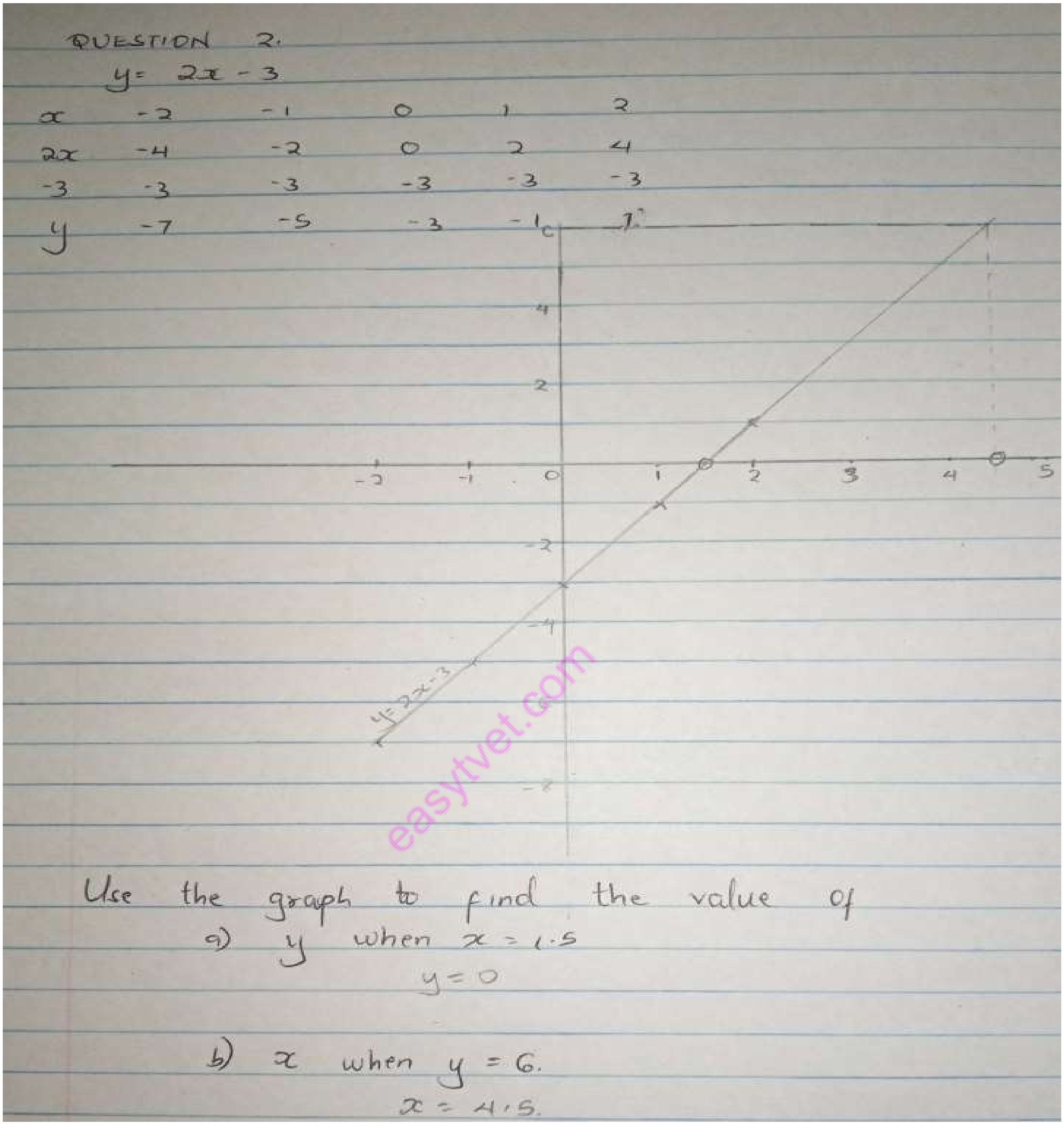
1.2.6.8 Model Answers

QUESTION ONE $y = 2x + 5$

x	-2	-1	0	1	2
$2x$	-4	-2	0	2	4
$+5$	5	5	5	5	5
y	1	3	5	7	9





1.2.7 Learning Outcome 7: Apply Indices and Logarithms

1.2.7.1 Introduction to the learning outcome

This unit describes the competencies required in applying basic mathematics on indices, logarithms and ratio.

1.2.7.2 Performance Standard

1. Converted numbers from one base to another
2. Applied the laws of indices in solving exponential equations
3. Applied the laws of logarithms in solving logarithmic equations

1.2.7.3 Information Sheet

Laws of indices

1. ***Law of Multiplication*** $a^n x a^m = a^{n+m}$
2. ***Law of division*** $a^n \div a^m = a^{n-m}$
3. ***Law of Power to Power*** $(a^n)^m = a^{nm}$

Examples

- $2^4 \times 2^8 = 2^{4+8} = 2^{12}$
- $5^4 \times 5^{-2} = 5^{4+-2} = 5^2$
- $3^9 \div 3^4 = 3^{9-4} = 3^5$
- $7^2 \div 7^5 = 7^{-3}$
- $2^{-3} = \frac{1}{2^{-3}} = \frac{1}{8}$
- $16^{1/2} = \sqrt{16} = 4$
- $8^{2/3} = (\sqrt[3]{8})^2 = 4$
- $5^0 = 1$

Laws of logarithms

I. $\log (A \times B) = \log A + \log B$

II. $\log \left(\frac{A}{B}\right) = \log A - \log B$

III. $\log A^n = n\log A$

Examples

1. **Simplify**; log 64 − log 128 + log 32

$64 = 2^6$, $128 = 2^7$, and $32 = 2^5$,

Hence log 64 − log 128 + log 32

$= \log 2^6, - \log 2^7 + \log 2^5,$

$= 6 \log 2 - 7 \log 2 + 5 \log 2$

by the third law of logarithms

$= 4 \log 2$

2. **Evaluate** $\frac{\log 25 - \log 125 + \frac{1}{2}\log 625}{3 \log 5}$

$\frac{\log 5^2 - \log 5^3 + \frac{1}{2}\log 5^4}{3 \log 5}$

$\frac{2\log 5 - 3\log 5 + 2\log 5}{3 \log 5}$

$\frac{1 \log 5}{3 \log 5} = \frac{1}{3}$

3. **Solve the equation**:

$\log (x - 1) + \log (x + 1) = 2 \log (x + 2)$

$\log (x - 1) + \log (x + 1) = \log (x - 1)(x + 1)$

from the first law of logarithms

$= \log (x^2 - 1)$

$2 \log (x + 2) = \log (x + 2)^2 = \log (x^2 + 4x + 4)$

Hence if $\log (x^2 - 1) = \log (x^2 + 4x + 4)$

then $x^2 - 1 = x^2 + 4x + 4$

$-1 = 4x + 4$

$-5 = 4x$

$x = -\frac{5}{4}$

Examples on Logarithms

1. **Express $5^3 = 125$ in logarithm form.**

Solution:

$5^3 = 125$

As we know,

$a^b = c \Rightarrow \log_a c = b$

Therefore;

$\log_5 125 = 3$

2. **Express $\log_{10} 1 = 0$ in exponential form.**

Solution:

Given, $\log_{10} 1 = 0$

By the rule, we know;

$\log_a c = b \Rightarrow a^b = c$

Hence,

$10^0 = 1$

3. **Find the log of 32 to the base 4.**

Solution: $\log_4 32 = x$

$4^x = 32$

$(2^2)^x = 2x2x2x2x2$

$2^{2x} = 2^5$

2x=5

x=5/2=2.5

4. **Find x if $\log_5(x-7)=1$.**

Solution: Given,

$\log_5(x-5) = 1$

Using logarithm rules, we can write;

$5^1 = x-7$

5 = x-7

x=5+7

x=12

5. **Solve for x if $\log(x-1)+\log(x+1)=\log_2 1$**

Solution: $\log(x-1)+\log(x+1)=\log_2 1$

$\log(x-1)+\log(x+1)=0$

$\log[(x-1)(x+1)]=0$

Since, log 1 = 0

(x-1)(x+1) = 1

$x^2-1=1$

$x^2=2$

$x=\pm\sqrt{2}$

Since, log of negative number is not defined.

Therefore, $x=\sqrt{2}$

3. **Find the value of x, if $\log_{10}(x-10)=1$.**

Solution: Given, $\log_{10}(x-10)=1$.

$\log_{10}(x-10) = \log_{10}10$

x-10 = 10

x=10+10

x=20

1.2.7.4 Learning Activities

In your groups, perform the following;

In Problems 1 to 11, evaluate the given expression:

1. $\log_{10} 10\,000$
2. $\log_2 16$
3. $\log_5 125$
4. $\log_2 \frac{1}{8}$
5. $\log_8 2$
6. $\log_7 343$
7. $\lg 100$
8. $\lg 0.01$
9. $\log_4 8$
10. $\log_{27} 3$
11. $\ln e^2$

In Problems 12 to 18 solve the equations:

12. $\log_{10} x = 4$
13. $\log x = 5$
14. $\log_3 x = 2$
15. $\log_4 x = -2\frac{1}{2}$
16. $\lg x = -2$
17. $\log_8 x = -\frac{4}{3}$
18. $\ln x = 3$

In Problems 19 to 22 write the given expressions in terms of log 2, log 3 and log 5 to any base:

19. $\log 60$
20. $\log 300$
21. $\log\left(\frac{16 \times \sqrt[4]{5}}{27}\right)$
22. $\log\left(\frac{125 \times \sqrt[4]{16}}{\sqrt[4]{81^3}}\right)$

Simplify the expressions given in Problems 23 to 25:

23. $\log 27 - \log 9 + \log 81$
24. $\log 64 + \log 32 - \log 128$
25. $\log 8 - \log 4 + \log 32$

Evaluate the expressions given in Problems 26 and 27:

26. $\dfrac{\frac{1}{2}\log 16 - \frac{1}{3}\log 8}{\log 4}$
27. $\dfrac{\log 9 - \log 3 + \frac{1}{2}\log 81}{2\log 3}$

Solve the equations given in Problems 28 to 30:

28. $\log x^4 - \log x^3 = \log 5x - \log 2x$
29. $\log 2t^3 - \log t = \log 16 + \log t$
30. $2\log b^2 - 3\log b = \log 8b - \log 4b$

1.2.7.5 Self-Assessment

1. $\text{Log}_6(216)$ + [log(42) - log(6)] / log(49)
2. $(3^{-1} - 9^{-1}) / 6)^{1/3}$
3. $(\log_x a)(\log_a b)$

4. $2 = \log_a e$

1.2.7.6 Tools, Equipment, Supplies and Materials

Tools, Equipment, Supplies and Materials

Tools/Equipment:	**Materials:**
• Scientific Calculators • Rulers • Pencils • Erasers • Computers with internet connection	• Charts with presentations of data • Graph books • Text books

1.2.7.7 References

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1.2.7.8 Model Answers

1.

a. $= \log_6(6^3) + \log(42/6) / \log(7^2)$
$= 3 + \log(7) /2 \log(7) = 3 + 1/2 = 7/2$

b. $= ((1/3 - 1/9) / 6)^{1/3}$
$= ((6 / 27) / 6)^{1/3} = 1/3$

c. $= \log_x a\ (\log_x b / \log_x a) = \log_x b$

2. $a^2 = e$

$\ln(a^2) = \ln e$

$2 \ln a = 1$

$a = e^{1/2}$

1.2.8 Learning Outcome 8: Perform business calculations

1.2.8.1 Introduction to the learning outcome

This unit describes the competencies required in applying basic mathematics on business calculations.

1.2.8.2 Performance Standard

1. Converted one currency to another
2. Calculated exchange rates
3. Calculated income
4. Calculated of taxes
5. Calculated average sales

1.2.8.3 Information Sheet

Business calculations refers to calculations and estimations related to transactions done in a Business. The medium of a Business Transaction is " The Currency". A countrys currency cannot be used to transact in another country. For international trading, currencies will have different values in relation to each other. These rates are not fixed and will fluctuate. When changing the Kenyan currency to a foreign currency, the bank sells to us and when changing a foreign currency to a Kenyan currency, the bank buys from us.

Use the table to answer the questions that follows;

Currency	Buying	Selling
1 US Dollar **$**	**78.4133**	78.4744
1 Sterling Pound **£**	**114.1616**	114.3034
1 Euro **€**	**73.4226**	73.52953
1 South African Rand	**7.8842**	7.9141
1 UAE Dirham	**21.3480**	21.3670
1 Indian Rupee	**1.5986**	1.5999

1. A tourist visited Kenya and exchanged **500** Euros for his use while in the country how much Kenya shillings did he get.

Solution

1 € = 73.4226 Ksh

500 **€ =?**

500 × 73.4226 = 36,711.30

2. An exporter bought sterling pounds equivalent to **ksh 500,000.** After settling bills worth **1000** Sterling Pound **£** He exchanged the balance for Euros. If he purchased goods worth **100** Euros, calculate his balance in Kenya shillings.

Ksh 114.3034 = 1 **€ hence**

Ksh 500,000 ÷114.3034= £ 4374.3230

£ 4374.3230- £ 1000 = £ 3374.3230

1**£** = 114.1616

3374.3230 = ?

3374.3230× 114.1616 = **ksh** 385218.1126

To Euros

ksh 73.52953 = 1 **€**

385,218.1126 **= ?**

(385,218.1126× 1) / 73.52953 = **€** 5238.9579

5238.9579 – 100 = 5138.9579

1 **€ = ksh** 73.4226

5138.9579 = ?

5138.9579 ×73.4226 = Ksh 384,657.9108

INCOME TAX

Tax on personal income is known as income tax. **Taxable income** is the amount on which tax is levied and includes salary plus allowances. Every employee in Kenya is entitled to an automatic personal relief of **ksh 1056** per month.

To find the tax payable by an individual we subdivide the income into tax brackets /slabs corresponding to the table of taxation in use such as

K£ per annum	Rate in ksh / £
1 – 5808	2
5809 – 11280	3
11281 – 16572	4
16573 – 22224	5
Excess over 22224	6

Example

Alison has a taxable income of k£ **18460** p.a. calculate how much **tax** she should pay per month if she claims personal relief.

Solution

Divide the income into tax slabs as follows

18460-(It's between 16573 – 22224)

1st 5808× 2 = Ksh11616

(11280-5808=5472)

2nd 5472 ×3 = Ksh 16416

(16572-11280=5472)

3rd 5472 × 4 = Ksh 21888

Remaining

18460-16572=1888

1888 ×5 = Ksh 9440

Gross tax per year **Ksh59,360 -**

Less personal relief 1056 × 12 = Ksh 12672

Net tax per year **Ksh46688**

Per month

46688÷12= Ksh 3890.67

Example 1

Mrs. Otieno earns a monthly salary of ksh **12,400**, a house allowance of ksh **8000** per month and a medical allowance of ksh **2,400** per month she claims a personal relief and contributes ksh **1000** towards a pension scheme. Calculate her net income.

Solution

Taxable income = salary plus allowances = 12,400 +8,000+2,400 = **22,800.**

(22,800×12) ÷20 = £13,680p.a.

Divide it into slabs

13680---

1st 5808 ×2 = ksh11,616

7872-

2nd 5472×3 = ksh16,414

Remainder 2400×4 = ksh 9,600

Gross tax…………………….... ksh 37,630

Less personal **relief ……………(1056× 12)=**ksh12,672

Net tax ………………………...ksh 24,958

Per month=24958÷ 12 = 2080

P.A.Y.E

Total deductions are ksh2080 (P.A.Y.E) + ksh1000 (pension scheme) = ksh3080

Net income = ksh22,800 – ksh3,080 = ksh19,720

Housing

If an employee is provided with a house by the employer (either freely or for nominal rent) then **15% of his salary is added** to his salary (less rent paid) for purposes of tax calculated.

Example 2

John earns k£ **13636** p.a. and is housed by his employer; calculate his P.A.Y.E and his net income.

Solution

Taxable income = 13636+(13636 × 15%) =15681.4

Tax brackets

15681.4 –

1st slab 5808 ×2 = Ksh11,616

9873-

2nd slab 5472 ×3 = Ksh16,416

remainder 4401.4 ×4= Ksh17,605.6

Gross tax Ksh 45,637.6

Less relief Ksh 12,672

Net tax Ksh 32,965.60

P.A.Y.E =(per month)32965.6÷12=2747

Net income = (13,636×20÷12) – 2747 = **19,980**

1.2.8.4 Learning Activities

In groups of three practice the following

Use the table of taxation below in this exercise

Income in £ /month	Rate %
1 – 484	10
485- 940	15
941- 1396	20
1397-1852	25
Excess over 1852	30

1. Mr Kenneth earns £ **40000** p.a. he is housed by the employer and pays a nominal rent of ksh **1000** . Calculate his **P.A.Y.E** and his **net income** .

2. Mrs Naliaka earns a monthly salary of **Ksh 14,800** a medical allowance of ksh**1200** per month and a travelling allowance of ksh**12,000** per month. She is housed and pays a nominal rent of ksh**700** per month, she contributes towards a retirement scheme towards which she pays k£**240** per annum. Calculate her P.A.YE. hence her net income.

1.2.8.5 Self-Assessment

Use the table below to answer the questions that follows;

Currency	Buying	Selling
1 US Dollar **$**	**78.4133**	78.4744
1 Sterling Pound **£**	**114.1616**	114.3034
1 Euro **€**	**73.4226**	73.52953
1 South African Rand	**7.8842**	7.9141

1 UAE Dirham	**21.3480**	21.3670
1 Indian Rupee	**1.5986**	1.5999

1.A Kenyan businessman purchased a commodity worthy US$100 to a company in the United States of America. The Kenyan can either pay through his account in Kenya or through his account in the United Kingdom. Which method is cheaper and by how much? Give your answer in Kenya shillings given that;(United Kingdom use Sterling Pound)

2.A tourist came to Kenya from London with 6000 Euros which he converted to Kenya shillings at a bank. While in Kenya he spent a total of Kshs.300,000 then converted the balance into sterling pounds at the Same bank. Calculate the amount in sterling pounds he received.

3. John intended to import a car worth € **15,000** from France. How much did he pay in **ksh** to acquire the Euros?

4. If he later changed his mind and instead re-converted the money to **ksh** , how much did he end up with ?

5. Mary raised **ksh** 500,000 for a study course in Britain. She bought an air ticket for **ksh** 80,000 and converted the balance to sterling pounds. Once in Britain she bought winter clothes worth £ 250 and paid **£2060** as tuition fees. How much in **ksh** did she end up with?

1.2.8.6 Tools, Equipment, Supplies and Materials

Tools/Equipment:	Materials:
• Scientific Calculators • Rulers • Pencils • Erasers • Computers with internet connection	• Charts with presentations of data • Graph books • Text books

1.2.8.7 References

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Question 1

Purchase Price = US$ 100

1 US$ = 78·4133 ksh

100$ =

100 × 78·4133

= Ksh 7841·33

If he pays via account in the UK.

1 £ = 114·16 ksh

? = 7841·33 ksh

= 68·69 £

1 £ = Ksh 114·3034

68·69 £ = ?

$\frac{68{\cdot}69\,£ \times 114{\cdot}3034}{1\,£}$ = Ksh 7830·34

Paying via his account in the UK is cheaper by 11 shillings.

Question 2

1 Euro = Ksh 73·52953

6000 Euros = ?

= sh 441175·8

less Spent sh 300000·0

141,175·80 Ksh

Converted to pounds

1 £ = Ksh 114·1616

? = Ksh 141175·80

= $\frac{141175{\cdot}80 \times 1\,£}{114{\cdot}1616}$

= £1236·6

Question 3

	Cash raised	= Sh	500 000
less	Air ticket		80,000
			420, 000

Balance converted to sterling pounds

1 £ = Ksh 114·1616

? = Sh 420,000

$= \frac{420000}{114 \cdot 1616}$

= £ 3678·99

less clothes	250
	3428·99
Tuition fee	2060
	1368·99

How much in ksh

1 £ = Ksh 114·3034

1368·99£ = ?

= 1368·99 × 114·3034

= Sh 156480·20

1.2.9 Learning Outcome 9: Apply Ratios

1.2.9.1. Introduction to the learning outcome

This unit describes the competencies required in applying basic mathematics on numbers and ratio.

1.2.9.2. Performance Standard

1. Differentiated between rational and irrational numbers
2. Expressed ratios as percentages

3. Solved problems involving direct and inverse proportions

1.2.9.3. Information Sheet

Rational numbers refer to a number that can be expressed in a ratio of two integers. An irrational number is one which can't be written as a ratio of two integers. Expressed in fraction, where denominator ≠ 0. Cannot be expressed in fraction.

Problems and Solution

Question 1: **Which of the following are Rational Numbers or Irrational Numbers?**

2, -0.45678…, 6.5, √ 3, √ 2

Solution: Rational Numbers – 2, 6.5 as these have terminating decimals.

Irrational Numbers; – -0.45678…, √ 3, √ 2 as these have a non-terminating non-repeating decimal expansion.

Question 2: Check if below numbers are rational or irrational.

2, 5/11, -5.12, 0.31

Solution: Since the decimal expansion of a rational number either terminates or repeats. So, 2, 5/11, -5.12, 0.31 are all rational numbers.

Rationalization of denominators in working

1. $\frac{7\sqrt{10}}{\sqrt{2}}$

$$
\begin{aligned}
\frac{7\sqrt{10}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} &= \frac{7\sqrt{20}}{\sqrt{4}} \\
&= \frac{7\sqrt{20}}{2} \\
&= \frac{7\sqrt{4 \cdot 5}}{2} \\
&= \frac{7\sqrt{4}\sqrt{5}}{2} \\
&= \frac{7 \cdot 2\sqrt{5}}{2} \\
&= \frac{{}^{7}\cancel{14}\sqrt{5}}{\cancel{2}\ {}_{1}} \\
&= 7\sqrt{5}
\end{aligned}
$$

Incase the denominator contains a sum of two numbers either both rational and irrational or all irrational, we use a conjugate;

2. $\frac{2}{3+\sqrt{3}}$

$$\frac{2}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{2\left(3-\sqrt{3}\right)}{\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)}$$

$$= \frac{6-2\sqrt{3}}{9-3\sqrt{3}+3\sqrt{3}-\sqrt{9}}$$

$$= \frac{6-2\sqrt{3}}{9-\cancel{3\sqrt{3}}+\cancel{3\sqrt{3}}-\sqrt{9}}$$

$$= \frac{6-2\sqrt{3}}{9-3}$$

$$= \frac{6-2\sqrt{3}}{6}$$

$$= \frac{2\left(3-\sqrt{3}\right)}{6}$$

$$= \frac{\overset{1}{\cancel{2}}\left(3-\sqrt{3}\right)}{\underset{3}{\cancel{6}}}$$

$$= \frac{3-\sqrt{3}}{3}$$

3. $\frac{3}{2-\sqrt{2}}$

$$\frac{3}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{3\left(2+\sqrt{2}\right)}{\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)}$$

$$= \frac{6+3\sqrt{2}}{4+2\sqrt{2}-2\sqrt{2}-\sqrt{4}}$$

$$= \frac{6+3\sqrt{2}}{4-\sqrt{4}}$$

$$= \frac{6+3\sqrt{2}}{4-2}$$

$$= \frac{6+3\sqrt{2}}{2}$$

Expressing ratios as percentages ang vice versa

1. For example, If the ratio is 12:4, convert it to the form $\frac{12}{4}$, which is an equation we can solve. After that, multiply the result by 100 to get the percentage.

12 ÷ 4 = 3

3 × 100 = 300%

Equation to solve the percentage for a ratio

If given a percentage, you can be converted back into a ratio using very simple steps

Examples

a. 75%

Step One: Convert the percentage to a decimal

75%=0.75

Step Two: Convert from decimal form to fraction

0.75= $\frac{3}{4}$

Step Three: Rewrite as a ratio

$\frac{3}{4}$=3:4

b. 300%

Step One: Convert the percentage to a decimal

300%=3

Step Two: Convert from decimal form to fraction

3= $\frac{3}{1}$

Step Three: Rewrite as a ratio

$\frac{3}{1}$=3:1

Direct and inverse proportion

Direct proportion

There is a direct proportion between two values when one is a multiple of the other. For example,**1cm=10 mm** . To convert cm to mm, the multiplier is always 10. Direct proportion is used to calculate the cost of petrol or exchange rates of foreign money.

The symbol for direct proportion is α.

The statement ‘t is directly proportional to r’ can be written using the proportionality symbol:

t αr

if y=2p then y is proportional to p and y can be calculated for p=7;

y=2x7=14

similarly, if y=60 then p can be calculated; 60=2p

to find p, divide 60 by 2:

60÷2=30

Finding the equation in direct proportion

Proportionality can be used to set up an equation.

There are four steps to do this:

1. write the proportional relationship
2. convert to an equation using a constant of proportionality
3. use given information to find the constant of proportionality
4. substitute the constant of proportionality into the equation

Example

The value e is directly proportional to p. When e=20, p=10. Find an equation relating e and p.

1. e ∝ p
2. e=k p

3. 20=10k so k= 20÷10 =2
4. e=2p

This equation can now be used to calculate other values of e and p.

If p=6 then, e=2×6=12.

Inverse proportion

It occurs when one value increases and the other decreases. For example, more workers on a job would reduce the time to complete the task. They are inversely proportional.

he statements 'b is inversely proportional to m' is written:

$b \propto \frac{1}{m}$

Equations involving inverse proportions can be used to calculate other values.

Using: $g=\frac{36}{w}$ (so g is inversely proportional to w).

If g=8 then find w.

$8=\frac{36}{w}$

$w=\frac{36}{8}$

Similarly, if w=6, find g.

$g=\frac{36}{6}$

g=6

Finding the equation in inverse proportion

Proportionality can be used to set up an equation.

There are four steps to do this:

1. write the proportional relationship
2. convert to an equation using a constant of proportionality
3. use given information to find the constant of proportionality
4. substitute the constant of proportionality into the equation

Example

If g is inversely proportional to w and when g=4, w=9, then form an equation relating g to w.

1. $g \propto \frac{1}{w}$

2. $g=k\times\frac{1}{w}=\frac{k}{w}$

3. $4=\frac{k}{9}$ so k=4×9=36

4. $g=\frac{36}{w}$

This equation can be used to calculate new values of g and w.

If g=8 then find w.

$8=\frac{36}{w}$

$w=\frac{36}{8}=4.5$

Similarly, if w=6, find g.

$g=\frac{36}{6}$

g=6

1.2.9.4. Learning Activities

Practice the following assessment

1. A student pays 20% more for his bus fare from home to school than he used to pay two years ago. If he pays sh. 30, how much was he paying then?
2. A hawker bought a glass for sh 24 and later sold it latter for sh.36. what was his percentage gain?
3. In an analysis, 3.5% of all parts of a machine were declared substandard. If there were 72 substandard parts, how many parts were analyzed?
4. What percentage is 0.002cm of 4cm?
5. Four men can build 32m long in 12 days. what length of wall can eight men working in the same rate build in eight days?
6. Three tractors, each working eight hours a day, can plough a field in five days. How many days would two such tractors, working 10 hours a day take to plough the same field?

1.2.9.5. Self-Assessment

1. Convert each of the following decimals into a percentage;

a) 0.32
b) 0.88
c) 0.02
d) 3.2

2. Convert the following percentages into decimal;
    a) 120%
    b) 200%
    c) 40%
    d) 25%

1.2.9.6. Tools, Equipment, Supplies and Materials

Tools, Equipment, Supplies and Materials

Tools/Equipment:	**Materials:**
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1.2.9.8. Model Answers

QUESTION 1

Convert the following decimals to percentages.

a) 0.32

$= \frac{32}{100} \times 100 = 32\%$

b) 0.88

$\frac{88}{100} \times 100 = 88\%$

c) 0.02

$\frac{02}{100} \times 100 = 2\%$

d) 3.2

$3\frac{2}{10} = \frac{32}{10} \times 100 = 320\%$

QUESTION 2

Convert the percentages to decimals.

a) 120%

$= \frac{120}{100} = 1.2$

b) 200%

$= \frac{200}{100} = 2.0$

c) 40%

$= \frac{40}{100} = 0.4$

d) 25%

$= \frac{25}{100} = 0.25$