CHAPTER 3: NUMERACY SKILLS/DEMONSTRATE NUMERACY SKILLS

3.1 Introduction

Demonstrate numeracy skills unit of competency is among the seven basic competencies offered in all the TVET level 6 qualification. The unit covers the knowledge, skills and behaviours required to effectivelly use numeracy skills by a worker inorder to apply a wide range of mathematical calculations for work, apply ratios, rates and propotions to solve problems in real life situatins. It entails identifying and using of geometry to draw and construct 2D and 3D shapes for work; collect, organize, present and interprate statistical data to help make decisions at the work place. The significance of numeracy skills to TVET level 6 curriculum is to develop logical thinking and reasoning strategies, in day today activities, in solving problems and making sense of numbers.

The critical aspects of competencies to be covered includes mathematical calculation for work, applying ratios through use of detailed maps to plan travel routes for work, geometry to draw and construct 2D and 3D shapes, collect, organize and interpret statistical data and also the use scientific calculators. The basic resources required include; calculators, rulers, pencils, graph books and dice. The unit of competency covers eight learning outcomes. Each of the learning outcome presents; learning activities that covers performance criteria statements, thus creating an opportunity to demonstrate knowledge and skills in the occupational standards and contents in the curriculum. Information sheet provides; defination of key terms, content and illustrations to guide in training. The competency may be assessed through written tests, demonstration, practical asignment and case study. Self assessment is provided at the end of each learning outcomes.

3.2 Performance Standard

Apply a wide range of mathematical calculations, use and apply ratios, rates and proportions, estimate, use detailed maps to plan travel routes, use geometry to draw 2D shapes and construct 3D shapes, collect, organize, and interpret statistical data and use routine formula tasks in accordance to workplace tasks and texts and mathematical language.

3.3 Learning Outcomes

3.3.1 List of learning outcomes

a) Apply a wide range of mathematic calculations for work
b) Apply ratios, rates and proportions to solve problems
c) Estimate measure and calculate measurements for work
d) Use detailed maps to plan travel routes for work
e) Use geometry to draw and construct 2D and 3D shapes for work
f) Collect, organize and interpret statistical data
g) Use routine formulae and algebraic expressions for work
h) Use common functions of a scientific calculator

3.3.2 Learning Outcome No 1: Apply a wide range of mathematical calculations for work

3.3.2.1 Learning Activities

Learning Outcome No 1: Apply a wide range of mathematical calculations for work	
Learning Activities	**Special Instructions**
1.1. Extract mathematical tasks and texts information embedded in a range of workplace. 1.2. Interpret and comprehend mathematical information 1.3. Select and use a range of mathematical and problem-solving processes 1.4. Use different forms of fractions, decimals and percentages. Performed calculation with positive and negative numbers 1.5. Express numbers as powers and roots in mathematical calculations 1.6. Use routine formulas to do calculations 1.7. Use estimation and assessment of processes to check outcomes 1.8. Discuss and explain mathematical language used for processes, results and implications of tasks	

3.3.2.2 Information Sheet No3/LO1: Apply a wide range of mathematic calculations for work

Introduction

The learning outcome uses and estimate decimals, fractions, percentages, powers and roots to calculate range of mathematical and problem solving in real life situations. It also entails addition and subtraction of integers.

Definition of key terms

Fraction: A fraction is a number that represents part of the whole.

Decimal: A decimal is number whose whole number part and the fractional part are separated by a decimal.

Percentages: A percentage is a number or ratio expressed as a fraction of 100.

Content/Procedures/Methods/Illustrations

1.1 **Mathematical information embedded in a range of workplace tasks and texts is Extracted**

Mathematics as a discipline include the study of number theory, algebra, geometry, statistics, calculus etc. Experts in mathematics explore and make use of sequence, pattern and series to discover new knowledge in mathematics. For one to be successful either in business and workplace, the knowledge of mathematics is a must.

1.2. Mathematics information is interpreted and comprehended

Understanding mathematics helps one to understand and do a wide range of mathematical concepts and factors and identify principles of a given area in mathematics that make things work. Many people have difficult clarifying mathematical ideas or solving problems that involves more than just calculations. Interpretation and understanding of mathematics are the key principle component to help visualize the concept of mathematics.

1.3 A range of mathematical and problem-solving processes are selected and used.

A mathematical solving and problem-solving process involves four key steps. The four steps include

- Details,
- Main idea,
- Strategy
- How to do.

A mathematician works through each step and organizes their ideas to provide evidence of their mathematical thinking and to show how to arrive at the solution.

At the idea stage, the mathematician looks to answer the following questions.

- What is the main idea in the question?
- What is he looking for?
- What is needed?

Ideals: This is where one reads the questions more than one time to keenly understand the idea well. The areas of interest here is the numbers used, words and statements. Facts are extracted to help answer the question. Hidden information at this stage is also looked as though not clearly indicated. The key areas required at this level is details needed, important details, and what is hidden to answer the question.

Strategy: Here a strategy (strategies) may be chosen to find a solution of the problem and that strategy to help find the answer to the problem.

How: This is to make sure that the answer got is reasonable and can be used to help understand the process clearly. This helps one to know which method was used to arrive at the problem.

The advantage of using the above method is because it leads to high level of thinking.

1.4. Types of Fractions

A fraction is a numeric quantity which is expressed such that it is not a whole number ($e.g\ ⅓ = 0.33 \ldots$). The upper part of a fraction is called a numerator and the lower part is called a denominator. There are three types of fractions

i. **Proper fractions** e.g. $\frac{4}{7}$ $where\ 4\ is\ smaller\ than\ 7\ in\ this\ fraction.$

The value of a proper fraction is always less than 1.

ii. **Improper fractions** e.g. $\frac{11}{6}$ $where\ 11\ is\ bigger\ than\ 6\ in\ this\ fraction.$

The value of an improper fraction is always equal to or greater than 1.

iii. **Mixed fractions** e.g. $3\frac{1}{3}$ (combination of a proper fraction and a whole number)

The improper fractions can be converted into mixed fractions and vice versa. An improper fraction is first a fraction where top number (numerator) is greater than or equal to the bottom number, the denominator.

NOTE: Operation on addition, subtraction, multiplication and division can be operated on fractions.

Steps involved in converting an improper fraction to proper fractions are;

- Divide the numerator by the denominator
- Write down the whole number answer
- Write down any remainder above the denominator.

Decimals

A decimal is a fraction which is expressed such that it has a denominator as a power of multiples of ten and whose numerator is expressed by numbers placed to the right of a decimal point. There are two types of Decimals

i. Terminating decimals: these are decimals which stop at a given number of decimal places e.g. 0.7 (1decimal place), 0.85 (2 decimal place) and so on.

ii. Recurring/non terminating decimals: these are decimals which do not stop at a given number of decimal place but which may have a repeating digit or repeating set of digits after the decimal point e.g. (a) $0.33333333 \ldots \ldots. = 0.\dot{3}$ (b)$0.463463 \ldots.. = 0.\dot{4}6\dot{3}$. Recurring decimals can also be converted in fractions.

Example1: convert 0.444444 in fractions Steps;

- Let x be the recurring number,
- Examine the recurring decimal to determine the repeating digits,
- Place the repeating digit(s) to the left of the decimal point,
- Subtract the left sides of the equations and the right sides, i.e.,

X = 0.444444……………. (1)
10x = 4.44444…………….(2)
9x = 4……………………. (2) – (1)
X = $\frac{4}{9}$

Percentages
The meaning of the word percentage is per hundred. This is a way to represent parts of one whole. In percent one whole is considered to be 100%.
Percentages can be converted into fractions and decimals or vice versa.
Working with percentages
Percentages are mentioned with regards to the base value, which is also known as whole or reference value. The percentage amount is a portion of the base value. Converting percentages to fractions.

Procedure:

i. Write down the percentage divided by 100,
ii. If the percentage is not a whole number, then multiply both top and bottom by 10 for every number after a decimal point,
iii. Simplify.

Powers: These are numbers into which a given number is raised to. They are also referred to as exponentials, indices or Logarithms e.g. a^m where m is the power.
Root: The root of a number y is another number, which when multiplied by it gives itself a given number of times equal to y.

1.5 Calculations performed with positive and negative number
Numbers below zero are called negative numbers. Numbers above zero are called positive numbers. There are rules that are used when adding, subtracting, multiplying or dividing positive and negative numbers. A number line is used to help illustrate how to add and subtract positive and negative numbers. When adding positive numbers, we count to the right. When subtracting positive numbers, we count to the left.
Using a number line to illustrate $5-3=2$

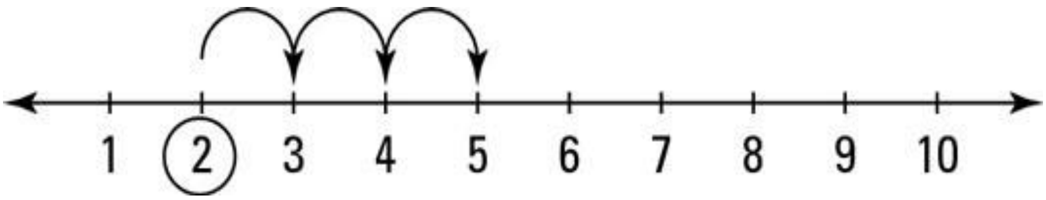

Figure 2. Number line

Here we imagine of moving up and down a number line to get the answer. Starting from zero we count up to5. Then subtract by moving six 6 steps to the left then subtract 4 to get -4.

1.6 Numbers are expressed as powers and roots are used in calculations

Powers are used when we want to multiply a number by itself repeatedly or we use powers (indices) to multiply a number by itself.

E.g. $8 \times 8 \times 8 \times 8$ is written as 8^4,

4 tells use the numbers of 8 (eights) to be multiplied together. Here the index (power) is 4. The number 7 is called the base.

Square roots

When 7 is squared we get 49. This means that $7^2 = 49$. The reverse of this process is called finding a square root.

The square root of 49 is 7 which is written as $\sqrt{49}$=7.

The square root of a perfect square has two roots e.g. $\sqrt{64}$=±8.

Generally, the square root of a member is a number which when squared give the original number.

NOTE: Negative numbers do not have square roots

1.7 Calculations done using routine formulas

All calculations used in mathematics follows a specific order. Formulas are used to provide mathematical solutions for real life problems. These formulas can be an equation, a principle or a logical relation. Addition, subtraction, multiplication, and division are easy. When we come across derivation, calculus and geometry, we need the use of formulas. There are many formulas used in mathematics which depends on a given situation.

1.8 Estimation and assessment processes are used to check outcome.

To achieve outcome in estimation and assessment we need to:

- Use a variety of estimation strategies to find the appropriate products and quotients of whole numbers.
- Analyze different estimation strategies for finding products.
- Identify essential elements of estimating quotient.

Some of the estimation strategy includes:

- Front end strategy
- Compensation strategy
- Rounding to multiples of 10
- Compatible numbers

1.9. Mathematical language used to discuss and explain processes.

Mathematics includes knowledge, skills and methods. Most language in mathematics are spoken and the symbols are vocal sounds. The language is a system of communication about mathematical objects such as numbers, sets, functions, operations and equations. The basic operation symbols in mathematics are $+(plus) - (minus)$

Conclusion

This learning outcome describes the skills and knowledge to extract, comprehend and analyze a broad range of mathematical information in fractions, decimals, percentages and whole numbers.

Further Reading

Purna Chandra Biswal (2009); *Probability and Statistics*; Prentice hall Of India Pvt Ltd

Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry*; Wesley –London

John bird (2006); high engineering mathematics; Elsevier ltd.

3.3.2.3 Self-Assessment

Written Assessment

1. Which one of the following contain a set of the fractions that are evenly spaced on a number line?
   a) $\frac{3}{4}, \frac{19}{24}, \frac{5}{6}$
   b) $\frac{3}{4}, \frac{19}{24}, \frac{7}{8}$
   c) $\frac{4}{5}, \frac{5}{6}, \frac{7}{8}$
2. How would you write the fraction $\frac{2}{10}$ as a decimal?
   a) 0.2
   b) 0.02
   c) 0.002
   d) 2
3. Choose the fraction equivalent to the decimal 0.09
   a) $\frac{9}{10}$

b) $\frac{90}{10}$
c) $\frac{90}{100}$
d) $\frac{9}{100}$

4. Convert $4\frac{3}{5}$ into an improper fraction.
5. Without using a calculator find the value of $\sqrt{36 \times 49}$
6. Find the square of $5\sqrt{2}$
7. A tank is to be filled from two taps. By itself, the first tap could fill the tank in $\frac{1}{2}$ hour. By itself, the second tap could fill the tank in $\frac{1}{3}$ hour. How long will they take together?

Oral Assessment

1. Differentiate between a decimal and a fraction.
2. What is an improper fraction?
3. What is a recurring decimal?

3.3.2.4 Tools, Equipment, Supplies and Materials

- Calculators
- Rulers, pencils and Erasers
- Charts with presentation of data
- Graph books
- Dice

3.3.2.5 References

John Bird (2006); *High engineering mathematics*; Elsevier Ltd
Purna Chandra Biswal (2009); *Probability and Statistics*; Prentice-hall Of India Pvt Ltd
Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry*; Wesley –London

3.3.3 Learning Outcome No 2: Use and apply ratios, rates and proportions for work

3.3.3.1 Learning Activities

Learning Outcome No 2: Use and apply ratios, rates and proportions for work	
Learning Activities	**Special Instructions**
2.1. Extract information regarding ratio, rates and proportions from a range of workplace tasks and texts. 2.2. Analyze mathematical information related to ratios, rates and proportions 2.3. Use problem solving processes to undertake the task 2.4. Simplify equivalence ratios and rates. 2.5. Calculate quantities using ratios, rates and proportions. 2.6. Construct graphs, charts or tables to represent ratios, rates and proportions 2.7. Review and check the outcomes. 2.8. Record information using mathematical language and symbols	Use textbooks

3.3.3.2 Information Sheet No3/LO2: Use and apply ratios, rates and proportions for work

Introduction

This learning outcome looks into the calculation of quantities using of ratios, rates, percentages and proportions in mathematics and also in real life situation. It also entails construction of graphs for ratios and proportions.

Definition of key terms

Rate: a rate is the ratio between rated quantities in different units. The word per is used to describe the units of two quantities measurements.

Ratio: ratio is the relationship between two numbers indicating how many times the first number contains the second.

Proportion: this is a part share or number considered in comparative relation to a whole.

Content/Procedures/Methods/Illustrations

2.1 Information regarding ratios, rates and proportions extracted from a range of workplace.

Task and texts: Here the skills and knowledge to apply and solve problems involving rates, ratios and proportions is described. This unit applies to individuals who need the skills of ratios, rates and proportions and how to apply them in real life situations.

2.2 Mathematical information related to ratios, rates and proportions are analyzed.

Rates

A rate is found by dividing one quantity by another. For example, a rate of pay consists of the money paid divided by the time worked. If a laborer receives Ksh200 for two hours work. His rate of pay is

$200 \div 2 = 100$ Shillings per hour

Here the rate is found by dividing the money by time.

Ratios

Ratios are helpful tool for comparing things to each other in mathematics and real life and therefore it is important to know what they mean and how to use them.

A ratio compares two quantities by division, with the dividend or number being divided termed antecedent and the divisor or number that is dividing termed the consequent.

Ratios occur frequently in daily life and help to simplify many of our interactions by putting numbers into perspective. Ratios allow us to measure and express quantities by making them easier to understand.

2.3 Problem solving processes are used to undertake the task.

Rates: a rate is the ratio between two related quantities in different units. In describing the units of a rate, the word "per" is used to separate the units of the two measurements used to calculate the rate.

Ratio

A good way of comparing quantities is by use of a ratio. Suppose that in a youth club there are 60 boys and 25 girls. The ratio of boys to the number of girls is spoken as '60 to 25' and can be written as

$$60:25$$

If you divide each part by 5, you get the simplified ratio.

$$12:5$$

This means that for every 12 boys there are 5 girls.

NOTE: You can only compare quantities using ratios if each quantity is measured in the same units

Proportions: To solve problems with percent we use the percent proportion shown in proportion and percentage.

$$\frac{k}{m} = \frac{x}{200} = r$$

$\frac{x}{200}$ Is called the rate.

$k = r.b$ This implies the $Percentage\ = rate\ \times\ time$

Percentages

Here we deal with 100% of a number. 100% means a whole thing and therefore 100% of a number is the same thing.

$$50\%\ of\ 20 = 10$$

$$20\%\ of\ 9 = \frac{9}{2}\ or\ 4\frac{1}{2}\ or\ 4.5$$

2.4 Equivalent ratios and rates are simplified

No matter how a ration is written, it is important that it be simplified down to its simplest form possible, just as with any fraction. This is done by finding the greatest common factor between the numbers and dividing accordingly. With a ratio comparing 16 to 24, for example, you find that both 16 and 24 can be divided by 8 which is the greatest common factor between the two numbers. This simplifies this ratio into 2 to 3 by dividing each by 8. Which is written as

$2{:}\,3$ or $\frac{2}{3}$ or 0.75

Note that a decimal is sometimes permissible, though less commonly used.

2.5 Quantities are calculated using ratios, rates and proportions

Ratios.

Ratios are used to compare amounts or quantities or describe a relationship between two amounts or quantities. For example, a ratio might be used to describe the cost of a month's rent as compared to the income earned in a month. We may also wish to compare the number of buffaloes to the total number of animals in a zoo, or the number of calories per serving in two different brands of ice cream.

Rates

Rates are special type of ratios used to describe a relationship between different units of measurement, such as speed, wages or even prices. An aircraft can be described as travelling 300 miles per hour; an earthmover might earn Ksh 40, 000.00 per hour; fuel price may be sold at Ksh112.00 per litre.

Proportions

If two fractions are equal, we say that the two given rations are equivalent. It two ration are equivalent, then the involved four quantities are said to be in proportion. Proportions can be used to solve problem involving scaling. For example, if two eggs are used to make 40 pancakes, then how many eggs do you require to make 200 pancakes?

For this to work the ration must be equivalent. The ratio is

$$\frac{Eggs}{Pancakes}$$

Using the above information, this can therefore be written as

$$\frac{2}{40} = \frac{x}{200}$$

From equivalent ratio $x = 10$

2.6 Graphs, charts or tables are constructed to represent ratios, rates and proportions

Rates, ratios and proportions can be represented on tables and graphs. The proportionality on a graph means that as x increases, y increases and as x decreases, y decrease and the ratios between them remains the same. Suppose we have the ratio of teachers to students in a given school as 1 to 5. This ratio can be represented on a table as well as on a graph. For this to be done we know that Ratios have additive and multiplicative reasoning pattern.

Table 4. Number of teachers and students

Teachers	1	2	3	4	5
Students	5	10	15	20	25

This information can be represented on a graph by either taking teachers to be on the x – axis and students to be on the y-axis or the vice versa.

2.7 The outcome reviewed and checked.

The number of pupils in a class increases from 30 to 36. We say that increase is in the ratio $36:30 = 6:5$

This means that the fraction

$$=\frac{New\ number\ of\ pupils}{old\ number\ of\ pupils} = \frac{6}{5}$$

The number of pupils has been multiplied by $\frac{6}{5} = 1.2$

This is called the multiplying factor.

2.8 Information is recorded using mathematical language and symbols

Mathematics is written in a symbolic language that is designed to express mathematical thoughts. English language is a source of knowledge, but is not designed for doing mathematics.

The symbolic language of mathematics is a special-purpose language. It has its own symbols and rules of grammar that are quite different from those of English (Uttam Kharde 2016).

Conclusion

This learning outcome describes the skills and knowledge required to solve real life problems using ratios, rates and proportions. Mathematics is a language which is developed using symbols and to study the mathematics, it is necessary to understand the language in which it is written.

Further Reading

- Purna Chandra Biswal (2009); *Probability and Statistics*; Prentice-hall Of India Pvt Ltd
- Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry*; Wesley London

3.3.2.3 Self-Assessment

Written Assessment

1. A ratio equivalent to 3:7 is?
   a) 3:9
   b) 6:9
   c) 9:21
   d) 18:49
2. The ratio 35:84 in simplest form is?
   a) 5:7
   b) 7:12
   c) 5:12
   d) None of these
3. In a class there are 20 boys and 15 girls. The ratio of boys to girls is?
   a) 4:3
   b) 3:4
   c) 4:5
   d) None of these
4. Mr. Kinyua spends 35% of his salary on rent. After paying the month's rent he had Ksh50, 000.00 left. How much does he earn each month now?
5. Three candidates stood for election to school council. The percentage votes for each person are given in the table below.

Table 5: Election results

STUDENTS	% OF VOTES
Kelvin	30
Juma	P
K'olender	Y

Only 60% of the school voted and those 20 papers were spoiled and did not count. Three times as many people voted for K'olender as for Juma there are 1800 pupils in the school. How many people voted for each of the candidates?

If $x:y = 4:1$ and $y = 3t$, find the ratio $x:y:t$

3.3.3.4 Tools, Equipment, Supplies and Materials

- Calculators
- Rulers, pencils and Erasers
- Charts with presentation of data
- Graph books
- Dice

3.3.3.5 References

J.K BackHouse, S.P.T Houldsworth; *Pure mathematics I;* Longman Group (FE) Ltd, Hong Kong

J.K BackHouse, S.P.T Houldsworth; *Pure mathematics I; Pure mathematics* I; Longman Group (FE) Ltd, Hong Kong

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Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry*; Wesley –London

3.3.4 Learning Outcome No 3: Estimate, measure and calculate measurement for work

3.3.4.1 Learning Activities

Learning Outcome No 3: Estimate, measure and calculate measurement for work	
Learning Activities	**Special Instructions**
3.1. Extract and estimate measurement information embedded in workplace texts and tasks 3.2. Identify and selected appropriate workplace measuring equipment 3.3. Estimate and make accurate measurements 3.4. Calculated the area of 2D shapes including compound shapes 3.5. Calculate using relevant formulas the volume of 3D shapes 3.6. Calculate using Pythagoras' theorem the sides of right-angled triangles 3.7. Perform conversion between unit's measurement 3.8. Problem solving processes are used to undertake the task 3.9. Review and check the measurement outcomes 3.10. Record appropriate information in using mathematical language and symbols in a given task	

3.3.4.2 Information Sheet No3/LO3: Estimate, measure and calculate measurement for work

Introduction

This learning outcome looks into the calculation of sides, area and volume of 2D and 3D shapes using relevant formulas. It also entails selection of measuring equipment and conversion of units of measurements.

Definition of key terms

Right angled triangle: This is a triangle whose one angle is equal to 90^0 and the other two angles are acute angles

2D shapes: These are flat planes figures or shape with two dimensions.

3D shapes: These are solids or objects with 3 dimensions. These dimensions are width, height and length.

Area: This is the quantity that expresses the extent of a two-dimension figure or shape, i.e., length and breadth.

Volume: The amount of space that a substance or object occupies

Content/Procedures/Methods/Illustrations

3.3.4.3 Measurement information embedded in workplace texts and tasks are extracted and interpreted.

This describes the skills and knowledge to estimate and measure quantities, to convert units within the metric system and between metric and non-metric units, to calculate area and volume including compound shapes and to use Pythagoras theorem. The unit helps to;

- Recognize common units of metric measurement for Length, Mass, Capacity and Temperature and use them appropriately in highly familiar situations.
- Identify and choose appropriate measurement tool and use it at a basic level in a limited range of highly familiar situations to measure and compare items.
- Recognize whole numbers into the hundreds related to measurement.
- Use common words for comparing measurements.

Required Knowledge

- Signs and prints, symbols represent measuring in measurement context such as on tools and packaging.
- Common units of metric measurement and their appropriate use.
- Abbreviations associated with highly familiar measurement and time.

3.2 Appropriate workplace measuring equipment are identified and selected

Length, Mass, Capacity and temperature have their standard measures given by Meters, Kilograms, Liters, and Degrees Celsius respectively. Familiar situations of measurement include reading and interpreting measures on advertising leaflets, notices, packaging etc. Appropriate measurement tool may include: rulers, tape measures, kitchen scales, measuring containers.

Common words used include:

- Long/short
- Big/small
- Thick/thin
- Short/tall
- Double/half

3.3 Accurate measurements are estimated and made

Estimation is a concept that one can incorporate in daily routine. To ensure accuracy of measurement it is important to introduce areas in measurement where practice can be done to enhance the art of accurate measurement.

This helps learners to use standardized units of measurements.

3.4. The area of 2D shapes including compound shapes are calculated

This involves skills and knowledge to develop the basic skills and confidence to perform simple and familiar numeracy task involving the identification, comparison and sketching of simple and familiar two-dimensional shapes.

Area of 2D Shapes

The area of a shape shows how much it covers. Here the area of shapes is calculated
For each length unit there is a corresponding area unit.
e.g.

1 square meter
1square centimeter
1 square kilometer etc.

Area of rectangles, parallelograms triangles and trapezia

Rectangle

The area of rectangle is found by multiplying its Length (x) and its breadth(y), which is given by

$$\boldsymbol{Area\ =\ xy}$$

Parallelogram

The area of a parallelogram is the product of its base and its height. If its base is b and the perpendicular is h then the area is $\boldsymbol{Area = bh}$

Triangle

The area of a triangle is half the product of the base and the height. If the base is b and the height is h. Then the area is

$$Area\ =\ ½\ x\ b\ x\ h = ½bh$$

Trapezium

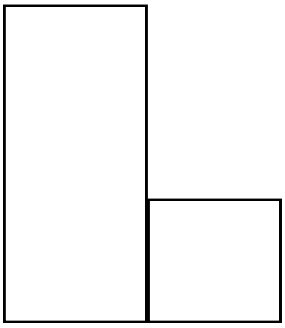
Suppose the parallel sides of a trapezium are a and b, and that its height is h. then the area is

$$½h\ (a + b).$$

Area of a Circle

If a circle has radius r, then its area is πr^2.

Compound Shapes
Some shapes are made from a combination of basic shapes. Consider the figure below.



The area of this L shape is found by dividing the shapes into different areas so that the area can be calculated.

Figure 3. L Shape

3.5. The volume of 3D shapes is calculated using relevant formulas
In mathematics (geometry) we often are required to calculate the surface area and volume of 3D shapes.
The attributes of a three-dimension figure are the faces, edges and vertices. The three dimensions compose of the edges of a 3D geometrical shape.

Examples of the 3D shapes includes

i. Sphere

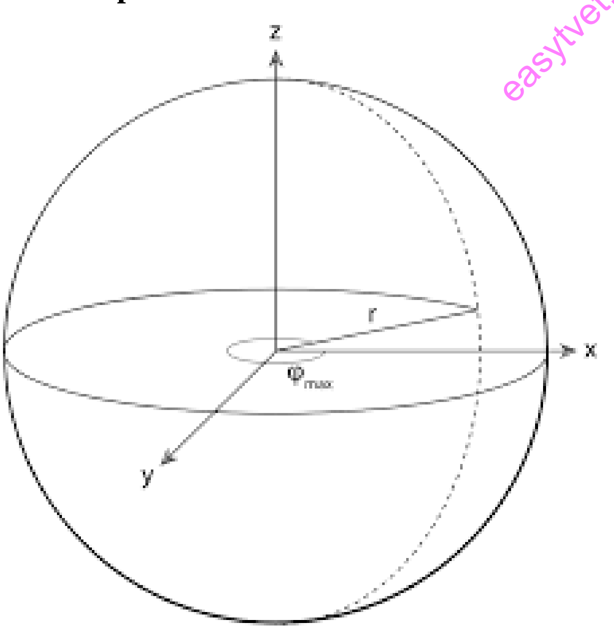


Figure 4. Sphere

The surface area is given by
A = $4\pi r^2$ and
Volume is given by
r = $\frac{4}{3}\pi r^3$

ii. Cone

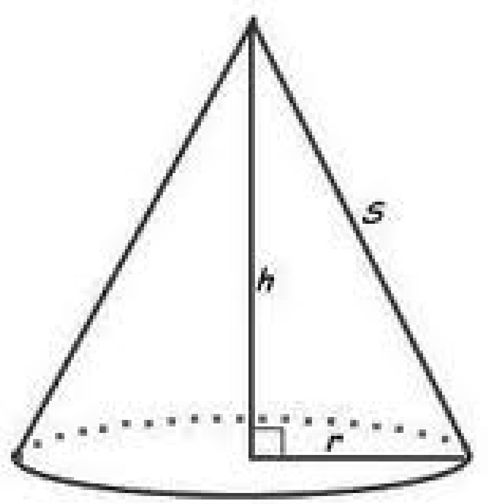


Figure 5. Cone

A Cone has a circular base and a slanting surface.

$$Surface\ area = \pi rl + \pi r^2$$

$$Volume = \tfrac{1}{3}\pi r^2 h$$

iii. Cylinder

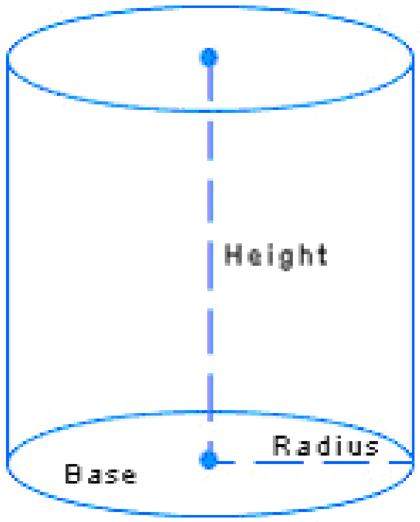


Figure 6. Cylinder

A closed cylinder has two circular circles and a curved surface.

The surface area is

$$S.A = 2\pi r^2 + 2\pi rh$$

The volume is

$$V = \pi r^2 h$$

iv. **Rectangular Prism:** this is a solid with side faces, where opposite sides are equal.

$$Surface\ area = 2(lh) + 2(lw) + 2(wh)$$
$$Volume = lhw$$

Other 3D shapes include Prisms.

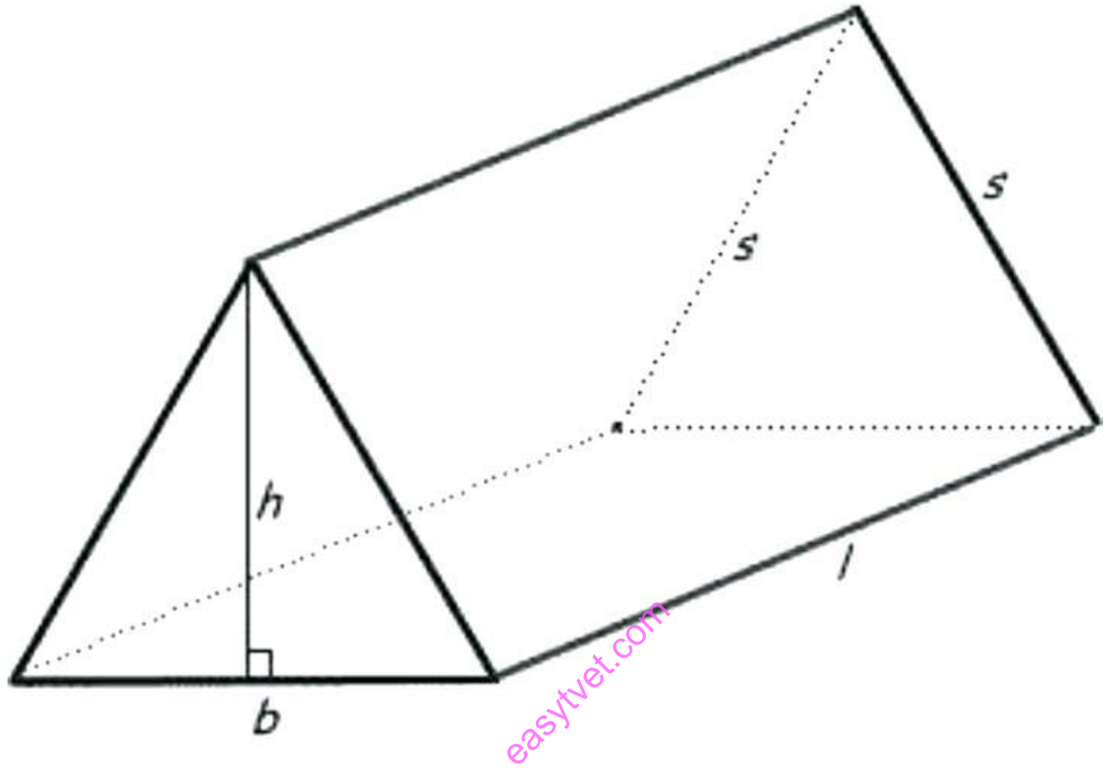


Figure 7. Rectangular prism

Prisms are three dimensional shapes with base of a triangle which can either be equilateral, isosceles or scalene triangle.

$$Surface\ area = bh + 2ls + lb$$
$$Volume = ½\,(bhl)$$

3.6 Sides of right-angled triangles are calculated using Pythagoras' theorem

Pythagoras theorem is one of the best-known mathematical formulas which provide us with the relationship between the sides of a right-angled triangle.

A right-angled triangle consists of two side meeting at 90(degrees) and a third side called the hypotenuse which is the longest side of the right-angled triangle.

The Pythagoras theorem state that the relationship in every triangle say

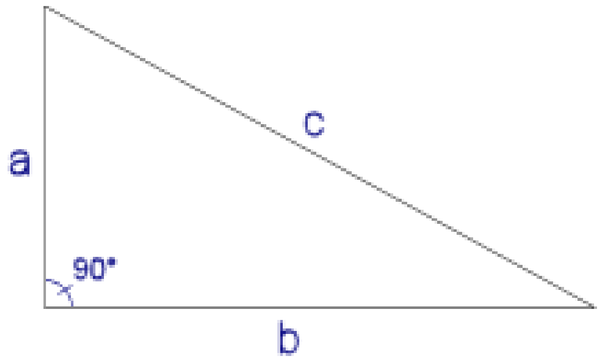


Figure 8. Right-angled triangle

Calculating the sides of the triangle requires the use of the right-angled triangle i.e. $c^2 = b^2 + a^2$ where

$$c = \sqrt{b^2 + a^2}$$

There are various types of a right-angled triangle which are categorized by the size of the acute angles. Examples are $30^0, 45^0, 60^0$, 90^0 etc.
Further concepts about right angled triangle can be done with trigonometry.

3.7 Conversions are performed between units of measurement

Measurements are ways that we tell others how much of an item we have. There are two systems of measuring objects i.e. English and Metric
English measurements are used only in USA, while metric measurements are used in nearly every other country.

Common Metric Measures

The metric system is also divided into different categories of measurement, but it has a base unit for each category. The categories under metric measurement are

Millimeters (mm) 1000mm = 1 m
Centimeters (cm) 100cm =1m
Meter (m) is the base unit
Kilometer (km) 1000m = 1km

Weight

Milligrams (mg) 100mg = 1g
Grams (g) base unit (1)
Kilograms (kg) 1000g = 1kg, 1000kg = 1tonne

Volume

Milliliters (ml) 1000ml = 1l
Liter (l) base unit

Temperature

Degrees Celsius (ºC)

Speed
Meters per seconds (m/s)

Time
Second (sec) 60 sec = 1 min
Minutes (60) min = 1 hr.
Hour 24hrs = 1 day

3.8 Problem solving processes are used to undertake the task
Measurement is the process of using a device or tool to find the dimensions, time, pressure amount, weight or mass of an object. We use measurement to help us solve problems in many real-life situations.

Perimeter and Area
To find the perimeter of an object or place, measure the distance around something.

Area: this is the amount of space within a boundary. The boundary can be a variety of shapes and calculating the area will depend on shape.

3.9 The measurement outcomes are reviewed and checked

Surface area
Rectangles
The surface area of a cuboid is equal to the sum of the area of all of its faces.
To find the surface area of any 3D shape, we can follow the process described below:
1. Draw a net of the 3D shape,
2. Calculate the area of each face,
3. Add up the area of all the faces to set the surface area (S.A) of the entire shape.

3.10 Information is recorded using mathematical language and symbols appropriate for the task
The 2D and 3D shapes can be identified using the name and describing the features using appropriate vocabulary. This includes:
- Names, identifies and classifies a range of simple 2D and 3D shapes.
- Using mathematic language to describe the properties of a range of common 2D shapes and 3D objects including size, face, edge, corner, base and angle.
- Identifies and recognizes 2D and 3D shapes by drawing.

Conclusion
This learning outcome is based calculating lengths, areas and volumes of 2D and 3D shapes to help solve real life problems.

Further Reading

Purna Chandra Biswal (2009); *Probability and Statistics;* Prentice-hall Of India Pvt Ltd

Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry;* Wesley –London

3.3.4.3 Self-Assessment

Written Assessment

1. A three-dimension shape can be called?
    a) Flat shape
    b) solid
    c) polygon
    d) hexagon
2. Vertices are also called?
    a) Edges
    b) corners
    c) faces
    d) sides
3. Which 3D shape has six rectangular faces?
    a) cube
    b) cuboid
    c) prism
    d) tetrahedron
4. A parallelogram has base 20cm and area$660cm^2$. What is its height?
5. The parallel side of a trapezium are 10cm and 12cm. The area is$55cm^2$. Find the height of the trapezium.
6. A paint roller is a cylinder which is 15cm long and with radius 3cm. Find the area of wall it can cover in one revolution.
7. A water tank is 0.8m high, 1.2m long and 1m wide. It is three-quarters full of water. How many bricks, 20cm by 10cm by 10cm, can be put into the tank before the water overflows?
8. A section of a path is 2m wide and 12m long. It is made from concrete 6cm thick. Find the volume of concrete, giving your answer in m^2

Oral Assessment

1. What is the difference between a kite and a rhombus?
2. How many sides do a nonagon has?

3.3.4.4 Tools, Equipment, Supplies and Materials

- Calculators'
- Rulers, pencils and Erasers'
- Charts with presentation of data
- Graph books
- Dice

3.3.4.5 References

J.K BackHouse, S.P.T Houldsworth; *Pure mathematics I*
J.K BackHouse, S.P.T Houldsworth; *Pure mathematics I*
John Bird (2006); *High engineering mathematics;* Elsevier Ltd
Purna Chandra Biswal (2009); *Probability And Statistics;* Prentice-hall Of India Pvt Ltd
Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry;* Wesley –London

3.3.5 Learning Outcome No 4: Use detailed maps to plan travel routes for work

3.3.5.1 Learning Activities

Learning Outcome No 4: Use detailed maps to plan travel routes for work	
Learning Activities	**Special Instructions**
4.1. Identify and interpret different types of maps 4.2. Identified key features of maps. 4.3. Identify and interpret scales. 4.4. Calculate and apply actual distances using scales. 4.5. Determine positions or locations using directional information 4.6. Determine distances, speeds and times using planned routes and directions. 4.7. Gather and identified relevant factors related to planning a route with checked information 4.8. Select and check relevant equipment for accuracy and operational effectiveness. 4.9. Plan and record specialized mathematical language and symbols appropriate for a given task.	Individual presentation

3.3.5.2 Information Sheet No3/LO4: Use detailed maps to plan travel routes for work

Introduction

The learning outcome makes use of scale, maps and routes to calculate distances, speeds and times of positions or locations. It also entails giving and receiving directions using formal and informal methods.

Definition of key terms

Contour: These are lines joining points of equal value. Each successive contour represents an increase or decrease in a constant value. They are usually associated with change in thickness.

Speed: The rate at which someone or something moves or operates.

Distance: This is the length between two points.

Scale drawing: This is a drawing where dimensions are proportional to the actual size of the object being drawn in a predetermined ratio.

Content/Procedures/Methods/Illustrations

4.1 Different types of maps are identified and interpreted

In geography maps are one of the most important tools researchers, cartographers, students and other can use to examine the entire earth or a specific part of it.

Maps simplify defines a picture of the earth surface.

Maps can be classified into two main groups' i.e.

- Reference maps
- Thematic maps

Reference Maps: They show the location of geographical boundaries, physical features of earth, or cultural features such as places, cities, water and roads. Examples are: political maps, physical road, topographic maps, time zone, geologic and Zip code maps. The figure below shows Kenya reference map showing the features and boundaries of provinces.

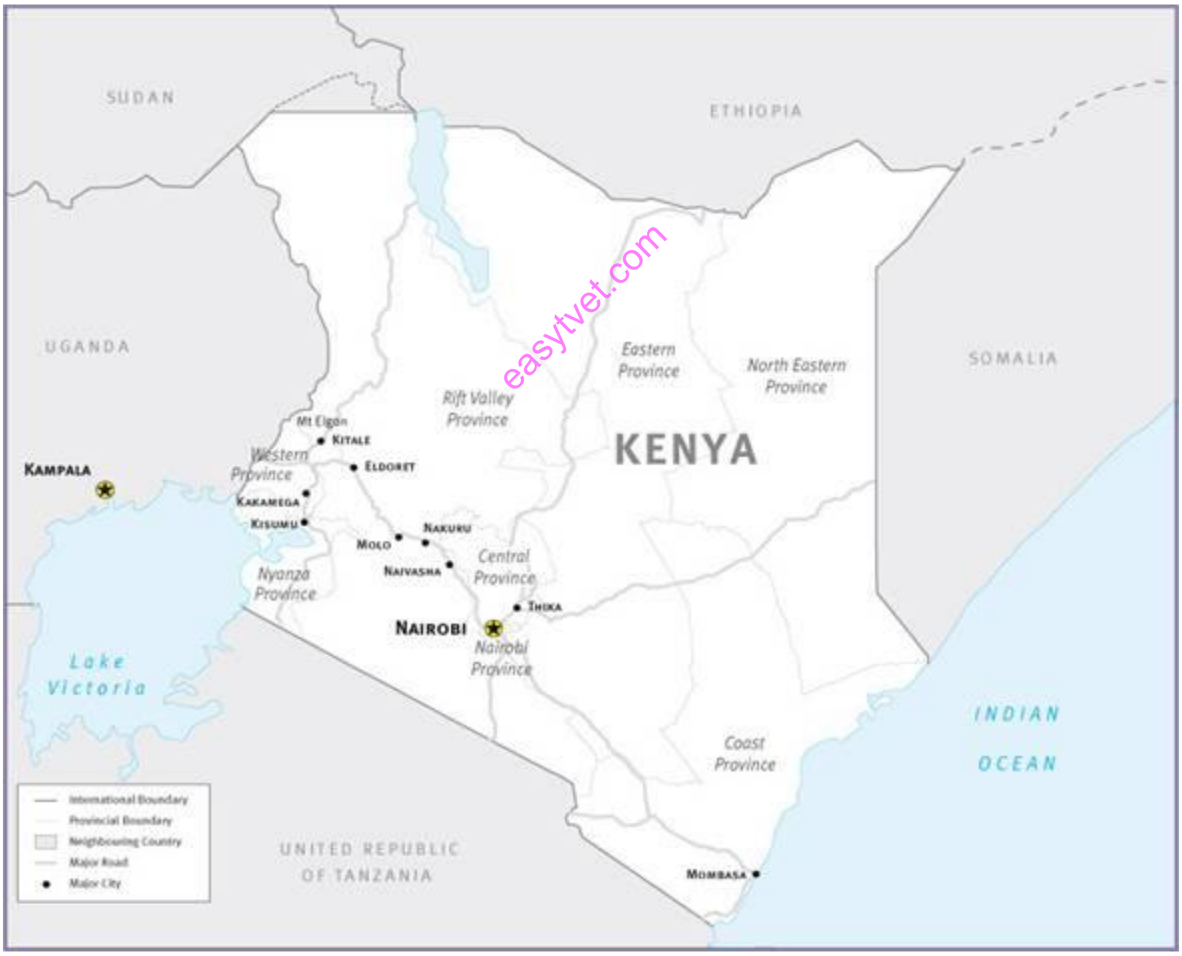


Figure 9. Reference map

Thematic Maps

They show the variation of a topic across a geographical area of weather maps, income map, resource maps etc. The figure shows an example of thematic map of Kenya indicating the distribution of ethnic groups.

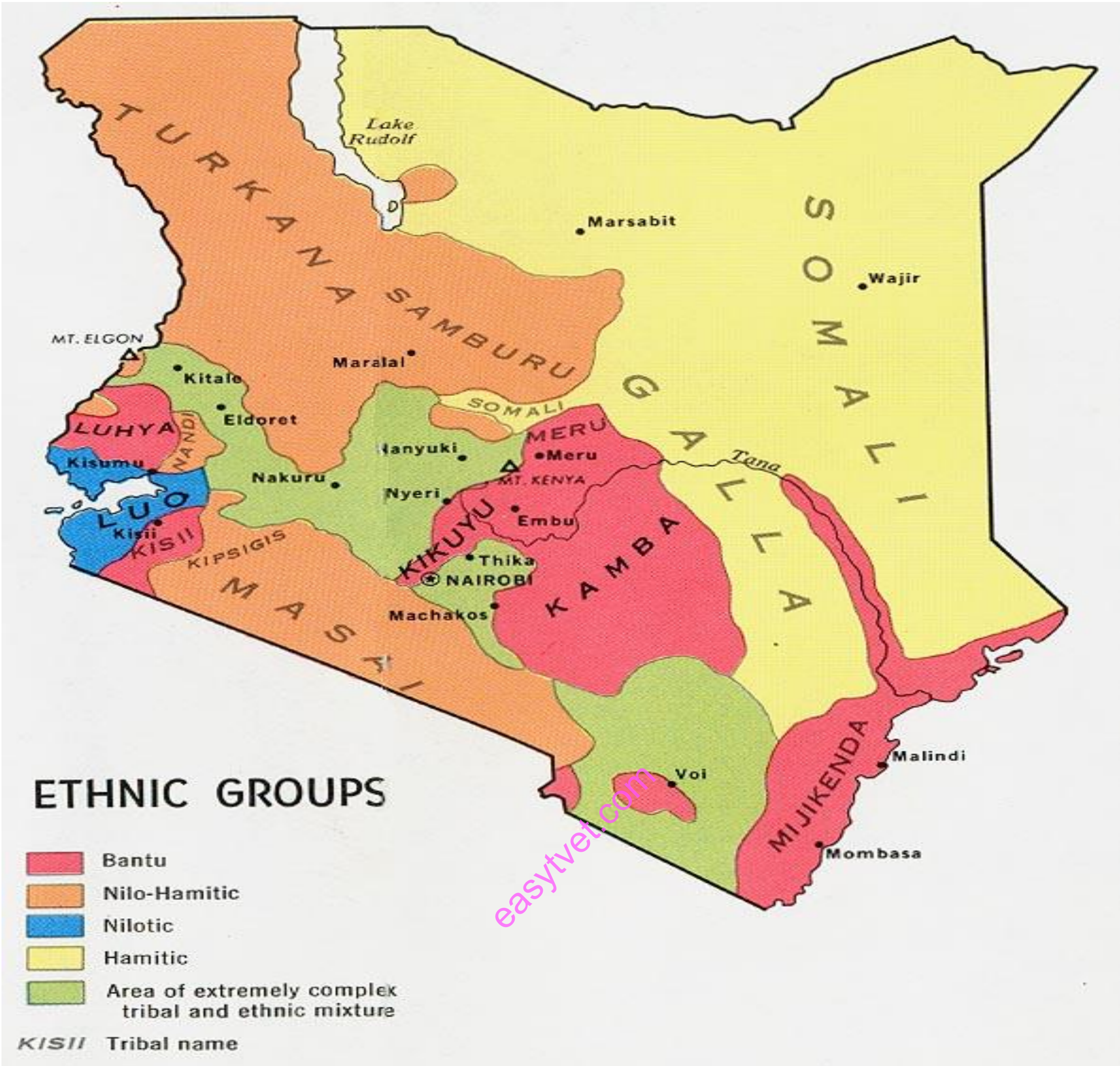


Figure 10. Thematic map

Other types of maps include;

Physical maps; they showing body of water masses, rivers seas etc. land features are colored according to their elevation

Topographical maps; they are like the physical maps where the difference is that they use contour lines instead of colors to show the changes in the landscape.

Road maps, or route map is a map that primarily displays roads and transport links rather than natural geographical information. It is a type of navigational map that commonly includes political boundaries and labels, making it also a type of political map

Interpretation of map

Maps are a 2-D representation of a 3-D world. They are a bird's eye view as if the viewer is flying above the land surface and looking down on it.

They show how objects are distributed and their relative size.

Map reading is the use of maps to gain the information they have to impart. It is an accomplishment that includes a thorough knowledge of the symbols and conventions used and the ability to interpret in terms of actual landscape the facts recorded. The knowledge of the symbols and conventions can be readily obtained by a little study. As a consequence, the map reader must supply many minor details from his general familiarity with map-making procedure and personal acquaintances with natural features.

Example

1. A map that shows the national and state boundaries, and cities is called a?
   a) Physical map
   b) Special map
   c) Population density map
   d) North Pole map
2. ___________shows the four cardinal directions and intermediate directions
   a) Key
   b) Compass rose
   c) Scale
   d) Symbol
3. State the five elements of a map.
4. Landforms such as mountains and volcanos can be found on a _________map.

1.2 Key features of maps are identified

A map usually contains the following elements

i. **Title (and subtitle):** They usually draw attention by virtue of its dominant size; it serves to focus attention on the primary content of the map.
ii. **Key (legend or explanation):** This is the principal reference to the map symbols. The legend describes all unknown or unique map symbols used. It is subordinate to the title.
iii. **Scales:** They provide the leader with important information regarding the linear relations on the map. A scale can be numeric or graphical.
iv. **Credits:** They includes the map source, the author, indication of the reliability of accuracy of the map, dates, or other explanatory materials.

The maps elements stated above are shown in the map given below.

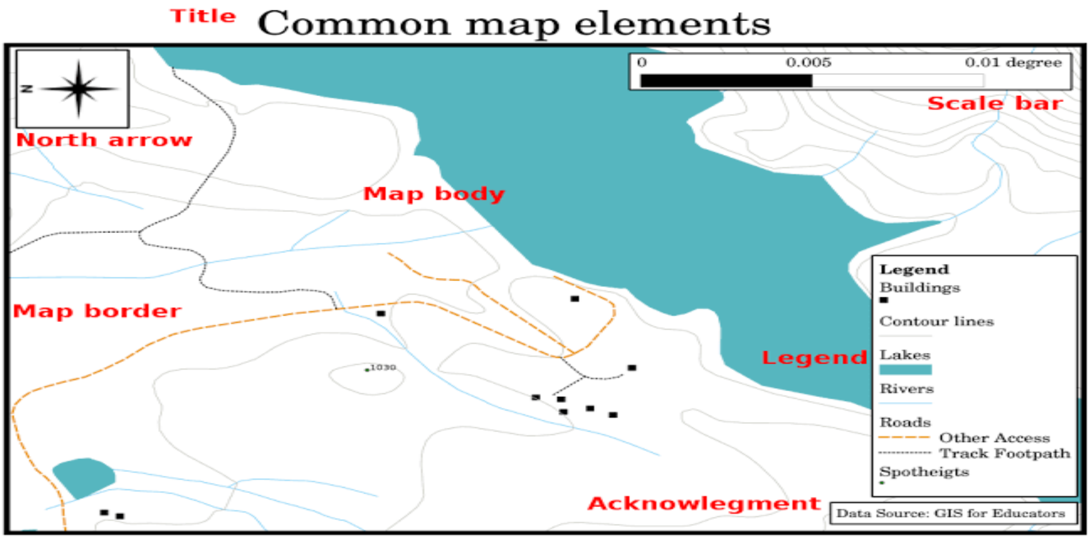


Figure 11. Features of a map

4.3 Scales are identified and interpreted

The scale of a map is the ratio of a distance on the map to the corresponding distance on the ground. There are 3 main ways that scale is indicated on a map;

- Verbal
- Representative fraction (RF)
- Bar scale

The use of scales;

- Use a given scale in conjunction with measurement on a plan/map to determine length.
- Dimension.
- Determine the scale of a map or plan.
- Use a given scale in conjunction with other content or skills to complete a project (e.g. use a given scale to determine the dimension in which to draw a 2-dimensional plan of an object, and the draw the plan)

Reading of maps

The following should be considered when reading maps;

- Identify the labels/names of national roads that must be travelled on to travel between two locations.
- Identify the number of the towns and the routes between two locations.
- Identify the scale of map.
- Identify the position of the two locations on a map and use given distance values on the map to determine the travelling distance between the two locations.
- Interpret a given set of directions and describe what location the directions lead to.
- Provide a set of directions to travel between two locations in a town using street names.

- Use a map in conjunction with a distance chart to determine the shortest route to travel between two locations.
- Estimate travelling times between two or more locations based on estimated travelling speed known or calculated distances.

Reading plan of maps

- Use a given key to identify the number of windows/rooms shown on the plan of the building.
- Identify on which plan a particular structure is shown (e.g. the door is shown on the north elevation plan)
- Measure dimensions on a plan and use a given scale to determine actual distances.

4.4 Scales are applied to calculate actual distances

Maps are very useful for more than just directions. They are used to determine the distances between two or more places. The scales on map can be of different types. The process of calculating actual distance from a map is as follows:

- Use a ruler to measure the distance between two points. If the line is curved, use a string to determine the distance and then measure the string.
- Get the scale for the map you are going to use which is either a ruler bar or written scale.
- If the scale is representative fraction for example (1/1000, 000), multiply the distance of the ruler by the denominator which denotes distance in the ruler units.
- If the scale is a ratio e.g. (1:100,000), you will multiply the map units by the number after the colon.

Any other form of a scale is calculated the same way. Decoding the scale is the key to determining the distance.

A scale of 1:100000 means that the real distance is 100000 times the length of 1 unit on the map of drawing.

Example1:

Write the scale 1 cm to 1 m in ratio form

Solution

1cm to 1m = 1cm: 1m

=1cm: 100cm

=1:100

NOTE: If the scale is 1:x then multiply the map distance by x to calculate the actual distance.

Example2:

A particular map shows a scale of 1:5000 what is the actual distance if the map distance is 8cm?

Solution

Scale = 1:5000 = 1cm: 500cm

Hence; map distance: actual distance = 1: 5000

Map distance is 8cm, let the actual distance be x,

Therefore,

8 = x = 1: 5000

8/x = 1/5000

x/8 = 5000/1 (multiply by 8)

x = 40000.

Hence, the actual distance is 40,000cm

40,000/100 = 400m

4.5 Positions or locations are determined using directional Information

Maps helps use to locate points on the earth's surface using a coordinate system which has both the $X - axis$ and $Y - axis$. The commonly used types of co-ordinate systems are geographical co-ordinates and Cartesian co-ordinate systems. The geographical coordinate system measures location of two points although the two points are described from a 3D surface using the polar axis. The latitude and longitude are used in locating points on earth surface. Latitudes are parallel to the equator and the longitudes are perpendicular to prime meridian. Both the equator and the prime meridian have a designated value0^0. Measurements of longitude are also defined as being either west or east of the prime meridian. Latitude measures the North-South position of location on earth surface.

Parallel of latitudes

They are;

- True east west lines
- Always parallel
- Any two are always equal distance apart

Meridians of longitudes

They are;

- All run in the true north-south direction
- Spaced farthest apart at the equator and converge to a point at the poles
- An infinite number can be created at the poles

The map in the figure below shows the locations of various points on the earth surface using the longitudes and latitudes.

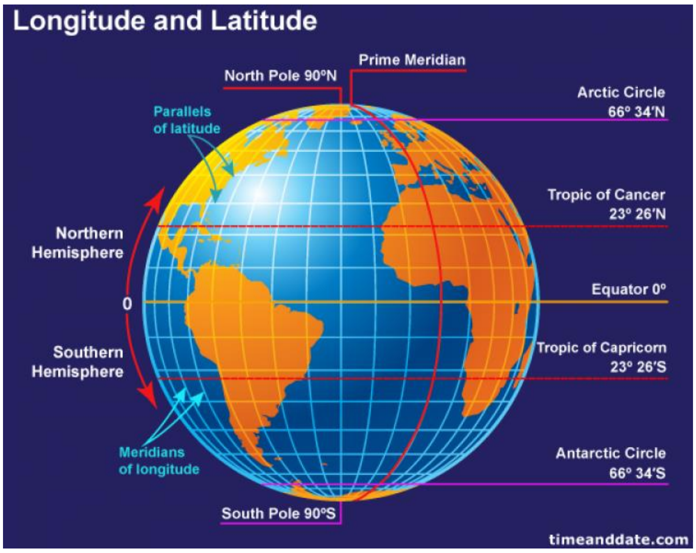


Figure 12. Map showing Longitude and Latitude Source: https://www.pixels.com/featured/world-map

The combination of meridian of longitudes and parallel of latitudes establishes a framework or grid by means of which exact positions can be determined in reference to prime meridian and the equator. A point described as 50^0, 25^0 is located 50^0 of arc north of the equator and 25^0 arch of west of the Greenwich meridian.

Example

1. Which of the following is the correct format for representing the location of a point on the earth surface?
   a) Longitude degrees N or S, Latitude degrees E or W
   b) Latitude degrees N or S, longitude degrees E or W
   c) Latitude degrees E or W, longitude degrees N or S
   d) Longitude degrees E or W, Latitude degrees N or S
2. If a city has location 35^0, 140^0E, which of the following statements is not true about the city?
   a) The city is 35^0 north of the equator
   b) The city is 140^0 east of the prime meridian
   c) The city lies on the equator
   d) The city lies above the equator

4.6 Routes are planned by determining directions and calculating distances, speeds and times

Routes take into account factors such as current traffic and the typical road speed on the requested day and time. The information includes the distance, travel time and a representation of the route geometry. When working with maps one may be required to calculate how long it would take to travel a specific distance at a certain speed. When using the formula involving distance, rate, and time it is essential that you pay attention to the units used in the problem. If the rate in the problem gives miles per hour (mph), then time must be in hours. If the time is given in minutes, divide by sixty to determine the number of hours prior to solving the equation. Distance, speed and time are calculated using the following formulas

- $Distance\ = speed\ \times\ time$
- $Time = \frac{Distance}{speed}$
- $speed = \frac{Distance}{time}$

Example

A train travelled 255 miles in 300 minutes. Determine the rate at which the train was traveling.

Solution

Hours=300mins/60=5hours

Rate=$distance/time$ =255/5=51 miles per hour

4.7 Information is gathered and identified and relevant factors related to planning a route checked

Route planning system drastically reduces the time it takes to plan a transport schedule. It also lower mileage and cut on fuel usage and increase customer service. Route planning is used to decide the route to take from one place to another. Consider the problem that you need to go from an office to a particular shop. Here, the office and the shop become the source and the destination respectively. The output of the route planning algorithm is the rout to take, which enlist all the roads and intersections to take to reach the destination from the source. Important factors and relevant to planning a route include;

- Distance to be covered
- Time to be taken
- Equipment for measuring distance and time

4.8 Relevant equipment is select and checked for accuracy and operational effectiveness

Equipment are inspected prior to start up in accordance with work place. Any aspects of e.g. or workplace that are found to be outside manufacturers or workplace specifications are reported to designated person for appropriate action. Equipment and component should be tested after start-up in accordance with manufacture specifications and workplace procedures.

Warning systems are all checked for operation effectiveness. Faults that may affect the safe operation of the equipment are reported to the appropriate personnel for specification. Results of inspection and testing are accurately reported in accordance with regulatory requirements, workplace policy and industry require. Records should be clear, then ambiguous and concisely kept in accordance with workplace policy. Clear reference is made to any items which may affect the future safety of the equipment.

Required Knowledge and Skills

- Operational safety requirement for the equipment concerned.
- Housekeeping standards procedures required in the workplace
- Site and layout and obstacles

Required of equipment operations

Communicate effectively with others when checking and assessing the operational capability of equipment. Read and interpret instructions, procedures, information's labels and signs relevant to the checking and assessing the operational capability of equipment. Interpret and follow operational instructions and prioritize work. Apply precautions and required actions to minimize, control, or eliminate hazards that may exist during work activities. Work may be conducted in;

- A range of work environment
- By day or night

Customers may be:

- Internal or external

Work place may comprise of:

- Large medium or small worksites

Work may be conducted in:

- Limited or restricted spaces
- Exposed conditions
- Control or open environment

4.9 Task is planned and recorded using specialized mathematical language and symbols appropriate for the task

Mathematics is written in a symbolic language that is designed to express mathematical thought. English language is a source of knowledge, but designed for doing mathematics.

Symbolic language.

The symbolic language of mathematics is a special purpose language. It has its own symbols and grammar that is quite different from English.

The vocabulary of Mathematics

The vocabulary mathematics consists of symbolic expression written the way mathematics traditionally writes them. A symbol is a typographical character such as $\lambda, \emptyset, \mu, \infty$. it includes symbols that are specific to mathematics, such as $\forall, \exists, \leq, \leftrightarrow, \beta, \infty$ which are expressions that stands for something.

A symbolic expression consists of symbols arranged according to specific rules.

Symbolic assertion is a complete statement that stands alone as a sentence.

Example, $\pi > 0$, $y > 0, x = 10$

Symbolic Statements

A symbolic statement is a symbolic assertion without variables. It is either true or false

- Example:$\pi r^2 > and\ 3^2 = 9^2$ are true symbolic statements
- π <0 and 2+4=8 are false symbolic statements. The false symbolic statements are also still regarded as symbolic statements.

Languages are ways of transforming information and meaning. Mathematics is a language which is developed using symbols. To study mathematics, it is necessary to understand the language in which the mathematics is written and read.

Conclusion

This learning outcome is based on scale drawing and determining positions, distances, speeds and time at various locations.

Further Reading

Purna Chandra Biswal (2009); *Probability and Statistics*; Prentice-hall Of India Pvt Ltd

Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry*; Wesley –London

3.3.5.3 Self-Assessment

Written Assessment

1. Calculate the speed of a car that travels 50km in an hour
    a) 50km/hr.
    b) 100km/hr.
    c) 200km/hr.
    d) 25km/hr.
2. A straight line on a d-t graph represents
    a) Slowing down
    b) Constant speed
    c) Speeding up
    d) No movement
3. The correct equation for calculating speed is
    a) v=t/d
    b) v=d×t
    c) v=d/t
    d) None

4. Convert these times in the 12-hour clock to the 24-hour clock
    a) 5.30a.m
    b) 1.40 pm
    c) 7.15p.m

5. A runner took 2 hours and 28 minutes to complete a race. If the ended at 10.15 a.m., when did he begin?
6. One plane leaves Nyeri on a bearing of100^0, a second plane leaves Nanyuki on a bearing of144^0. Where do their paths cross?
7. A point on the valley floor is 500m from the foot of a cliff. From the point, the angle of elevation of the top of the cliff is10^0. How high is the cliff?

Oral Assessment

1. Differentiate between the angle of elevation and angle of depression.
2. Differentiate between latitude and longitude as used in locating a point on the earth surface

3.3.5.4 Tools, Equipment, Supplies and Materials

- Calculators
- Rulers, pencils and Erasers
- Charts with presentation of data
- Graph books
- Dice
- Handheld Global positioning

3.3.5.5 References

J.K BackHouse, S.P.T Houldsworth; *Pure mathematics I*
J.K BackHouse, S.P.T Houldsworth; *Pure mathematics I*
John Bird (2006); *High engineering mathematics;* Elsevier Ltd
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Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry;* Wesley –London

3.3.6 Learning Outcome No 5: Use geometry to draw and construct 2D and 3D shapes for work

3.3.6.1 Learning Activities

Learning Outcome No 5: Use geometry to draw and construct 2D and 3D shapes for work	
Learning Activities	**Special Instructions**
5.1. Identify a range of 2D shapes, 3D shapes and their uses in work contexts. 5.2. Name and describe features of 2D and 3D shapes. 5.3. Identify types of angles in 2D and 3D shapes. 5.4. Estimate and measure angles drawn using geometric instruments. 5.5. Name and identify angle properties of 2D shapes. 5.6. Evaluate unknown angle properties in shapes. 5.7. Apply properties of perpendicular and parallel lines to shapes. 5.8. Demonstrate and understand the use of symmetry. 5.9. Demonstrate and understand the use of similarity 5.10. Identify the workplace tasks and mathematical processes required. 5.11. Draw 2D shapes for work 5.12. Construct 3D shapes for work 5.13. Review and check the outcomes 5.14. Use specialized mathematical language and symbols appropriate for a task.	Individual presentation.

3.3.6.2 Information Sheet No3/LO5: Use geometry to draw and construct 2D and 3D shapes for work

Introduction

The unit deals with the use of geometrical instruments to draw and construct lines, angles, 2D and 3D shapes. It also entails estimation and calculation of angles of two- and three-dimension shapes.

Definition of key terms

Angle: This is the space between two intersecting lines or surfaces at or close to the point where they meet.

Similarity: This is state or fact of being similar.

Symmetry: This is the quality of being made up of exactly similar parts facing each other or around an axis.

Content/Procedures/Methods/Illustrations

5.1 A range of 2D shapes and 3D shapes and their uses in work contexts is identified

Two (2) Dimensional shapes and designs

- Circles
- Square/rectangles
- Triangles
- Parallelogram
- Trapezium
- Pentagon
- Hexagon
- Nonagon
- Octagon
- Rhombus

Three (3) Dimensional shapes and designs includes

- Spheres
- Cubes
- Cylinders
- Prisms
- Triangular prisms

5.2 Features of 2D and 3D shapes are named and described

The features of 2D and 3D shapes are described by the number of faces, edges, vertices. The faces are the flat parts of the shape. The edges are the line where two faces meet. The vertices are the points where two or more edges meet.

Example- the 3D shape with 6 faces has 12 edges and vertices. It is very important that we can handle 3D shapes in order to be able to count their faces, edges and vertices so they will help to construct their own 3D shapes from nets in order to find the surface areas.

5.3 Types of angles in 2D and 3D shapes are identified

2D shapes

Angles that make up the 2D shapes include right angled triangles (90°) found in rectangles, squares. Other angles that can be constructed to form 2D shapes include, 30°, 60°, 120°, and 150°.

5.4 Angles are drawn, estimated and measured using geometric instruments

Some of the instrument that are used for measuring angles of 2D and 3D shapes includes

Protractors

This is the most common device to measure angles. It is used to measure small angles and sometimes big ones. It is very useful in construction, engineering and architecture. The protractor which is commonly found in stationery stores is a half-circle with marked degrees from 0° to 180°.

Hand Square

Hand squares and set squares are devices used to measure angles in geometry. They are used for measuring of large angles as they have degrees from 0° to 360°. They are used for construction of stairs, frames and rafters. Framing squares are used for measuring angles as they are usually L-Shaped devices.

Compass

A compass is geometry is a hinged set of arms, where one arm has a pointed end and the other holds a pencil. There are more devices used for measuring angles and they are

- Navigational Plotter
- The sextant
- The theodolite
- Miter saw
- Inclinometer
- Goniometer

5.5 Angle properties of 2D shapes are named and identified

The properties of 2D shapes include details such as its angles and the number of its sides.

Examples

Squares

- All the four angles are the same.
- Opposite sides are parallel and equal.
- Diagonal intersect at 90°.
- Diagonal are equal in length.
- Angles add up to 360°.
- Have four sides.

Others are:

Properties of 2D Shapes

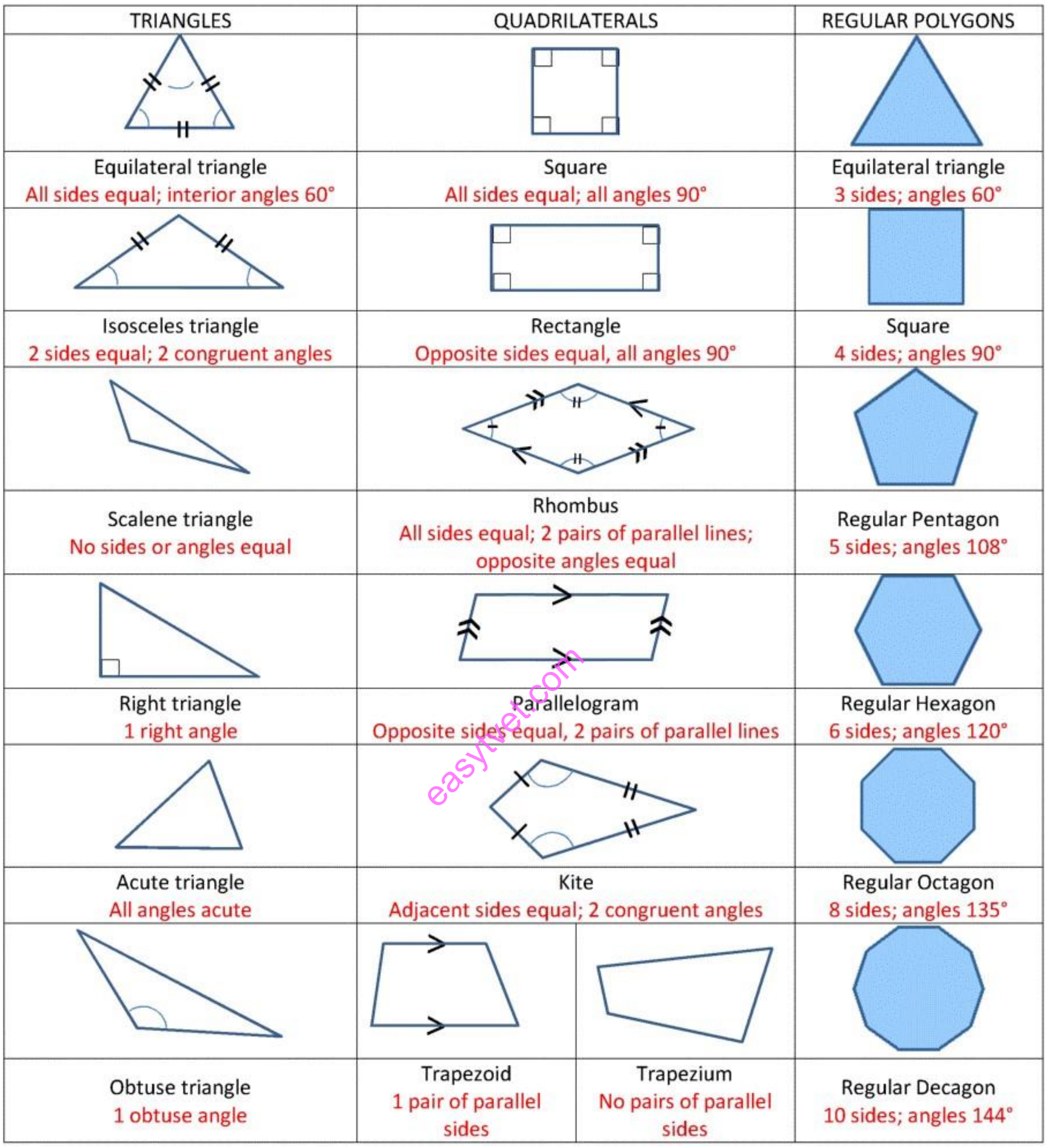


Figure 13. Geometric shapes Source: www.math-salamanda.com

3D shapes

These shapes can be complex as they involve talking about its vertices, faces and edges. The edges of 3D are the lines where 2 faces meet, so a cube has 12 edges. The faces of a cube and a cuboid all meet at 90°. The vertices are the points of a cube where 2 or more edges meet. The edge in a cube or cuboid meets at 90°.

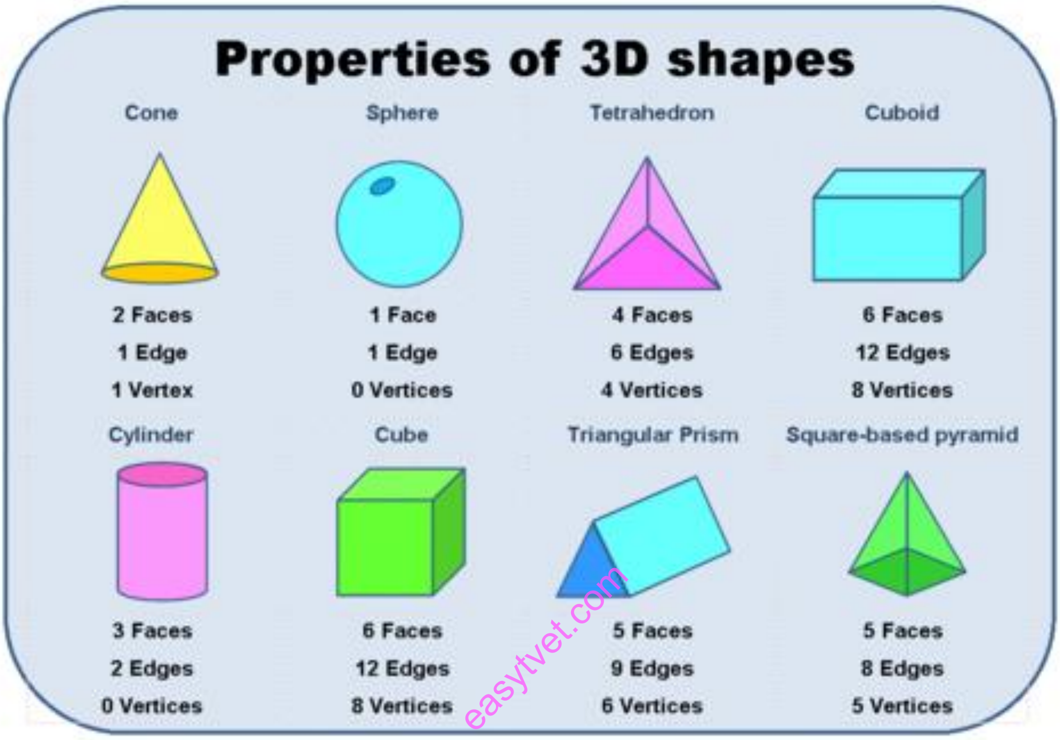


Figure 14. Properties of 3D shapes

5.6 Angle properties are used to evaluate unknown angles in shapes

Unknown angles can be calculated from various shapes if angle properties are known. E.g. given the a right angled find the value of x. If 50-degree angle is known and the remaining angle being 8x.

$$8x + 50° = 90°$$
$$8x = 40°$$

The missing angle is $8 \times 5 = 40^0$

5.7 Properties of perpendicular and parallel lines are applied to shapes

Two lines are perpendicular if the angle between them is 90°. The perpendicular bisector of a line cuts it in half at 90°. To construct shapes either in 2D or 3D one must know how to construct perpendicular lines. Examples are;

- Construction of a perpendicular line from a given point to a given line
- Construction of a perpendicular at a point on a line
- Construction of perpendicular bisectors of a line
- Construction of parallel lines

Other methods include

- Using a straight edge and compasses.
- Using a set square.

5.8 Understanding and use of symmetry is demonstrated

Symmetry is the quality of being made up of exactly similar parts facing each other around an axis. The role of symmetry in geometry is universally recognized. Scientist regard symmetry breaking to be the process of new pattern formation. Broken symmetry is important because they help in classifying unexpected changes form. Through the process of symmetry breaking new patterns are formed. An object has rotational symmetry if the object can be rotated about a fixed point without changing the overall shape. The figure below illustrates the ideal behind symmetry. Triangle ABC is similar to Triangle AEF. The line $x = 1$ is the line of symmetry.

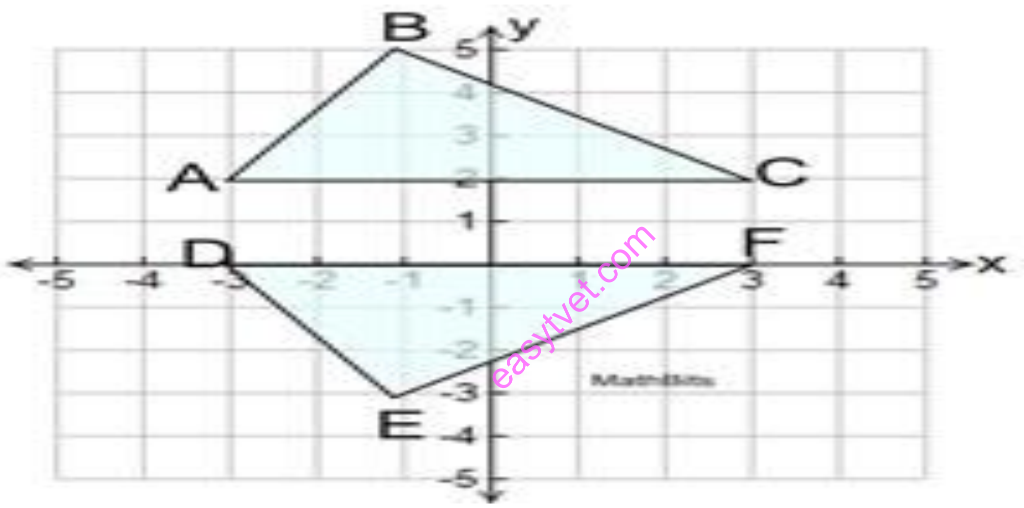


Figure 15. Symmetry demonstration

2D and 3D have different line of symmetry

Square has 4 lines of symmetry

Rectangle has two lines of symmetry

Isosceles triangle has one line of symmetry

Equilateral triangle has three lines of symmetry

Scalene triangle has zero lines of symmetry

1.9 Understanding and use of similarity is demonstrated

The term congruence means that two figures or objects are of the same size and shape. Similarity means that two figures or objects are of the same shape, though not of the same size. Two circles will always be similar because they have the same shape. Triangle ABC and triangle $A'B'C'$ are similar because the corresponding sides and angles are equal.

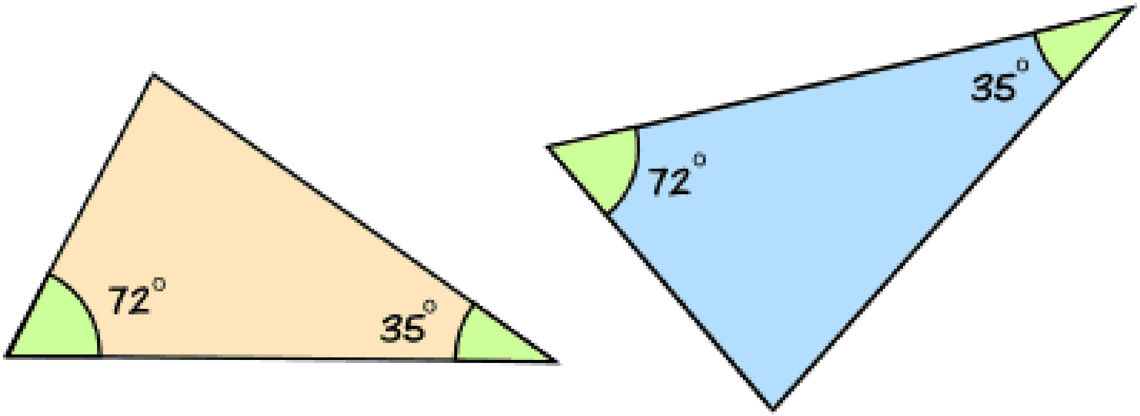
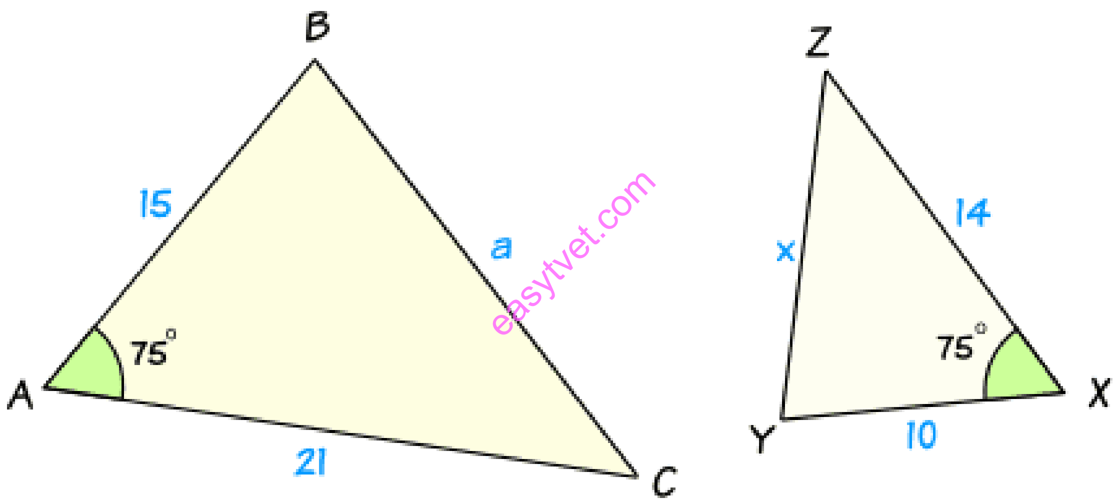


Figure 16. Congruent figures

If the two of their angles are equal, then the third angle must be equal. Angles of a triangle always add to 180^0

In this case, the missing angle is $180^0 – (72^0 + 35^{0)} = 73^0$



In this example we can see that:

- One pair of sides is in the ratio of 21: 14 = 3: 2
- Another pair of sides is in the ratio of 15: 10 = 3 : 2
- There is a matching angle of 75^0 in between them

So, there is enough information to tell us that the two triangles are similar.

5.10 The workplace tasks and mathematical processes required are identified

The task and mathematical estimates required are;

- Carrying out calculations
- Construction of 2D and 3D shapes using mathematical equipment.
- Performing the processes of enlargement, rotation, reflection, using symmetry and similarity.
- Making of nets for various 3D shapes to help calculate surface area.

5.11 2D shapes is drawn for work

Examples are;

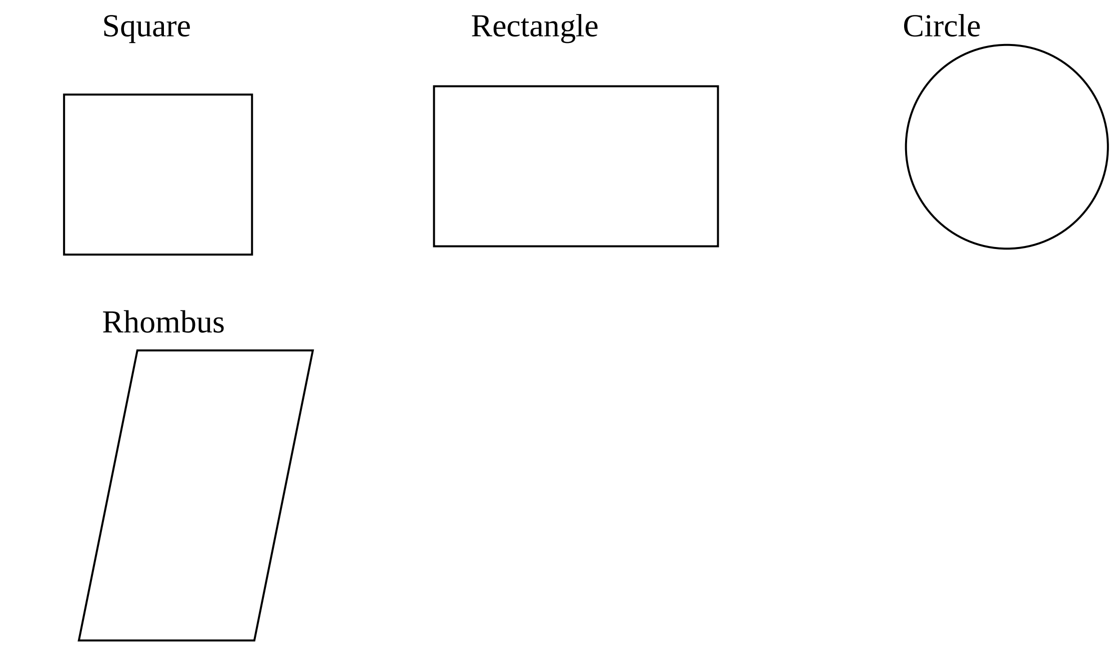
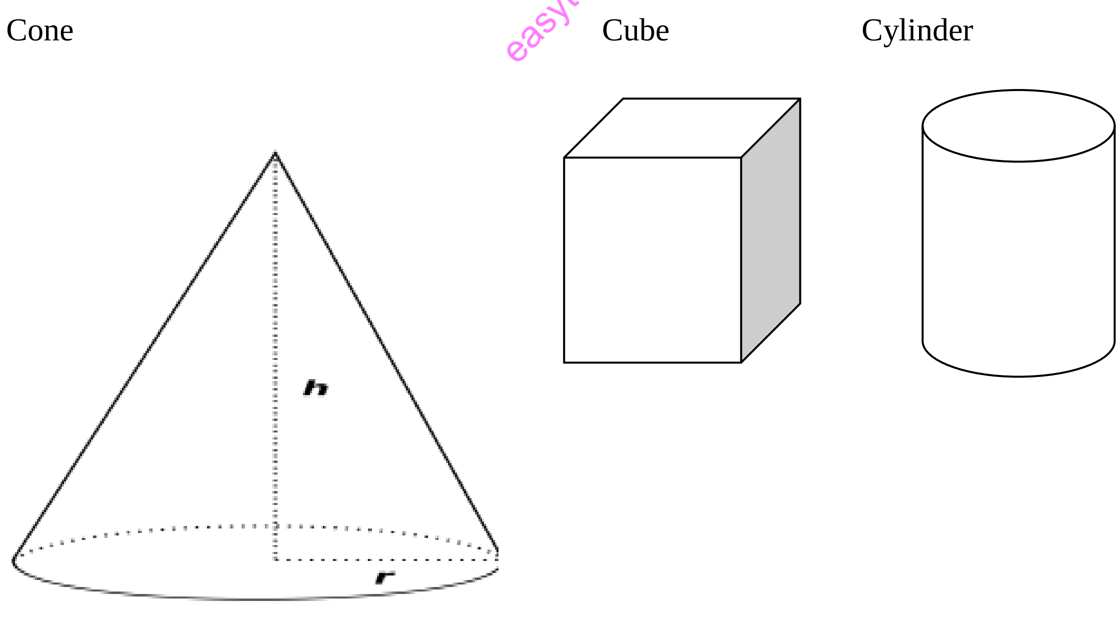


Figure 17. Examples of 2D shapes

5.12 3D shapes are constructed for work



Where h is the height and r is the radius.

Figure 18. Examples of 3D shapes

5.13 The outcomes are reviewed and checked

The construction of 2D and 3D shapes helps in calculating the surface area and volume. Marking he nets of the 3D shapes can help to calculate the surface area and this can help to verify the formulas used for the calculations of these areas

5.14 Specialized mathematical language and symbols appropriate for the task are used

Symbols appropriate for the tasks are used. Mathematics has its own language which is appropriately used to help achieve designed results.

In this learning outcome some of the term used to help achieve the desired tasks are;

- Faces of 2D and 3D shapes
- Vertices
- Edges
- Lines of symmetry
- Nets of solids

Conclusion

This learning outcome is based is based on use of geometry to construct 2D and 3D and to demonstrate the understanding of symmetry and similarity. The construction of is also demonstrated.

Further Reading

Purna Chandra Biswal (2009); *Probability and Statistics;* Prentice hall Of India Pvt Ltd

Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry;* Wesley –London

3.3.6.3 Self-Assessment

Written Assessment

1. Which three-dimension figure will be created if a rectangle is rotated about one of its lines of symmetry?
    a) Cone
    b) cube
    c) cylinder
    d) triangular prism

2. A right circular cone has a radius of 6 inches and a slant side length of 10 inches. A right cylinder has a radius of 8 inches and a height of 12 inches. How many cones full of water are needed to fill the cylinder
    a) 4
    b) 8
    c) 10
    d) 12
3. Find the magnitude of $\overline{AB}$ where A (-4, -6) and B (1, -3)
    a) $\sqrt{106}$
    b) $\sqrt{90}$
    c) $\sqrt{34}$
    d) $\sqrt{18}$
4. Find the surface area and volume of a cube of side 6cm.
5. A prism is 8cm long and its cross-section is a right-angled triangle with sides 5cm, 12cm and 13cm. Find its surface area?
6. The area of a trapezium is $36m^2$. Its height is 9m and one of the parallel sides is 5m. Find the length of the other parallel side?

Oral Assessment

1. What is a tetrahedron?
2. How many vertices does a triangular prism has?

3.3.6.4 Tools, Equipment, Supplies and Materials

- Calculators
- Rulers, pencils and Erasers
- compass
- Charts with presentation of data
- Graph books
- Dice

3.3.6.5 References

J.K BackHouse, S.P.T Houldsworth; *Pure mathematics I*
J.K BackHouse, S.P.T Houldsworth; *Pure mathematics I*
John Bird (2006); *High engineering mathematics;* Elsevier Ltd
Purna Chandra Biswal (2009); *Probability and Statistics;* Prentice hall Of India Pvt Ltd
Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry;* Wesley –London

3.3.7 Learning Outcome No 6: Collect, organize and interpret statistical data

3.3.7.1 Learning Activities

Learning Outcome No 6: Collect, organize and interpret statistical data	
Learning Activities	**Special Instructions**
6.1. Identify workplace issue requiring investigation 6.2. Determine audience, population and sample unit 6.3. Identify data to be collected 6.4. Select data collection method 6.5. Collect and organize appropriate statistical data 6.6. Illustrate data in appropriate formats 6.7. Compare the effectiveness of different types of graphs 6.8. Calculate the summary statistics for collected data 6.9. Interpret the results and findings 6.10. Check data to ensure that it meets the expected results and content 6.11. Extract and interpret results and information from tables, graphs and summary statistics. 6.12. Investigate mathematical language and symbols are used in reporting results	Individual presentation

3.3.7.2 Information Sheet No3/LO6: Collect, organize and interpret statistical data

Introduction

This learning outcome describes the collection, organization, presentation and interpretation of data for making valid decisions. It also looks into methods of selecting a sample from a population as well as errors made during sampling.

Definition of key terms

Data: Is facts and statistics collected together with desired aim to help make conclusions
Population: Is the whole number of people or objects in a given area
Sample: Is a carefully chosen part of the population
Census: This is complete official count of a given population
Sampling: Is the process of selecting a sample from a population

Content/Procedures/Methods/Illustrations

6.1 Workplace issue requiring investigation are identified

Every activity requires data for statistical reasons. The reason why we collect data is

- To get tangible returns.
- To grow existing database.
- Improving of existing products.
- For customer feedback.
- To understand the audience.
- Increase engagement

6.2 Audience / population / sample unit is determined

There are two main measures of a population. This includes

- Population size (the number of individuals)
- Population density: this is the number of individuals per unit area or volume.

A sample is usually used to help get the characteristics of a given population. The reason why a sample is used is because sampling method is cheaper and faster. There are various methods of choosing a sample from a population. These methods are random and non-random.

Random or probability methods are

- Simple random sampling
- Stratified sampling
- Systematic sampling
- Multistage sampling etc.

Non-random methods are

- Judgment sampling
- Quota sampling
- Cluster sampling etc.

6.3 Data to be collected is identified

Data collection is a process of gathering and measuring information on various variables of interest, which enables one to answer stated research question, test of hypothesis concerning a certain situation. Data can be classified according to the different measurement scales. They are:

a) Nominal data

This is data which has been measured at the weakest and lowest level of measurements. Such a level entails a classification of data qualitatively or by name (hence the term nominal). If you observe a group of students in a college, they fall in two categories, male or female. We cannot add or subtract if no numbers have been assigned to them. If this data has not been coded in a personal file and be available to computer programming, one might use numeral zero (0) to represent one (1) to represent married people.

b) Ordinal data

This type of data is acquired in a way that categorizes it, so that each category has a known or specific relationship to every other category. There are some underlying dimensions with respect to the quality of each data category. Like ranking of data along a specific scale. Measure like big, bigger, and biggest or disagree strongly, disagree neutral, agree, agree strongly.

Other examples of Ordinal classifications are like secretary grade 1, secretary grade 2, secretary grade 3 etc.

c) Interval data

This kind of data is acquired through a process of measurement where equal measuring units are employed. The movement in magnitude between one measure to the one above it or below is identical in the subject population under consideration. In interval data, the quantity zero exists in theory but does not matter in practice. E.g., zero degrees Fahrenheit does not represent the complete absence of temperature.eg consider the Fahrenheit the scale of temperature. The difference between 30 degrees and 40 degrees represent the same temperature difference as the difference between 80 degrees and 90 degrees. This is because each 10 degrees interval has the same physical meaning.

d) Ratio data

Ratio data have all the characteristics of interval data. The zero position indicates the absence of the quality being measured. The Fahrenheit scale for temperature has an arbitrary zero point and is therefore not a ratio scale. However, zero on Kelvin scale is absolute zero hence a ratio scale.

6.4 Data collection method is selected

There are various data collection methods. They are categorized into either for collecting primary or secondary data. Primary data is data which is collected for the first time by the investigator. There are various methods of collecting primary data. They include;

- Observation
- Interviews
- Questionnaires
- Sampling etc.

6.5 Appropriate statistical data is collected and organized

Large amounts of data normally should be summarized. In preparation for the summarization, it is often useful to distribute the data into groups or classes or categories and then determine the number of individuals belonging to each group, class or category. Such a number is called the **class frequency**. If the data is arranged in classes together with the corresponding class frequency for each class in a table, we call such a frequency distribution or frequency table.

6.6 Data is illustrated in appropriate formats

Data can be presented in tables. Tables are the formats in which most numerical data are initially stored and analyses. Other method in which data is represented are;

- Bar graphs
- Line graph
- Pie chart
- Histogram (frequency polygon)
- The stem and leaf plot

The stem and leaf plot

This is a method of representing data which was devised by Turkey in 1977 and it involves three essential steps.

i. Find the range of the given data.
ii. Choose an appropriate class width.
iii. Make a new table which looks like a histogram, but which preserves the original data.

6.7 The effectiveness of different types of graphs are compared

There were times when data was limited and thus data presentation was also limited. Currently there is massive data and thus there are numerous methods of presenting data.

Tables are the best way to present data for reference purposes and can include very complex information. This type of information can be presented clearly by using an appropriate label and displaying the data in suitable groups. Bar graphs are good for comparisons, while line charts are good for relationships and distributions.

Pie charts are only used for simple compositions but cannot be used for comparisons or distributions.

Data is best presented in a consolidate manner and thus helps leaders to get an overview of most of the important findings of your work without having to go through the entire manuscript. Visuals are introduced to present data due to clarity. Therefore, various types of graphs for data presentation are used for comparing different set off data.

6.8 The summary statistics for collected data is calculated

Summary statistics for data is calculated by use of measures of central tendency and measures of dispersion. Measures of central tendency include;

i. **Arithmetic mean**

Given the set of numbers $x_1, x_2, \ldots, x_n$ the mean which is dented by $\bar{X}$ is defined by

$$\frac{X_1 + \cdots + X_n}{n} = \frac{\sum x_i}{n}$$

However if the numbers $x_1, x_2, \ldots, x_n$ occur with frequencies $f_1, f_2, \ldots, f_n$ then the mean is given by $\bar{X} = \frac{\sum f_i x_i}{\sum f_i}$

ii. **Mode**

This is an observation which occurs more times than any other observation.

iii. **Median**

If a set of data is arranged in ascending or descending order the median is the middle number if the number of observations is old.

Measures of dispersion

This measures the extent to which a set of data spread about an average are the mean. There are various measures of dispersion.

i. **Range**

This is the difference between the largest and the smallest values in each data

ii. **Mean Absolute Deviation (Mad)**

The sum of the deviation is zero. Since the total of the deviation is zero the mean of the absolute deviation is the one that is used as a measure of dispersion.

$$MAD = \frac{\sum |X_i - \bar{x}|}{n}$$

iii. **Inter Quartile Range**

This is the difference between 3rd quartile and 1st quartile

$Q_3 - Q_1$

iv. **Semi-interquartile range**

This is given by $\frac{1}{2}(Q_3 - Q_1)$

v. **Coefficient of Quartile Deviation**

A relative measure of dispersion based on the quartile deviation is called the coefficient of quartile deviation. It is defined as

Coefficient of Quartile Deviation

Coefficient of quartile deviation$=\frac{\frac{Q_3 - Q_1}{2}}{\frac{Q_3 + Q_1}{2}} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

It is pure number free of any units of measurement. It can be used for comparing the dispersion

vi. Variance

The variance of a set of data is given by

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

and the standard deviation is the square root of variance.

$$\sigma^2 = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

For data with frequencies the variance is given by

$$\sigma^2 = \frac{\sum_{i=1}^{n} f_i(x_i - \bar{x})^2}{\sum f_i} \quad \text{or } \sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

6.9 The results / findings are interpreted

Data interpretation refers to the implementation of processes through which data is reviewed for the purpose of arriving at an information conclusion. The information of data assigns a meaning to the information analyzed and determines it significance and implications.

Analysis involves estimating the values of unknown parameters of the population and testing of hypothesis for drawing inferences.

6.10 Data is checked to ensure that it meets the expected results and content

To check whether the data meet the expected results and content, the following should be considered;

a) **Error**: This is the collective noun for any departure of the results from the true value. Analytical error can be;
   - Random or unpredicted
   - Systematic or predicted
   - Constant
   - proportional

b) **Accuracy:** This is the trueness or closeness of analytical result to the value

c) **Precision:** The closeness with which results of replicate analyses of a sample agree

d) **Bias:** The consistent deviation of analytical results from the true value caused by systematic errors in a procedure

6.11 Information from the results including tables, graphs and summary statistics is extracted and interpreted

When data is presented on all forms of data presentations, information can be extracted from these forms. Some of the information that can be extracted is the measures of central tendency (mean, mode, variance etc.). Other measures that can be extracted are measures of dispersion which includes (range, quartiles, etc.)

6.12 Mathematical language and symbols are used to report results of investigation

Statistics has become the universal language of the sciences and data analysis which leads to powerful results. There is a wide variety of symbols used in statistics. The Greek letters are usually used. Some of these are

- Σ summation
- μ population mean
- β beta
- σ standard deviation
- π product

Conclusion

This learning outcome is based collection, organization, summarization, and presentation of data to make varied decisions and conclusions.

Further Reading

Purna Chandra Biswal (2009); *Probability and Statistics;* Prentice-hall Of India Pvt Ltd

Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry;* Wesley –London

3.3.7.3 Self-Assessment

Written Assessment

1. A numerical value used as a summary measure for a sample, such a sample mean is known as?
    a) Population parameter
    b) Sample parameter
    c) Population means
    d) Sample statistics
2. Mμ is an example of
    a) Population parameter
    b) Sample statistics
    c) Population variance
    d) Mode

3. The sum of percentages frequencies for all classes will always equal to
    a) One
    b) the number of classes
    c) the number of items in the study
    d) 100
4. Consider the table below

0-9	10-19	20-29	30-39
40	50	70	40

    What is the class size for this distribution?
    a) 9
    b) 10
    c) 11
    d) Varies from class to class
5. The difference between the largest and the smallest value is the?
    a) Variance
    b) Interquartile range
    c) Range
    d) Coefficient of variation
6. The age of distribution of 70 workers in a factory is given below.

Ages (years)	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Frequency	2	10	12	X	10	8	2	3

    i. Find the value of X.
    ii. Calculate the mode, median and quartile deviation of the above data
7. For a throw of a fair tetrahedron die with its faces marked 1,2,3,4 respectively, the score X represents the number of the face which lands on the floor. Find $\boldsymbol{E}\ (\boldsymbol{X})$and variance of x
8. The median and standard deviation of 50 observations are 25 and 6 respectively. Find the second Pearson measure of skewness if the sum of the observations is 900. Comment on your answer.
9. The expenditure of 100 persons is given below

Expenditure ($)	0-10	10-20	20-30	30-40	40-50
Number of persons	14	f_1	27	f_2	15

The mode of the distribution is 24. Show that;

i. The values of f_1 and f_2 are 23 and 21 respectively.
ii. Obtain the median of the distribution.

Oral Assessment

1. State four primary data collection methods
2. Differentiate between a sample and a population.

Practical Assessment

Correct statistic data of mathematics trainee attendance in your class, organize, analyze, and present it in an orderly manner.

3.3.7.4 Tools, Equipment, Supplies and Materials

- Calculators
- Rulers, pencils and Erasers
- Charts with presentation of data
- Graph books
- Dice

3.3.7.5 References

J.K BackHouse, S.P.T Houldsworth; *Pure mathematics I*
J.K BackHouse, S.P.T Houldsworth; *Pure mathematics I*
John Bird (2006); *High engineering mathematics;* Elsevier Ltd
Purna Chandra Biswal (2009); *Probability and Statistics;* Prentice hall Of India Pvt Ltd
Thomas G.B and Finney R.L (2008); *Calculus and Analytical geometry;* Wesley –London

3.3.8 Learning Outcome No 7: Use routine formula and algebraic expressions for work

3.3.8.1 Learning Activities

Learning Outcome No 7: Use routine formula and algebraic expressions for work	
Learning Activities	**Special Instructions**
7.1. Demonstrate the understanding of informal and symbolic notation, representation and conventions of algebraic expressions 7.2. Develop simple algebraic expressions and equations 7.3. Operate on algebraic expressions 7.4. Simplify algebraic expressions 7.5. Do substitution of simple routine equations 7.6. Identify and comprehend routine formulas used for work tasks 7.7. Evaluate routine formulas by substitution 7.8. Transpose routine formulas 7.9. Identify and use appropriate formulas for work related tasks 7.10. Check and use outcomes of calculated result	• Individual presentation/ Assignment • Group work

3.3.8.2 INFORMATION SHEET No.3 /LO7: Use routine formula and algebraic expressions for work

Introduction

The learning outcome covers the use of routine formula and algebraic expressions for work. It entails understanding of informal and symbolic notation, representation and conventions of algebraic expressions. In addition, it also involves operations on algebraic expression by substituting and simplification of the algebraic expressions.

Definition of key terms

Formula: This is a mathematical relationship or symbols expressed as a rule.

Linear equation: Refers to an equation between two variables that gives a straight line when plotted on a graph.

Content/Procedures/Methods/Illustrations

7.1 **Informal and symbolic notations of algebraic expressions**

Special notations are used in algebra which is a critical aspect. Symbols, also known as variables, are used to represent numbers. Variables are represented by letters such as a and b or x, y, z. Special symbols such as =, ≥, ≤ among others are used to denote relationships. Other symbols are used to denote the familiar arithmetic operations: + for addition; -for subtraction; or x for multiplication; ÷ for division; and for positive square root. Superscripts such as the 2 in x^2, are used to denote repeated multiplications. For example (x^2 is a shorthand for$x.x$, and x^3 is a shorthand for$x.x.x$). The symbol π is used to represent the special number 3. 142. Thus algebra is full of expressions such as

$$3 \times 2 + 2\text{x} + 4$$

$$\frac{a+b}{2}$$

$$a + c$$

And equations (pairs of expressions separated by an equality sign, (=)

$$y = 2x^3 + 2x + 7$$

$$mn = \text{nm}$$

$$(x + a)(x - a) = x^2 - a^2$$

$$\text{A} = r^2$$

7.2 Simple algebraic expressions

In algebra, just like arithmetic, numbers are used as symbols for values and quantities but in case where numbers are unknown, we use letters to represent them. For example, when we say that a teacher went to class with 5 books, we know the exact number of books he/she had. Suppose we say that the teacher went to class with x books. What does x represent? The letter p can represent a number.

Examples

1. Geoffrey had k books and John had g books. How many books did the two have?

Solution: Total number books equals (k + g) books.

2. John had q mangoes and divided from equally among his t children. How many mangoes did each child get?

Solution: There are q mangoes, and there are t children, each child got $\frac{q}{t}$ mangoes.

Note that you should state what the letters represent when you form an algebraic expression from a mathematical problem. You can choose any letter except the letter o. This is because this letter can be mistaken for the number 0.

7.3 Operations on algebraic expression
Subtraction and Addition
In addition, or subtraction is done by checking the like terms. Like terms refers to same variables or variables raised in the same power.

Example 1: $x + 3x + 5y - 2 + 5$
Group all the similar terms

$$x + 3x + 5y - 2 + 5$$

Combine all like terms and same variables.

$$x + 3x + 5y - 2 + 5$$

$$4x + 5y + 3$$

Example2: $5x + x^2 - 3x + 5$
Group all similar terms

$$5x + x^2 - 3x + 5$$

Combine like terms

$$x^2 + 5x - 3x + 5$$

$$x^2 + 2x + 5$$

Observe that x^2 is not added to$2x$.

x^2Is in second degree while $2x$ is not. They are same variable but different degree.

Multiplication of algebraic Expression
Multiplying an algebraic expression involves distributive property and index law.

Example 1: 5 multiplied to x equals 5x.
Example 2: $x(x - 1)$

$$x(x - 1)$$

x(x) + x (1) $\times (x) - x(1)$

$$x^2 + x$$

Example 3: $(x-2)^2$
$(x-2)^2$ This equation means that (x – 2) is multiplied with (x – 2)

$$(x-2)(x-2)$$

$$x(X)+x(-2)-2(x)-2(-2)$$

x^2 – 2x – 2x + 4

x^2 – 4x + 4

Division of algebraic expression

Example: $30a^3 3b^2$ divided by $5a^2 2b^3$

$$\frac{30a^3 3b^2}{5a^2 2b^3}$$

$$\frac{30aaabb}{5aabbb}$$

Divide the constant to obtain 6ab

7.4 Simplifying algebraic expression
Simplification of an algebraic expression means writing it in the most efficient manner, without changing the value of the expression. This mainly involves collecting like terms. The rule here is that only like terms can be added together.

Like (or similar) terms
Like terms are those terms which contain the same powers of same variables.
Examples of like terms:

$3x$, $6x$, and $-4x$
$3yx^2$, $-6yx^2$
$8xy^2$, $5y^2 x$, and $3xy^2$
Examples of unlike terms
Xy^2 and $x^2 y$
2x+3y

Combining like terms

$$3x^2+5x^2=(3+5)x^2=8x^2$$

Distributive law was used in reverse we undistributed a common factor of x^2 from each term.
Example: $x^2 + 2x + 3x^2 + 2 + 4x + 7$

Starting with the highest power of x, we see that there are four x-squared in all ($1x^2 + 3x^2$).

Then we collect the first powers of x and see that there are six of them ($2x + 4x$).

The only thing left is the constants 2 + 7 = 9. Putting this all together we get

$x^2 + 2x + 3x^2 + 2 + 4x + 7$

$= 4x^2 + 6x + 9$

Parentheses

Parentheses must be multiplied out before collecting like terms**.** If there is some factor multiplying the parentheses, then the only way to get rid of the parentheses is to multiply using the distributive law.

Example: $3x + 2(x – 4)$

$3x + 2x – 8$

$= 5x – 8$

7.5 Substitution into simple routine equations

Examples

When x=2,
What is $10/x + 4$?
Replace 2 where x is:
10/2 + 4 = 5 + 4 = 9

When x=5,
What is $x + x/2$?
Replace 5 where x is:
5 + 5/2 = 5 + 2.5 = 7.5

If x=3 and y=4,
What is $x^2 + x$y?
Replace 3 where x is, and 4 where y is:
3^2 + 3×4 = 3×3 + 12 = 21

If x=3
What is $x^2 + x$y?
Replace 3 where x is:
32 + 3y = 9 + 3

7.6 Linear equation

A linear equation looks like any other equation. It is made up of two expressions set equal. A linear equation is special because:

- It has one or two variables (x and y).
- No variable in a linear equation is raised to a power greater than 1 or used as the denominator of a fraction.
- When you find pairs of values that make the linear equation true and plot those pairs on a coordinate grid, all the points for any one equation lie on the same line. Linear equations graph as straight lines.

A linear equation in two variables describes a relationship in which the value of one of the variables depends on the value of the other variable. In a linear equation in x and y, x is called x is the independent variable and y depends on it. We call "y" the dependent variable. When you assign a value to the independent variable, x, you can compute the value of the dependent variable, "y". You can then plot the points named by each (x, y) pair on a coordinate grid.

7.7 Slope

Slope is how steep the line is with respect to the *y* axis.

Procedures for plotting a straight line

Pick any two points on the line.

Find how fast *y* is changing, subtract the *y* value of the second point from the *y* value of the first point ($y_2 - y_1$).

Find how fast *x* is changing, subtract the *x* value of the second point from the *x* value of the first point ($x_2 - x_1$).

Find the rate at which *y* is changing with respect to the change in *x*, write your results as a ratio: $(y_2 - y_1)/ (x_2 - x_1)$.

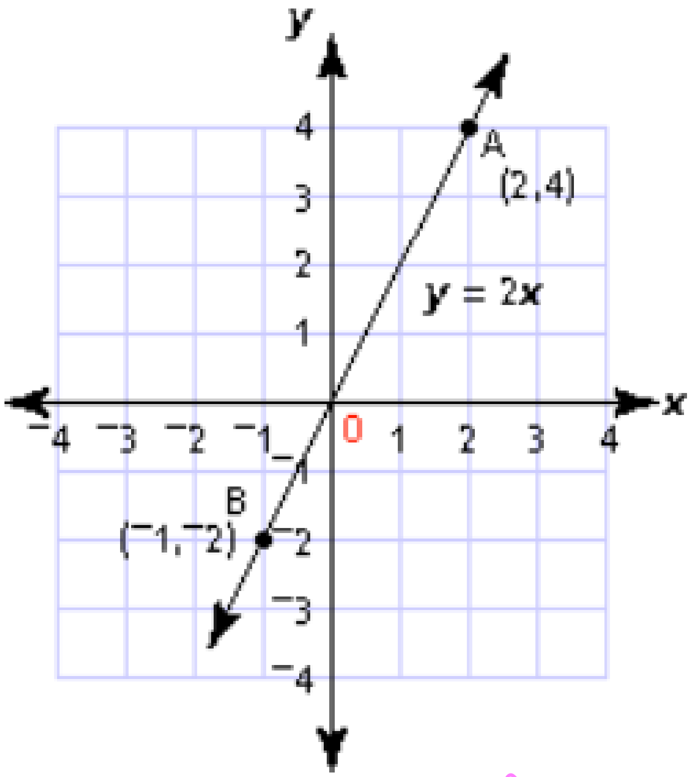


Figure 19. Plotting a straight line

If we designate Point *A* as the first point and Point *B* as the second point, the slope of the line is (‐2 – 4)/ (- 1 – 2) = -6/-3. Or 2.

If we designate Point *B* as the first point and Point *A* as the second point, the value of the slope is the same: (4 – -2)/(2 – -1) = 6/3, or 2. The value is constant along the line and if you pick different other points will give u same result. The number obtained is a gradient.

7.8 Intercept form

The slope-intercept form of a linear equation is $y = mx + b$.

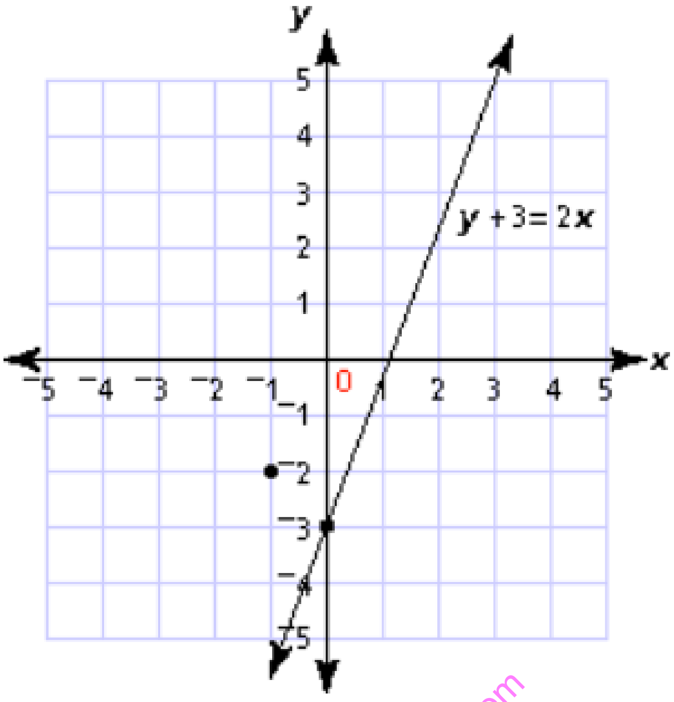


Figure 20. Slope-intercept form of a linear equation

In the equation, x and y are the variables. The numbers m and b give the slope of the line (m) and the value of y when x is 0 (b).

The value of y when x is 0 is called the y-intercept because (0, y) is the point at which the line crosses the y axis.

You can draw the line for an equation in this form by plotting (0, b), then using m to find another point. For example, if m is 1/2, count +2 on the x axis, then +1 on the y axis to get to another point (1, b + 2)

The equation for this line is $y + 3 = 2x$. In slope-intercept form, the equation is $y = 2x – 3$. You can see that the slope m = 2 and the slope really is 2 since for every positive2 change in y, there is a positive1 change in x.

Now look at b in the equation: -3 should be the y value where x = 0 and it is.

7.9 Zero Slope

When there is no change in *y* as *x* changes, the graph of the line is horizontal. A horizontal line has a slope of zero.

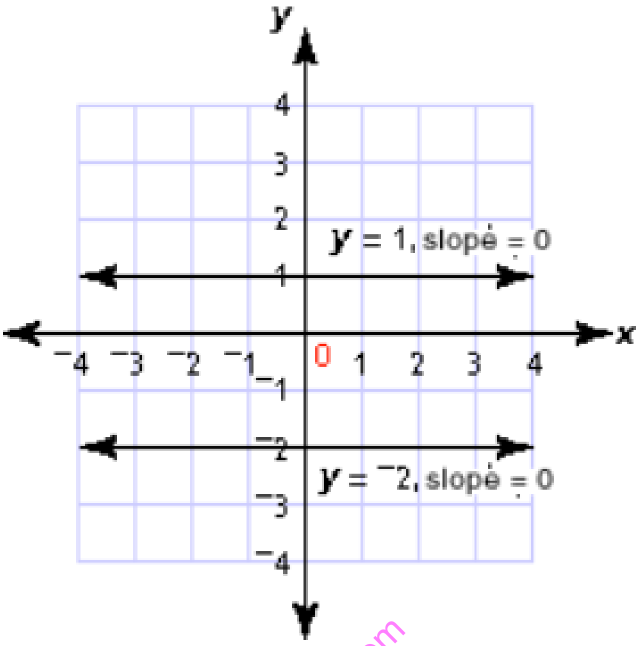


Figure 21. Zero slope

7.10 Undefined Slope

When there is no change in x as y changes, the graph of the line is vertical. You could not compute the slope of this line, because you would need to divide by 0. These lines have undefined slope.

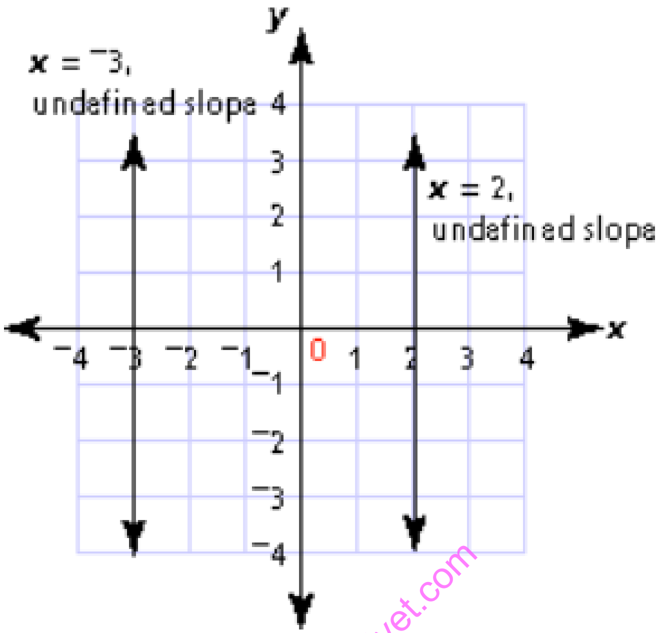


Figure 22. Undefined slope

Conclusion

This unit deals with forming, simplifying and solving for the unknowns in a given equation. The unit also deals with solving linear equation.

Further Reading

McNeil, N. M., Weinberg, A., Hattikudur, S., Stephens, A. C., Asquith, P., Knuth, E. J., &Alibali, M. W. (2010). *A is for apple: Mnemonic symbols hinder the interpretation of algebraic expressions. Journal of Educational Psychology*, *102*(3), 625.

3.3.8.3 Self-Assessment

Written Assessment

1. If x=−3, what is the value of 2x−1x+1?
   a) −4
   b) −2
   c) 2
   d) 323
2. Which of the following is equivalent to 3a+4b− (−6a−3b)?
   a) 16ab
   b) –3a+b
   c) –3a+7b
   d) 9a+b
   e) 9a+7b

3. If 2(x−5) =−11, then x=?
   a) −212
   b) −8
   c) −112
   d) −3
   e) −0.5

4. What is the slope of the line with the equation 2x+3y+6=0?
   a) −6
   b) −3
   c) −2
   d) −23
   e) 23
5. For all x≠−4, which of the following is equivalent to the expression below?

$$\frac{x^2 + 12x + 3}{x + 4}$$

   a) x+3
   b) x+8
   c) x+11
   d) x+16
   e) x+28
6. What is a linear equation?
7. Provide three characteristics of a linear equation
8. Using a graph paper, draw a line y=x

3.3.8.4 Tools, Equipment, Supplies and Materials

- Calculators
- Graph books
- Group discussions
- Practical examples

3.3.8.5 References

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3.3.9 Learning Outcome No 8: Use Common functions of a scientific calculator for work

3.3.9.1Learning Activities

Learning Outcome No 8: Use Common functions of a scientific calculator for work	
Learning Activities	**Special Instructions**
8.1. Locate required numerical information to perform tasks 8.2. Determine the order of operations and function keys necessary to solve mathematical calculations 8.3. Identify and use function keys on a scientific calculator 8.4. Estimations are referred to check reasonableness of problem-solving process 8.5. Appropriate mathematical language, symbols and conventions are used to report results	• Individual assignment • Group work

3.3.9.2 Information Sheet No.3 /LO8: Apply a wide range of mathematic calculations for work

Introduction

The learning outcome covers the common functions of a scientific calculator for work. It entails locating the required numerical information to perform tasks, determining the order of operations and functions of keys necessary to solve mathematical calculation, identifying and using function keys on a scientific calculator, and reporting results using appropriate mathematical language, symbols and conventions.

Definition of key terms

Calculator: It is an electronic gadget that is portable and typically used to perform calculations ranging from basic arithmetic to complex mathematics.

Numerical: Refers to expressions involving numbers or associated with numbers.

Problem solving: This is a process of trying to find solution to a problem through working in depth to extract details to steer towards getting a better solution.

Content/Procedures/Methods/Illustrations

8.1 Numerical information

Numerical information is what can be measured or something that has measurements. It is often collected as a number form, even though there are other types of numbers that may appear in the number from. For instance, the number of people that attended the movie theater over the course of the month. Numerical data can be identified by just seeing or checking whether the data can be added together. Besides, an individual should be able to perform any operation of mathematics on the numerical data. The data can also be rearranged in ascending and descending order. Information can only be numerical if the answers can also be represented in fraction and decimal form. If the numerical information is put to various groups, then it is considered categorical. If you were to measure the height of four ladders, you could average the heights, you could add the heights, and you could put them in ascending or descending order. That's because the height of the ladders is numerical data

8.2 Order of operations

When you have a math problem that involves more than one operation—for example, addition and subtraction, or subtraction and multiplication—which do you do first?

Example 1:

6 – 3 x 2 =?

In cases like these, we follow the order of operations. The order in which operations should be done is abbreviated as PEMDAS:

- Parentheses
- Exponents
- Multiplication and Division (from left to right)
- Addition and
- Subtraction (from left to right)

In the above example, we're dealing with multiplication and subtraction. Multiplication comes a step before Subtraction, so first we multiply 3 x 2, and then subtract the sum from 6, leaving 0.

Example 2:

30 ÷ 5 x 2 + 1 =?

There are no Parentheses.

There are no Exponents.

We start with the Multiplication and Division, working from left to right.NOTE: Even though Multiplication comes before Division in PEMDAS, the two are done in the same step, from left to right. Addition and Subtraction are also done in the same step.

30 ÷ 5 = 6, leaving us with 6 x 2 + 1 =?

6 x 2 = 12, leaving us with 12 + 1 =?

We then do the Addition: 12 + 1 = 13

Note that if we'd done the multiplication before the division, we'd have ended up with the wrong answer:

5 x 2 = 10, leaving 30 ÷ 10 + 1 =?
30 ÷ 10 = 3, leaving 3 + 1 =?
3 + 1 = 4 (off by 9!)
One last example for advanced students, using all six operations:

8.3 Functional keys on a scientific calculator

The buttons used on the scientific calculator are labelled differently depending on the manufacturer. However, below are some of the common functions of keys and the meaning.

Table 6: Functional Keys

Operation	Mathematical Function
+	Addition or plus
-	Subtraction or minus: note that there is a different button that can make a positive number to be negative, often marked as (-) or negation (NEG).
×	Multiply or ties
/ or ÷	Over, divide, division by
^	Raised to the power of
y^x or x^y	X raised to power y or y raised to the power x
√	Square root
e^x	Exponent
LN	Log of or natural logarithm
SIN	sine function
SIN^{-1}	Arcsine or inverse sine function
COS	Cosine function
COS^{-1}	Arccosine or inverse cosine function
TAN	Tangent function
TAN^{-1}	Arctangent or inverse tangent function
()	Parenthesis
Store	Place a number in memory for later use
Recall	Recover the number from memory for immediate use

Identification of keys to use depends on the kind of numerical information offered and operations instructed. For instance, 3+5× 4 you know, according to the order of operations, the 5 and the 4 should be multiplied by each other before adding the 3. Your calculator may or may not know this. If you press 3 + 5 x 4, some calculators will give you the answer 32 and others will give you 23 (which is correct). Find out what your calculator does. If you see an issue with the order of operations, you can either enter 5 x 4 + 3 (to get the multiplication out of the way) or use parentheses 3 + (5 x 4).

8.4 Estimations

Estimation is a procedure by which we assign a numerical value or numerical values to the: population parameter based on the information collected from a sample, population parameter based on the information collected from a population sample, statistic based on the information collected from a sample, statistic based on the information collected from a population. In mathematics estimations are commonly done by rounding off or expressing numbers to the nearest unit, ones, tens, hundreds, thousands, etc. Example: rounding off the 345 to the nearest hundreds we obtain 300.

8.5 mathematical language, symbols and convention

Mathematical convention

A mathematical convention is a fact, notation, name or usage which is a consensus among mathematicians. For example, the fact that one evaluates multiplication before addition in the expression 3+7×8 is merely conventional. There is nothing inherently significant about the order of operations.

Example 1

	Examples	Meaning
Start of the alphabet	a, b, c,….	Fived values (constants)
From i to n	i, j, k, l, m, n	Positive integers
End of the alphabet	…..x, y, z	variables

Example 2

$y = ax + b$ It is assumed a and b are fixed values, x and y are assumed to vary or change (variables)

Mathematical symbols

Mathematics uses more of symbols rather than words:

- There are the 10 digits: 0, 1, 2, ... 9
- There are symbols for operations: +, −, ×, /,
- And symbols that "stand in" for values: x, y,
- And many special symbols: π, =, <, ≤,

Conclusion

The learning outcome equips learners with competency numerical skills of handling numbers. It further provides basic knowledge on handling a scientific calculator.

Further Reading

Kincaid, D., Kincaid, D. R., & Cheney, E. W. (2009). *Numerical analysis: mathematics of scientific computing* (Vol. 2). American Mathematical Soc.

3.3.9.3 Self-Assessment

Written Assessment

Find:

1. $-10 + 3 - (-4) + 5 =$
    a) 2
    b) -12
    c) -4
    d) 16
2. $-96 \div -6 \div 8 =$
    a) 2
    b) 12
    c) -12
    d) -2
3. $5x - 2 - (8 - 12) + 16 \div -8 =$
    a) 6
    b) -8
    c) -16
    d) -6

4. Define the term numerical information.
5. Provide some of the symbols used in mathematics.

3.3.9.4 Tools, Equipment, Supplies and Materials

- Calculators
- Rulers, pencils and Erasers
- Charts with presentation of data
- Graph books
- Dice

3.3.9.5 References

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